ADDITIONAL MATHEMATICS (US)

Paper 0459/01
Paper 1

Key Messages

In order to be successful in this paper, candidates need to have covered, and be confident with, the entire syllabus.

General Comments

There were many apparent gaps in candidates' knowledge. These are indicated in the comments on individual questions. Even in topics that were familiar to them, many candidates tended to show a lack of ability in handling the relevant mathematics.

Presentation was often poor, with working scattered about the page and answers not clearly indicated. Candidates should be encouraged to set out their working in a clear, logical order; this ensures that if they do not have the correct answer, method marks can be awarded for correct working.

Some candidates lost marks through premature rounding. Candidates should be encouraged to work in fractions or in decimals to several more decimal places than are required for the answer. If the question asks for an intermediate answer and the candidate gives this answer to three significant figures, more than three significant figures should be used when this answer is used in further working.

Comments on Specific Questions

Question 1

Candidates mostly attempted to form three simultaneous equations and to solve them. However, they were unable to handle the algebra required to solve these. It had been expected that candidates would use the simpler method of using the points of intersection with the x-axis to find the factors of the given polynomial.

Answer: $y = x^3 - 2x^2 - 5x + 6$

Question 2

Most candidates successfully factorised the numerator and denominator of the main fraction, but they did not seem to be aware that dividing a fraction by an expression is the same as multiplying the denominator by this expression.

Answer: $\dfrac{1}{(x-2)(x-1)}$

Question 3

(i) Few candidates scored any marks on this question. The majority of candidates generally found three separate probabilities, i.e. P(female), P(junior) and P(junior female), and either incorrectly added these or gave three separate answers.

(ii) Most candidates did not understand what was required in this part question. Candida have found P(F) and $\mathrm{P}\left(F|J\right)$ and, as they are the same, conclude that F and J are indepen

Answers: **(i)** $\frac{97}{132}$ **(ii)** Yes

Question 4

(i) Candidates did not appear to be aware of the need for compatibility of matrices before multiplying.

(ii) Some candidates achieved the correct numerical answer, but only by using bogus matrix methods. Some of these lost the only mark available to them, because of premature rounding.

Answers: **(i)** $\mathbf{A} = (2.25 \ \ 2.35)$, $\mathbf{C} = \begin{pmatrix} 8500 \\ 9400 \end{pmatrix}$ **(ii)** \$780.12

Question 5

(i) About half of the candidates understood what was required and succeeded in finding the correct number. Many candidates made no realistic attempt at all.

(ii) This part was found difficult, with many candidates omitting it completely.

(iii) Many candidates omitted this part. The rest correctly found the coordinates of the mid-point, but some failed to convert these coordinates to complex number form.

Answers: **(i)** $2 + 2\mathrm{i}$ **(ii)** $\sqrt{2}\,\mathrm{i}$ **(iii)** $1 + \frac{2+\sqrt{2}}{2}\mathrm{i}$

Question 6

(i) Candidates appeared to be unfamiliar with the cartesian equation of a circle. Some found the coordinates of the centre, but then either substituted them into the given equation and found a set of values of a, b and c that satisfied the equation, or used them as the values of a and b with the radius as c.

(ii) Some candidates showed understanding of the gradients of perpendicular lines, although a few omitted the "x" in the equation. Others showed understanding of $y = mx + c$ but were unable to find the relevant gradient.

Answers: **(i)** $x^2 + y^2 + 8x - 6y + 20 = 0$ **(ii)** $y = -\frac{1}{2}x - \frac{3}{2}$

Question 7

Candidates did not appear to be familiar with this topic, with very few showing any understanding. Only a few candidates correctly found E(X) in part **(ii)**.

Answers: **(i)** $\frac{1}{10}, \frac{1}{5}, \frac{3}{10}$ **(ii)** $\mathrm{E}(X) = 4$, $\mathrm{Var}(X) = 1$

Question 8

All candidates showed some familiarity with the manipulation of surds, although only a few candidates achieved the correct answer. Some correctly expanded the denominator, but attempted to multiply the numerator and denominator by the denominator without first changing the sign of its irrational part.

Answer: $16 - 11\sqrt{2}$

Question 9

About half of the candidates showed good algebraic skills and succeeded, or almost succeeded, in rea[ching] the correct answers. Other candidates did not translate the problem into algebra, but attempte[d a] diagrammatic solution, unfortunately by means of extremely poor sketch graphs, which gained no marks.

Answers: (–1, 0) and $\left(\frac{5}{4}, \frac{45}{4}\right)$

Question 10

(a) The majority of candidates needed more practice in handling the trigonometry of non-acute angles. Of those who showed more understanding, the majority found negative solutions rather than those within the given domain.

