## Published

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## MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

## Types of mark

M Method marks, awarded for a valid method applied to the problem.
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.
When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

## Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 1(i) | $\begin{aligned} & k x-5=x^{2}+4 x \\ & x^{2}+(4-k) x+5=0 \end{aligned}$ | M1 | equating line and curve equation and collecting terms to form an equation of the form $a x^{2}+b x+c=0$ $x$ terms must be gathered together, maybe implied by later work |
|  | For a tangent $(4-k)^{2}=20$ | DM1 | correct use of discriminant |
|  | $k=4+2 \sqrt{5}$ | A1 | Accept $k=4+\sqrt{20}$ |
|  | Alternative <br> Gradient of line $=k$ <br> Gradient of curve $=\frac{\mathrm{d} y}{\mathrm{~d} x}=2 x+4$ <br> Equating: $k=2 x+4$ | M1 |  |
|  | substitution of $k=2 x+4$ or $x=\frac{k-4}{2}$ in $k x-5=x^{2}+4$ and simplify to a quadratic equation in $k$ or $x$ | DM1 |  |
|  | $k=4+2 \sqrt{5}$ | A1 | Accept $k=4+\sqrt{20}$ |
| 1(ii) | Normal gradient $=-\frac{1}{4+2 \sqrt{5}} \times \frac{4-2 \sqrt{5}}{4-2 \sqrt{5}}$ | M1 | use of negative reciprocal and attempt to rationalise using a form of $a-b \sqrt{5}$ or $a-\sqrt{20}$ or their equivalent from (i) |
|  | $\begin{aligned} & =-\frac{4-2 \sqrt{5}}{-4} \mathrm{oe} \\ & =1-\frac{\sqrt{5}}{2} \end{aligned}$ | A1 | $-\frac{4-2 \sqrt{5}}{-4}$ oe leading to $1-\frac{\sqrt{5}}{2}$ |
| 2 | $\mathrm{p}(3)=27+9 a+3 b-48$ | M1 | attempt to find $\mathrm{p}(3)$ |
|  | $3 a+b=9$ oe | A1 |  |
|  | $\begin{aligned} & \mathrm{p}^{\prime}(x)=3 x^{2}+2 a x+b \\ & \mathrm{p}^{\prime}(1)=3+2 a+b \end{aligned}$ | M1 | attempt to differentiate and find $\mathrm{p}^{\prime}(1)$ must have 2 terms correct |
|  | $2 a+b=-3$ oe | A1 |  |
|  | $a=12, b=-27$ | A1 | for both |
| 3(a) | $x^{3} y^{7}$ | B2 | B1 for each term |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 3(b)(i) | for $(t-2)^{\frac{3}{2}}=(t-2)^{\frac{1}{2}}(t-2)$ soi | M1 |  |
|  | $(t-2)^{\frac{1}{2}}(4+5(t-2))$ | A1 |  |
|  | $(t-2)^{\frac{1}{2}}(5 t-6)$ | A1 |  |
| 3(b)(ii) | $2 \text { and } \frac{6}{5}$ | B1 | FT on their $(t-2)^{\frac{1}{2}}(5 t-6)$, must have 2 |
| 4(a)(i) | $\mathrm{f}>5, \mathrm{f}(x)>5$ | B1 |  |
| 4(a)(ii) | $\frac{y-5}{3}=\mathrm{e}^{-4 x} \text { or } \frac{x-5}{3}=\mathrm{e}^{-4 y}$ | B1 |  |
|  | $-4 x=\ln \left(\frac{y-5}{3}\right) \text { or }-4 y=\ln \left(\frac{x-5}{3}\right)$ | B1 |  |
|  | leading to $\mathrm{f}^{-1}(x)=-\frac{1}{4} \ln \left(\frac{x-5}{3}\right)$ <br> or $\mathrm{f}^{-1}(x)=\frac{1}{4} \ln \left(\frac{3}{x-5}\right)$ <br> or $\mathrm{f}^{-1}(x)=\frac{1}{4}(\ln 3-\ln (x-5))$ <br> or $\mathrm{f}^{-1}(x)=-\frac{1}{4}(\ln (x-5)-\ln 3)$ | B1 |  |
|  | Domain $x>5$ | B1 |  |
| 4(b) | $\ln \left(x^{2}+5\right)=2$ | B1 |  |
|  | $x^{2}+5=\mathrm{e}^{2}$ | B1 |  |
|  | $x=1.55$ or better or $\sqrt{\mathrm{e}^{2}-5}$ | B1 |  |
| 5(a)(i) | $\overrightarrow{O M}=\overrightarrow{O C}+\frac{1}{2}(\overrightarrow{O A}-\overrightarrow{O C}) \text { oe }$ | M1 | may be implied by correct answer. |
|  | $\frac{1}{2}(\mathbf{a}+\mathbf{c})$ | A1 |  |


