## Cambridge Assessment International Education

Cambridge International General Certificate of Secondary Education

## CANDIDATE NAME

CENTRE NUMBER


CANDIDATE NUMBER $\square$

## CAMBRIDGE INTERNATIONAL MATHEMATICS

Additional Materials: Graphics Calculator

## READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO NOT WRITE IN ANY BARCODES.
Answer all the questions.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 24 .

Answer all the questions.

## INVESTIGATION

## JUMPING FROGS

This investigation looks at the number of different ways that a frog can jump between stones in a line.
The stones are always 1 unit apart.
The frog always jumps

- from left to right
- from the first stone to the last stone.

The diagram shows a frog sitting on a stone in a pond.


This frog has a jump length of $\mathbf{1}$ unit. This is enough to move from one stone to the next stone.
There is only 1 way to jump between two stones.


There is only 1 way to jump between three stones.

When there are more than three stones in a line, there is always only 1 way for this frog to jump from the first stone to the last stone.

1 A different frog has a maximum jump length of 2 units.
There is still only 1 way to jump between two stones.


There are now 2 ways to jump between three stones.

(a) Complete the diagrams below to show the 3 ways that this frog can jump between four stones.



(b) Complete the diagrams below to show the 5 ways that this frog can jump between five stones.












(c) The table shows the number of ways to jump between stones when the maximum jump length is 2 units.

| Number of stones | Number of ways |
| :---: | :---: |
| 2 | 1 |
| 3 | 2 |
| 4 | 3 |
| 5 | 5 |
| 7 | 13 |
| 7 |  |

The numbers in the last column of the table form a sequence.
(i) Write down the addition rule to find further terms in this sequence.
$\qquad$
$\qquad$
(ii) Use your rule to show that the number of ways for this frog to jump between 8 stones is 21 .

2 Another frog has a maximum jump length of $\mathbf{3}$ units.
There is only 1 way for this frog to jump between two stones.


There are 2 ways to jump between three stones.

(a) There are 4 ways to jump between four stones.

These are the same 3 ways as in question 1(a) and 1 new way.
Draw the new way on the diagram below.

(b) There are 7 ways to jump between five stones.

These are the same 5 ways as in question 1(b) and 2 new ways.
Draw the 2 new ways on the diagrams below.

(c) The table shows the number of ways to jump between stones when the maximum jump length is 3 units.

| Number of stones | Number of ways |
| :---: | :---: |
| 2 | 1 |
| 3 | 2 |
| 4 | 4 |
| 5 | 7 |
| 6 | 13 |
| 7 | 24 |

The numbers in the last column of the table form a sequence.
(i) Write down the rule to find further terms in this sequence.
$\qquad$
$\qquad$
(ii) Use your rule to find the number of ways for this frog to jump between 8 stones.

3 The table shows the number of ways to jump between 2 to 8 stones when the maximum jump length is 1 to 6 units.

| Number of stones | Maximum jump length |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 unit | 2 units | 3 units | 4 units | 5 units | 6 units |  |
| 2 | 1 | 1 | 1 | 1 | 1 | 1 |  |
| 3 | 1 | 2 | 2 | 2 | 2 | 2 |  |
| 4 | 1 | 3 | 4 | 4 | 4 | 4 |  |
| 5 | 1 | 5 | 7 | 8 | 8 | 8 |  |
| 6 | 1 | 8 | 13 | 15 | 16 | 16 |  |
| 7 | 1 | 13 | 24 | 29 | 31 | 32 |  |
| 8 | 1 | 21 |  |  |  |  |  |

(a) Complete the table.
(b) The numbers in the column for maximum jump length of 4 units form a sequence.

Write down the rule for finding further terms in this sequence.
$\qquad$
$\qquad$
(c) The numbers in the column for the maximum jump length of 5 units also form a sequence.

Write down the rule for finding further terms in this sequence.
$\qquad$
$\qquad$

4 This is the same table as in question 3, with some numbers missing.
Some of the numbers have been written as powers of 2 .

| Number of stones | Maximum jump length |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 unit | 2 units | 3 units | 4 units | 5 units | 6 units |  |
| 2 | $2^{0}$ |  |  |  |  |  |  |
| 3 | 1 | $2^{1}$ | $2^{1}$ | $2^{1}$ | $2^{1}$ | $2^{1}$ |  |
| 4 | 1 | 3 |  |  |  |  |  |
| 5 | 1 | 5 | 7 |  |  |  |  |
| 6 | 1 | 8 | 13 | 15 |  |  |  |
| 7 | 1 | 13 | 24 | 29 | 31 |  |  |
| 8 | 1 | 21 |  |  |  |  |  |

(a) Copy your values from the table in question $\mathbf{3}$ into the row for 8 stones.

Complete the table by writing in the remaining values as powers of 2 .
(b) Use patterns from your table to write down, as a power of 2, the number of ways of jumping between 30 stones when the maximum jump length is 29 units.
(c) Find the number of ways of jumping between 30 stones when the maximum jump length is 28 units.
(d) From the table, the largest number of ways of jumping between 3 stones is $2^{1}$. Write down, as a power of 2 , the largest number of ways of jumping between
(i) 5 stones,
(ii) 20 stones,
(iii) $x$ stones.
(e) (i) Find the smallest jump length that gives your answer in part (d)(i).
(ii) Find an expression in terms of $x$ for the smallest jump length that gives your answer in part (d)(iii).

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