UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

International General Certificate of Secondary Education

MARK SCHEME for the October/November 2009 question paper

for the guidance of teachers

0580 MATHEMATICS

0580/04   Paper 4 (Extended), maximum raw mark 130

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Abbreviations

cao	correct answer only
cso	correct solution only
dep	dependent
ft	follow through
isw	ignore subsequent working
oe	or equivalent
SC	Special Case
soi	seen or implied
www	without wrong working

1 **(a) (i)**	**8.4(0)**	**B2**	**B1** for 1.2 or 3.6 seen or **SC1** for figs 84 in answer	
(ii)	$\dfrac{their\,(i)}{20}\times100$ oe **42** ft www2	**M1** **A1ft**	ft their 8.4×5 After 0 scored **SC1 ft** for 58% or $\dfrac{20-their\,(i)}{20}\times100$ correctly given	
(b)	**6**	**B2**	**M1** for 9 or $8 \div (1 + 8 + 3)$ soi	
(c)	$\dfrac{2.4}{2}\times3$ oe (= 3.6 seen) or their **(a) (i)** ÷ 7 × 3 $\dfrac{3}{12}\times9$ oe (= 2.25 seen) **1.6(0)** cao www3	**M1** **M1** **A1**		
(d)	$\dfrac{2.40}{1.25}$ oe **1.92** www2	**M1** **A1**	Implied by figs 192 [11]	

2 **(a) (i)**	Reflection (M), $x = 1$	**B1,B1**	If extra transformations given in part **(a)** then zero scored
(ii)	Rotation (R) 180 (centre) (1, 0)	**B1** **B1** **B1**	Must be "rotation". Allow half turn for 180. Allow other clear forms of (1, 0)
(iii)	Enlargement (E) (centre) (6, 4) (scale factor) 3	**B1** **B1** **B1**	Must be "enlargement" Allow other clear forms of (6, 4) e.g. vector Accept 3 : 1 or 1 : 3
(iv)	Shear (H) y-axis invariant oe (factor) –1	**B1** **B1** **B1**	Must be "shear" Allow other explanation for invariant but not "parallel to" isw after y-axis invariant seen

(b) (i)	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	**B2**	**B1** for correct right–hand column matrix
(ii)	$\begin{pmatrix} 1 & 0 \\ -1ft & 1 \end{pmatrix}$	**B2ft**	Ft only their factor in **(a) (iv)** provided not zero **B1ft** for left-hand column in 2 by 2 matrix provided shear factor is not zero or **SC1** for $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ if not ft **[15]**

3	**(a) (i)**	1	**B1**	Penalty of –1 in question if any answers given as decimals or percentages (to 3sf) alone, but isw cancelling/conversion after correct answer
	(ii)	$\dfrac{3}{6}$ oe	**B1**	
	(b) (i)	$\dfrac{2}{30}$ oe www2	**B2**	**M1** for $\dfrac{2}{6} \times \dfrac{1}{5}$
	(ii)	6–12 and 12–6 and 7–11 and 11–7 soi $k \times \dfrac{1}{6} \times \dfrac{1}{5}$ for k = integer $\dfrac{4}{30}$ oe www3	**M1** **M1** **A1**	Evidence of all pairs adding up to 18 but no extras e.g. 4/6 × 1/6 Without seeing the first M, $\dfrac{4}{6} \times \dfrac{1}{5}$ oe scores **M2**, $\dfrac{2}{6} \times \dfrac{1}{5}$ oe scores **M1**
	(iii)	$\dfrac{4}{6} \times \dfrac{2}{5}$ $\dfrac{8}{30}$ oe www2	**M1** **A1**	
	(c)	$\dfrac{2}{6} + \dfrac{4}{6} \times \dfrac{2}{5}$ oe $\dfrac{18}{30}$ oe cao www2	**M1** **A1**	$\dfrac{2}{6}$ + their **(b) (iii)**
	(d)	4	**B2**	**M1** for $(1 + 1 + 6 + 7 + 11 + 12 + x) \div 7 = 6$ or better **[13]**

