UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
International General Certificate of Secondary Education

ADDITIONAL MATHEMATICS **0606/23**

Paper 2 **May/June 2010**

2 hours

Additional Materials: Answer Booklet/Paper
Graph paper (2 sheets)
Electronic calculator

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Booklet/Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **6** printed pages and **2** blank pages.

DC (LEO/KN) 25707

UNIVERSITY *of* CAMBRIDGE
International Examinations

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2}a^{n-2} b^2 + \ldots + \binom{n}{r}a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for ΔABC

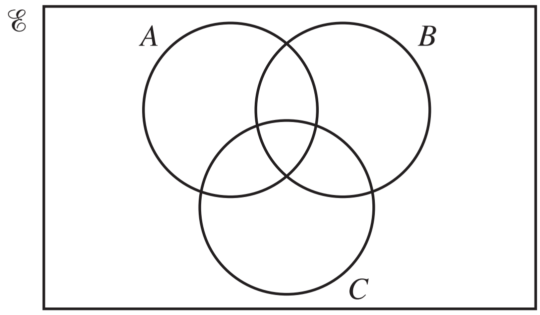
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

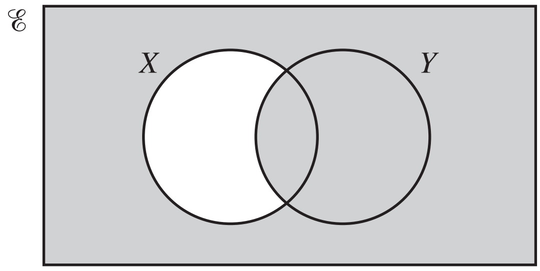
1 Find $\int \left(2 + 5x - \frac{1}{(x-2)^2}\right) dx$.

2 **(a)**



Copy the diagram and shade the region which represents the set $A \cup (B \cap C')$. [1]

(b)

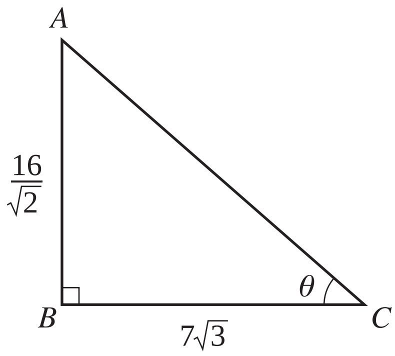


Express, in set notation, the set represented by the shaded region. [1]

(c) The universal set $\mathscr{E}$ and the sets P and Q are such that $n(\mathscr{E}) = 30$, $n(P) = 18$ and $n(Q) = 16$. Given that $n(P \cup Q)' = 2$, find $n(P \cap Q)$. [2]

3 The volume $V\,\text{cm}^3$ of a spherical ball of radius $r\,\text{cm}$ is given by $V = \frac{4}{3}\pi r^3$. Given that the radius is increasing at a constant rate of $\frac{1}{\pi}\,\text{cm}\,\text{s}^{-1}$, find the rate at which the volume is increasing when $V = 288\pi$. [4]

4



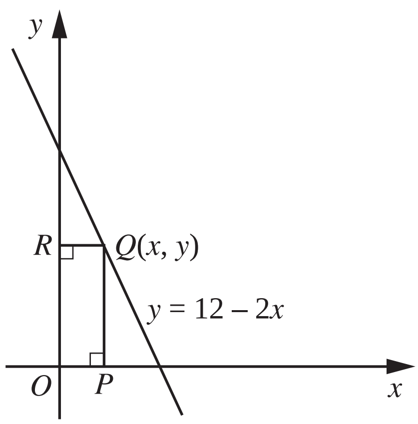
The diagram shows a right-angled triangle ABC in which the length of AB is $\frac{16}{\sqrt{2}}$, the length of BC is $7\sqrt{3}$ and angle BCA is θ.

(i) Find $\tan\theta$ in the form $\frac{a\sqrt{6}}{b}$, where a and b are integers. [2]

(ii) Calculate the length of AC, giving your answer in the form $c\sqrt{d}$, where c and d are integers and d is as small as possible. [3]

5 Solve the equation $2x^3 - 3x^2 - 11x + 6 = 0$. [6]

6



The diagram shows part of the line $y = 12 - 2x$. The point $Q(x, y)$ lies on this line and the points P and R lie on the coordinate axes such that $OPQR$ is a rectangle.

(i) Write down an expression, in terms of x, for the area A of the rectangle $OPQR$. [2]

(ii) Given that x can vary, find the value of x for which A has a stationary value. [3]

(iii) Find this stationary value of A and determine its nature. [2]

7 **(i)** Sketch the graph of $y = |3x + 9|$ for $-5 < x < 2$, showing the coordinates of the po… graph meets the axes.

(ii) On the same diagram, sketch the graph of $y = x + 6$.

(iii) Solve the equation $|3x + 9| = x + 6$. [3]

8 **(a)** **(i)** Write down the first 4 terms, in ascending powers of x, of the expansion of $(1 - 3x)^7$. [3]

(ii) Find the coefficient of x^3 in the expansion of $(5 + 2x)(1 - 3x)^7$. [2]

(b) Find the term which is independent of x in the expansion of $\left(x^2 + \dfrac{2}{x}\right)^9$. [3]

9 **(i)** Given that $y = \dfrac{x + 2}{(4x + 12)^{\frac{1}{2}}}$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{k(x + 4)}{(4x + 12)^{\frac{3}{2}}}$, where k is a constant to be found. [5]

(ii) Hence evaluate $\displaystyle\int_1^{13} \frac{x + 4}{(4x + 12)^{\frac{3}{2}}}\,\mathrm{d}x$. [3]

10 **(a)** Given that $\log_p X = 6$ and $\log_p Y = 4$, find the value of

(i) $\log_p \left(\dfrac{X^2}{Y}\right)$, [2]

(ii) $\log_Y X$. [2]

(b) Find the value of 2^z, where $z = 5 + \log_2 3$. [3]

(c) Express $\sqrt{512}$ as a power of 4. [2]

11 **(a)** Solve, for $0 < x < 3$ radians, the equation $4\sin x - 3 = 0$, giving your answers correct to 2 decimal places. [3]

(b) Solve, for $0° < y < 360°$, the equation $4\operatorname{cosec} y = 6\sin y + \cot y$. [6]

12 Answer only **one** of the following two alternatives.

EITHER

It is given that $f(x) = 4x^2 + kx + k$.

(i) Find the set of values of k for which the equation $f(x) = 3$ has no real roots. [5]

In the case where $k = 10$,

(ii) express $f(x)$ in the form $(ax + b)^2 + c$, [3]

(iii) find the least value of $f(x)$ and the value of x for which this least value occurs. [2]

OR

The functions f, g and h are defined, for $x \in \mathbb{R}$, by

$$f(x) = x^2 + 1,$$

$$g(x) = 2x - 5,$$

$$h(x) = 2^x.$$

(i) Write down the range of f. [1]

(ii) Find the value of $gf(3)$. [2]

(iii) Solve the equation $fg(x) = g^{-1}(15)$. [5]

(iv) On the same axes, sketch the graph of $y = h(x)$ and the graph of the inverse function $y = h^{-1}(x)$, indicating clearly which graph represents h and which graph represents h^{-1}. [2]

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