(b) Candidates did not appear to know the relevant formulas for this part of the question. The incorrect expression $\sin\left(\frac{3}{5}+\frac{5}{13}\right)$ was frequently seen in part **(i)**.

Answers: **(a)** $105°$, $165°$ **(b)(i)** $\frac{63}{65}$ **(b)(ii)** $\frac{29}{2}$

Question 11

Candidates generally seemed unfamiliar with the topic of functions, and there was very little evidence of familiarity with the concept of domain and range. In part **(iii)** the majority of candidates were familiar with the process of finding an inverse function, but some found the algebra difficult. Few were able to deal with the modulus in part **(iv)**.

Answers: **(i)** –2 **(ii)** $f \geq -9$ **(iii)** $f^{-1}(x) = -2 + \sqrt{x+9}$

Question 12

There were many candidates who appeared to have little understanding of vectors in any of the four parts. This is an area where candidates would benefit from extra practice.

(i) Those candidates who attempted the question generally answered this correctly.

(ii) Candidates were able to find vector $\overrightarrow{BC}$ correctly, but did not appreciate that the multiple is both negative and fractional.

(iii) Many candidates recognised the fact that the points are collinear.

(iv) Very few candidates could handle the addition of vectors and the ratio, but those that could mostly succeeded in gaining the correct answer.

Answers: **(i)** $\sqrt{20}$ **(ii)** $-\frac{1}{3}$ **(iii)** They are collinear **(iv)** $1.6\mathbf{i} - 0.8\mathbf{j}$

Question 13

(a) Few candidates showed any understanding of logarithms. The minority that did, correctly the law: $\log(a)^n = n\log(a)$, but thought that $\log a - \log b = \log a \div \log b$.

(b) The majority of candidates, in general, seemed unfamiliar with equations with the unknown in the index. None recognised the quadratic equation in 3^x. Most tried to take the logarithm of each term.

Answers: **(i)** $\log_b\left(\frac{a^3}{c}\right)$ **(ii)** 1.26

ADDITIONAL MATHEMATICS (US)

Paper 0459/02
Paper 2

Key Messages

In order to do well in this examination, candidates needed to give clear and well thought out answers to questions, with sufficient method being shown so that marks can be awarded. Full syllabus coverage is required.

General Comments

Candidates gave clearly presented answers on the whole. In order to improve, other candidates need to understand that their working must be detailed enough to show their method clearly. This is even more important if they make an error. Showing clear method is also very important if a question starts with the words "Show that…". This indicates that the answer has been given to the candidates and that the marks will be awarded for showing how that answer has been found. The need for this was highlighted in **Question 3(i)** in this examination. Presentation in this question could have been improved in some cases. Occasionally candidates showed that they had not fully read the information in the question. This is vital. This was exemplified in **Question 6(iii)**.

Comments on Specific Questions

Question 1

Both parts of this question were generally well done by virtually all candidates. In part **(ii)**, some used synthetic division and some equated coefficients, but it proved to be a very accessible start to the paper.

Answers: **(i)** 4 **(ii)** $(x - 3)(x^2 + x + 2) + 4$

Question 2

(i) This question was very well answered. Some candidates gave rather more explanation than was required to establish whether each expression was real or imaginary, but all gave clear calculated evidence of method.

(ii) This was the first part of the paper that gave candidates any real challenge. Some did correctly divide using the complex conjugate and did so successfully. However, the method required to complete the question – namely setting $(a^2 - 4) = 0$ was not attempted by the majority of candidates.

Answers: **(i) (a)** $2a$ real **(b)** $a^2 + 4$ real **(c)** 4i imaginary **(ii)** $a = \pm 2$

Question 3

(i) It was common, in this question, for candidates to rearrange each equation into the form $y = mx + c$. This was the first key step in the method. However, generally candidates did not go on and state the slopes from these equations. This was the next key step and was required to score full marks. Some did observe that the final key part of the explanation was that two of the lines were parallel and these were perpendicular to the remaining two lines, which were also parallel.

(ii) In order to improve here, candidates should give more thought to the presentation Some candidates clearly were attempting to use a correct method but were not able to successfully, as they did not realise that they had made an error. In this part, method h seen – if the only evidence of finding the points of intersection was a diagram, then the marks not awarded as the question specifically instructs that solutions by accurate drawing would not accepted.

(iii) Some candidates found the midpoint of one of the sides of the rectangle, not realising that they were using adjoining points.