4	(a)	(i)	Accurate triangle with 2 arcs seen, 2 mm accuracy for lines AC and BC	B2	SC1 if accurate but no arcs or one arc and BC are wrong way round with arcs
		(ii)	Accurate bisector of angle **ACB**, 2° accuracy and both pairs of arcs shown (accept equidistant marks on edges for 1st set of arcs) + must meet *AB*	B2ft	Ft their triangle **SC1ft** if accurate but no/one pair of arcs or short with arcs In both **(ii)** and **(iii)** isw
		(iii)	Accurate perpendicular bisector of **AD** 2 mm accuracy at mid-point and 2° for right angle and shows both sets of arcs + must meet *AC*	B2ft	ft their *D*, which must be on *AB* **SC1ft** if accurate but no/one pair of arcs or short with arcs
		(iv)	Correct region shaded cao	B1	Dependent on correct triangle, accurate bisectors of angle *ACB* and side *AD* with correct *D*
	(b)	(i)	$(\cos C) = \dfrac{140^2 + 180^2 - 240^2}{2 \times 140 \times 180}$ oe	M2	(–5600/50400 or –14/126) Allow use of 7, 9 and 12 **M1** for correct implicit statement Verification using 96.4 scores **M2** max
			– 0.111(1)…or better or 96.37 to 96.38	E1	Accept $-\frac{1}{9}$ but not a non-reduced fraction
		(ii)	0.5 × 140 × 180 sin (their 96.4) oe **12521 to 12523 or 12 500 or 12520** cao www2	M1 A1	(s = 280), allow use of 7, 9 (31.3…)
		(iii)	$(\text{Sin } B =) \dfrac{140 \sin(their 96.4)}{240}$ oe **35.4** or **35.42 to 35.44** cao www3	M2 A1	Allow use of 7, 12 **M1** for correct implicit statement **SC2** for correct answer by other method [15]

5	(a)	(i)	$(x + 3)(2x + 5) – x(x + 4) = 59$ oe $2x^2 + 6x + 5x + 15 – x^2 – 4x = 59$ oe $x^2 + 7x – 44 = 0$	M1 A1 E1	Implies **M1** (allow 11*x* for 6*x* + 5*x*) Correct conclusion – no errors or omissions
		(ii)	$(x + 11)(x – 4)$	B2	**SC1** any other $(x + a)(x + b)$ where $a \times b = – 44$ or $a + b = 7$
		(iii)	**–11, 4** www ft	B1ft	Strict ft dep on at least **SC1** in **(ii)** allow recovery if new working seen
		(iv)	$\tan = \dfrac{(their + ve\ root) + 3}{2(their + ve\ root) + 5}$ oe **28.3** (00…) ft www2	M1 A1ft	Could be alt trig method oe **M1** where trig function is explicit ft one of their positive roots (27.4° (27.40 – 27.41) from *x* = 11)

(b) (i)	$\dfrac{2x+5}{x+4} = \dfrac{x+3}{x}$ oe	M1	Must be seen. Allow ratio or correct p...
	$x^2 + 4x + 3x + 12 = 2x^2 + 5x$	A1	Correct expansion of brackets seen (allow ... $4x + 3x$)
	$x^2 - 2x - 12 = 0$	E1	Correct conclusion – no errors or omissions **M1** must be seen
(ii)	$\dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-12)}}{2(1)}$	B1,B1	In square root **B1** for $(-2)^2 - 4(1)(-12)$ or better. If in form $\dfrac{p+\sqrt{q}}{r}$ or $\dfrac{p-\sqrt{q}}{r}$, **B1** for $-(-2)$ and $2(1)$ or better
	or $(x-1)^2 -12 -1$ (B1) and $x - 1 = \pm\sqrt{13}$ (B1)		
	– 2.61, 4.61 final answers www4	B1,B1	If B0, **SC1** for –2.6 **and** 4.6 or **both** answers correct to 2 or more dps rot – 2.6055…, 4.6055….
(iii)	**26.4 (26.42…. to 26.44….)** ft	B1ft	ft $4 \times$ a positive root + 8
			[16]

6	**(a) (i)**	**–16**	B1	
	(ii)	**18 to 19**	B1	
	(b) (i)	**–4.3 to –4.2, 1.5 to 1.6**	B1,B1	
	(ii)	**–4.5 to –4.4 , 1.3 to 1.4**	B1,B1	
	(iii)	**–4.5 to –4.4 $< x <$ 1.3 to 1.4** ft	B1ft	Ft their **(ii)**. Allow clear worded explanations and condone $\leqslant$ signs
	(c)	$-\dfrac{30}{7}$ oe isw conversion	B2	Accept $-4\tfrac{2}{7}$, 30/–7. **M1** for 30/7 oe fracts, isw conversion **or** for –30/7 oe soi
	(d)	Ruled line passing within 2 mm of $(-5, 30)$ and $(2, 0)$	B2	**B1** for ruled line parallel to g(x). By eye (21° to 25° to horizontal if in doubt) allow broken line
	(e) (i)	Ruled horizontal line through $(-3, -27)$	B1	No daylight, not chord (allow broken)
	(ii)	$y = -27$	B1	
	(f)	Ruled lines $x = -3$, $x = -2$, $y = 40$	B1	Long enough to be boundary of region – allow broken or solid ruled lines
		Region enclosed by lines $x = -3$, $x = -2$, $y = 40$ and $y = $ g(x)	B1	Allow any clear indication
				[15]