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| 5(a)(ii) | $\begin{aligned} & \mathbf{b}=\frac{5}{2} \overrightarrow{O M} \text { oe, } \frac{5}{2}(\text { their }(\mathrm{i})) \\ & \text { or } \overrightarrow{O M}=\frac{2}{3}(\mathbf{b}-\overrightarrow{O M}) \end{aligned}$ | M1 | dealing with ratio correctly to relate $\mathbf{b}$ or $\overrightarrow{O B}$ to $\overrightarrow{O M}$ |
|  | $=\frac{5}{4}(\mathbf{a}+\mathbf{c})$ | A1 |  |
| 5(b)(i) | $\begin{aligned} & \|-10 \mathbf{i}+24 \mathbf{j}\|=26 \\ & \mathbf{p}=\frac{39}{26}(-10 \mathbf{i}+24 \mathbf{j}) \end{aligned}$ | M1 | magnitude of $-10 \mathbf{i}+24 \mathbf{j}$ and use with 39 |
|  | $\mathbf{p}=-15 \mathbf{i}+36 \mathbf{j}$ | A1 |  |
| 5(b)(ii) | If parallel to the $y$-axis, $\mathbf{i}$ component is zero | M1 | realising $\mathbf{i}$ component is zero |
|  | so $2 \mathbf{p}+\mathbf{q}=12 \mathbf{j}$ | DM1 | use of 12 |
|  | $\mathbf{q}=30 \mathbf{i}-60 \mathbf{j}$ | A1 |  |
| 5(b)(iii) | $\|\mathbf{q}\|=30 \sqrt{1^{2}+(-2)^{2}}$ or $\sqrt{900} \times \sqrt{5}$ | M1 | attempt at magnitude of their $\mathbf{q}$ |
|  | $\|\mathbf{q}\|=30 \sqrt{5}$ | A1 | Answer Given: must have full and correct working |
| 6(i) | $\frac{1}{2} \times 12^{2} \times \theta=150$ | M1 | use of sector area |
|  | $\theta=2.083$, so $\theta=2.08$ to 2 dp | A1 |  |
| 6(ii) | Area of triangle $A O B=\frac{1}{2} \times 12^{2} \sin 2.08$ | M1 | correct method for area of triangle |
|  | $\text { Area of segment }=150-\frac{1}{2} \times 12^{2} \times \sin 2.08$ | A1 | allow unsimplified, using $\theta=2.08,2.083 \text { or } \frac{150}{72}$ |
|  | $\sin 1.04=\frac{\frac{A B}{2}}{12}$ | M1 | correct trigonometric statement using $\theta=2.08,2.083$ or $\frac{150}{72}$ with attempt to obtain $A B$ |
|  | $A B=$ awrt 20.7 | A1 |  |
|  | Shaded area $=$ their $A B \times 8-$ their segment area | M1 | execution of a correct 'plan'(rectangle segment) |
|  | awrt 78.4 or 78.5 | A1 |  |


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| 6(iii) | Arc $A B=25$ or 24.96 | B1 |  |
|  | Perimeter $=25+$ their $A B+16$ | M1 | correct 'plan' ( arc + their $A B+2 \times 8$ ) |
|  | awrt 61.7 | A1 |  |
| 7 | differentiation to obtain answer in the form $p\left(3 x^{2}+8\right)^{\frac{2}{3}}$ or $q x\left(3 x^{2}+8\right)^{\frac{2}{3}}$ | M1 |  |
|  | $6 x\left(3 x^{2}+8\right)^{\frac{2}{3}}$ | B1 |  |
|  | $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{5}{3} \times 6 x\left(3 x^{2}+8\right)^{\frac{2}{3}}$ | A1 | all correct |
|  | When $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$ only solution is $x=0$ | DM1 | $q x\left(3 x^{2}+8\right)^{\frac{2}{3}}=0$ and attempt to solve |
|  | $x=0$ and $3 x^{2}+8=0$ has no solutions | A1 |  |
|  | Stationary point at ( 0,32 ) | A1 |  |
|  | correct gradient method with substitution of $x$ values either side of zero or equivalent valid method | M1 |  |
|  | correct conclusion from correct work using a correct $\frac{\mathrm{d} y}{\mathrm{~d} x}$ | A1 |  |
| 8(i) |  | B5 | B1 for shape of modulus function <br> B1 for $y$ intercept $=5$ (for modulus graph only) <br> B1 for $x$ intercept $=2.5$ at the V of a modulus graph <br> B1 for shape of quadratic function for $-1 \leqslant x \leqslant 6$ <br> B1 for intercepts at $x=0$ and $x=5$ for a quadratic graph |
| 8(ii) | $2 x-5= \pm 4$ | B1 | one correct answer |
|  | $x=\frac{9}{2}$ | M1 | solution of two different correct linear equations or solution of an equation obtained from squaring both sides or use of symmetry from first solution. |
|  | $x=\frac{1}{2}$ | A1 | second correct solution |