Answers: **(ii)** $\frac{\sqrt{125}}{2}$ **(iii)** (3.75, 1.5)

Question 4

Candidates clearly need a great deal more practice at this style of question, as it was not well answered. Some candidates attempted to create 2 by 2 matrices to work with, rather than using general matrix algebra.

Question 5

(i) Most candidates established $\frac{5}{6}$ as the key probability and about half of these went on to evaluate the required probability correctly. Some lost marks through prematurely approximating the figures and arriving at an answer of 0.0104. Those who made little progress often attempted to solve $\frac{5}{6}=\frac{x}{25}$.

(ii) About half of the candidates gave good and clear explanations and scored both marks here. Some did not make a comparison between the effectiveness without and with the new drug and simply commented on the new drug itself.

Answer: **(i)** 0.0105

Question 6

(i) This question was reasonably well answered, with most candidates scoring at least 1 mark. Comments regarding the outlier in class *B* were rare, but some good and clear observations were made regarding closeness of scores and evenness of spread.

(ii) Candidates' understanding of how changes to the data set affect average and dispersion generally could be improved.

(a) Candidates struggled with this question with most suggesting that the median would increase.

(b) Again, not well answered with most suggesting that the standard deviation would remain the same or increase.

(iii) (a) Most scored at least 1 mark in this question, with a good number scoring 2. Work was well presented and method generally shown.

(b) A small number of candidates gave clear and fully correct answers. However, the majority thought that *B* was better than *A* in both cases. The information given at the start of this part of the question was key to success here and some candidates ignored it or misinterpreted it. Candidates would be well advised to read each question carefully and reread each question, should time permit.

Answers: **(ii)(a)** unchanged **(b)** decrease **(iii) (a)** $\frac{218}{39}$

Question 7

Candidates found this question challenging.

Those who scored, generally earned 1 mark in part **(i)** for stating the sides that were equal in length from information given in the question and made no further progress.

In parts **(ii)** and **(iii)** some candidates were attempting to use the fact that triangles *BCA* and *DCE* were congruent with the reasoning Side – Angle – Side, but often there were key elements that were incomplete or vaguely stated.

In order to improve their performance, candidates need to practice similar questions and perhaps consider the use of a two column approach to proof to ensure that each statement is supported by a valid reason, as it was expected that clear and valid reasons be given to support statements made. Many were quoting postulates such as SAS or SSS without relating them to any particular triangle.

Question 8

Candidates were unable to apply their knowledge to this question. It is possible that candidates may improve should application of this section of the syllabus be practiced more extensively.

Answers: **(i)** 5 **(ii)** 16.5 **(iii)** 681

Question 9

A good sketch is generally the key to being successful in this type of question. Candidates generally were not able to make a reasonable sketch of the information given and therefore found the question challenging.

A correct triangle of the vector representing the boat's course and the current vector resulting in the correct vector from the first marker buoy to the second was generally not shown. Some candidates were able to draw the correct bearing and mark on some correct information, but generally did not position the vector for the current correctly in their diagram and were therefore unable to complete a relevant triangle of forces.

Part **(ii)** of this question was insufficiently answered to be able to comment further.

Answers: **(i)** 132° **(ii)** 0.37 hours or 22.4 minutes

Question 10

Candidates seemed universally unfamiliar with this part of the syllabus and therefore were unable to make any progress, regardless of the fact that the question was straightforward for its type.

Answers: **(i)** amplitude 3; period 90 **(ii)** $f(x) = 1$ **(iv)** 0, 45, 90, 135, 180

Question 11

(i)/(ii) These two parts were very well answered by all candidates with almost all scoring full marks for both parts.

(iii) Generally candidates did not comment on the slope being the rate of increase of percentage marks per hour of study, which was the response that was expected. Commenting that, as the number of hours of study went up so did the score, was not sufficient. Candidates need to be aware that comments of this type should relate specifically to the value of their slope, not just to a slope in general.

(iv) Very well answered – candidates knew how to use the line of best fit to make predictions without difficulty.

(v) Most candidates thought that the claim made was correct, although a small proportion did comment that eventually the score would reach a plateau regardless of the time spent revising.

Answers: **(iv)** 43%

Question 12

(a) A good proportion of candidates formed and solved a correct pair of equations. Some ca did not attempt this question.

(b) Performance in this question would have improved if candidates had thought more about th structure of the calculations needed to find the answer to the problem before trying to construct the recursive function. Those who attempted this question generally attempted to build a function using the figures in the question, regardless of the validity of the structure of what they had created.

Answers: **(a)** $a = 4$, $b = 5$ **(b)** $f(0) = 1650$ $f(n+1) = 1.035 \times f(n)$ $n \geq 0$