7	(a) (i)	$\dfrac{60}{360} \times \pi \times 2 \times 24$ oe **25.1 (25.12 to 25.14)** www2	M1 A1	Accept 8π
	(ii)	$\dfrac{60}{360} \times \pi \times 24^2$ oe **301 or 302 or 301.4 to 301.7** www2	M1 A1	Accept 96π
	(b) (i)	$\pi d = $ *their* **(a) (i)** oe **4 (3.99 – 4.01)** cao www2	M1 A1	
	(ii)	$24^2 - $ (their radius)^2 **23.7 (23.66 to 23.67)** cao www2	M1 A1	Alt trig method for *h* explicit Accept $\sqrt{560}, 2\sqrt{140}, 4\sqrt{35}$
	(iii)	$\frac{1}{3} \times \pi \times$ (their r)$^2 \times$ (their h) **394 – 398** cao www2	M1 A1	Not for $h = 24$
	(c) (i)	**27W**	B1	
	(ii)	**4W**	B1	If B0, B0 in (c), **SC1** for 27 and 4 alone **[12]**

8	(a)	**5.5 < t $\leqslant$ 6**	B1	Condone poor notation
	(b)	4.25, 4.75, 5.25, 5.75, 6.25, 6.75 $(2 \times 4.25 + 7 \times 4.75 + 8 \times 5.25 + 18 \times 5.75$ $+ 10 \times 6.25 + 5 \times 6.75)$ $(= 283.5)$ $\div$ 50 or their $\sum f$ **5.67** www4	M1 M1 M1 A1	At least 5 correct mid-values seen $\sum fx$ where x is in the correct interval allow one further slip Depend on second method After **M3** allow 5.7 isw conversion to mins/secs and reference to classes
	(c) (i)	**17, 15**	B1	
	(ii)	Rectangular bars of heights **11.3** and **15** Correct widths of 1.5 and 1 – no gaps	B1ft B1ft B1	ft their 17 divided by 1.5 ft their 15 11.3 plot between 11 and 12 include lines and 15 to be touching the 15 line
	(iii)	**2.5** cao	B1	**[10]**

9	(a)	$3(m-3)+4(m+4)=-7\times12$	**M2**	Allow <u>all over</u> 12 at this stage **M1** for $3(m-3)+4(m+4)$ seen
		$3m-9+4m+16=-84$	**A1**	Allow <u>all over</u> 12 at this stage May be seen in stages
		–13 www4	**A1**	
	(b) (i)	0.5 oe	**B1**	
	(ii)	$\dfrac{3(x+3)-2(x-1)}{(x-1)(x+3)}$	**M1**	If brackets not seen allow $3x+9-2x\pm2$ as numerator with a correct denominator
		$\dfrac{x+11}{(x-1)(x+3)}$ final answer	**A1**	isw incorrect expansion of denominator if correct brackets seen
	(iii)	$\dfrac{x(x+11)}{(x-1)(x+3)}=1$ ft **or**		
		$x+11=\dfrac{1}{x}(x-1)(x+3)$ or better ft	**M1**	Must clear one denominator correctly Ft their **(b)(ii)** dep on fraction in **(ii)** with $(x-1)(x+3)$ oe as denominator
		$x^2+11x=x^2+3x-x-3$	**M1**	Depend on previous **M1**
		$-\dfrac{1}{3}$ oe cso www3	**A1**	$-0.33(33\ldots)$
	(c)	$p(q-1)=t$ oe $pq=t+p$	**M1** **M1**	Multiplying by $(q-1)$ Ft their first step e.g. pq only term on one side
		$\dfrac{t+p}{p}$ oe final answer www3	**M1**	Ft their 2nd step e.g. dividing by p Note: $q-1=\dfrac{t}{p}$ is **M2** and then $q=\dfrac{t}{p}+1$ is **M1**
				[13]

10	(a)	$21+23+25+27+29=125$ $31+33+35+37+39+41=216$	**B1** **B1**	
	(b)	Cubes	**B1**	
	(c) (i)	n oe	**B1**	
	(ii)	n^3 oe	**B1**	
	(d)	$4^2-4+1=13$ www	**E1**	Allow 16 for 4^2, otherwise all must be seen
	(e)	$7\times43+2+4+6+8+10+12$	**B1**	All must be seen
	(f)	$n(n-1)$ final answer oe	**B1**	
	(g)	$n(n^2-n+1)+$ their **(f)** $n^3-n^2+n+n^2-n=n^3$	**M1** **E1**	All must be seen, no errors or omissions
				[10]