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| :---: | :---: | :---: | :---: |
| 8(iii) | $\begin{aligned} & 16\left(\frac{1}{2}\right)^{2}-80\left(\frac{1}{2}\right)+36=4 \\ & \text { and } 16\left(\frac{9}{2}\right)^{2}-80\left(\frac{9}{2}\right)+36=4 \end{aligned}$ | B1 | verification using both $x$ values or for forming and solving $16 x^{2}-80 x+36=0$ |
| 8(iv) | using their values from (ii) in an equality of the form $a \leqslant x \leqslant b$ or $a<x<b$ | M1 |  |
|  | $\frac{1}{2} \leqslant x \leqslant \frac{9}{2}$ cao | A1 |  |
| 9(i) | $5+4\left(\sec ^{2}\left(\frac{x}{3}\right)-1\right)$ leading to given answer | B1 | use of correct identity |
| 9(ii) | $3 \tan \left(\frac{x}{3}\right)(+c)$ | B1 |  |
| 9(iii) | attempt to integrate using (i) and/or (ii) | M1 |  |
|  | Area $=\int_{\frac{\pi}{2}}^{\pi} 4 \sec ^{2}\left(\frac{x}{3}\right)+1 \mathrm{~d} x$ | A1 | all correct |
|  | $\left[12 \tan \left(\frac{x}{3}\right)+x\right]_{\frac{\pi}{2}}^{\pi}$ | DM1 | correct method for evaluation using limits in correct order |
|  | $=\left(12 \tan \frac{\pi}{3}+\pi\right)-\left(12 \tan \frac{\pi}{6}+\frac{\pi}{2}\right)$ | A1 |  |
|  | $=8 \sqrt{3}+\frac{\pi}{2}$ | A1 |  |
| 10(a) | differentiation of a quotient or equivalent product | M1 |  |
|  | correct differentiation of $\mathrm{e}^{3 x}$ | B1 |  |
|  | $\begin{aligned} & \frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{3 \mathrm{e}^{3 x}\left(4 x^{2}+1\right)-8 x \mathrm{e}^{3 x}}{\left(4 x^{2}+1\right)^{2}} \\ & \text { or } \frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{3 \mathrm{e}^{3 x}}{4 x^{2}+1}-\frac{8 x \mathrm{e}^{3 x}}{\left(4 x^{2}+1\right)^{2}} \end{aligned}$ | A1 | everything else correct including brackets where needed, allow unsimplified |


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| :--- | :--- | ---: | ---: |
| $10(\mathrm{~b})(\mathrm{i})$ | one term differentiated correctly | M1 | A1 |
|  | $\frac{\mathrm{d} y}{\mathrm{~d} x}=-4 \sin \left(x+\frac{\pi}{3}\right)+2 \sqrt{3} \cos \left(x+\frac{\pi}{3}\right)$ | all correct |  |
|  | When $x=\frac{\pi}{2}, \frac{\mathrm{~d} y}{\mathrm{~d} x}=-5$ | M1 | correct use of rates of change |
|  | $\frac{\mathrm{d} y}{\mathrm{~d} x} \times \frac{\mathrm{d} x}{\mathrm{~d} t}=\frac{\mathrm{d} y}{\mathrm{~d} t}$ |  |  |
| $-5 \times \frac{\mathrm{d} x}{\mathrm{~d} t}=10$ oe | A1 | FT answer to (i) |  |
|  |  |  |  |

