ADDITIONAL MATHEMATICS

Paper 0606/11
Paper 11

Key messages

This paper gave candidates the opportunity to recall and use a range of mathematical techniques and to devise mathematical arguments, presenting those arguments precisely and logically. Good responses would be set out clearly and demonstrate a good understanding of fundamental techniques. They would also demonstrate a thorough understanding of mathematical language and notation including those relating to sets and functions. Shorter questions required recall of an appropriate technique but in longer questions a structured solution bringing together various techniques was required.

General comments

The paper provided a good range of responses showing that many candidates had worked hard and understood the syllabus objectives, being able to apply them appropriately. Candidates appeared to have no timing issues and most candidates attempted all the questions.

The paper contained some questions requiring results to be shown and there were some good responses to these, demonstrating a logical progression with all steps clearly written down. For these questions, candidates should be encouraged to keep their argument in a clear flow rather than having parts appearing out of order in separate places.

There were some topics where candidates appeared to be less familiar with the techniques required and candidates would benefit from practice in answering questions from all areas of the syllabus.

Candidates should be aware that for some questions not all solutions obtained are valid answers in the context of the question.

Comments on specific questions

Question 1

(a) Candidates answered the first part well, showing a good understanding of union and intersection of sets. The second part was not well answered, and candidates would benefit from practice in constructing Venn diagrams involving complements of sets. In both parts all responses were clear and neat.

(b) Many good responses were seen, but some candidates did not understand the set notation used in this question. $P \cap Q = \varnothing$ was clearly understood but practice is required in understanding the notation and representation of subsets. Candidates should be aware that clear labelling of sets is required.

Question 2

(i) Candidates demonstrated that they were familiar with the amplitude of a sine function and had been well prepared to answer this type of question.

(ii) Candidates demonstrated that they were familiar with the period of a sine function and had been well prepared to answer this type of question.

(iii) Some completely correct graphs were seen but for the majority more attention to detail was required, particularly as a grid was provided. Although good responses had been made to the previous parts, they appeared not to have been taken into consideration in this part. The straightforward approach of using a calculator to obtain and plot points was not always successfully accomplished with many candidates not calculating the y values at the end points. The curve had to be clearly a sine curve and show the symmetrical properties associated with a sine curve. Candidates should be encouraged to pay attention to the drawing of the curves at the maximum, minimum and end points.

Question 3

(i) The answer to this part should have been easily obtained by inspection. Many candidates did not realise that $-k + k = 0$ and embarked on unnecessary and inaccurate manipulation.

(ii) Nearly all candidates knew to substitute $x = -3$. Those employing the expected approach using $(2 \times -3 - 1)(k - 3) - 12 = 23$ were usually successful but occasional mistakes were made in simplification. Candidates who chose to expand and then use $x = -3$ were more likely to make errors in expansion and simplification. Losing the –12 from the polynomial was a common error. A few candidates used long division but should be advised that use of the remainder theorem is an easier approach.

(iii) Most candidates knew to equate the polynomial to –25 but errors were made in both the expansion of the brackets and through loss of the –12. An equation in the form of a quadratic expression equated to zero had to be obtained and candidates should be advised to take care that it is seen in this form. Most candidates knew that they had to use the discriminant, but a proper conclusion was required in addition to its calculation.

Question 4

(i) Many good solutions were given with the majority of candidates showing a good understanding of the application and use of the binomial theorem. The evaluation of b proved difficult for some as not all candidates realised that they had to equate the coefficient of the second term in their binomial expansion to 256 to obtain an equation to find b. Candidates were often unsuccessful in finding c as either b was used instead of b^2 or an incorrect value of b was being used.

(ii) An incorrect expansion of $\left(2x - \frac{3}{x}\right)^2$ was a common cause of loss of marks in this question. Candidates should be advised to take care when executing this type of expansion. Common mistakes were to obtain $2x^2$ or $-\frac{9}{x^2}$ or not to recognise that there was a numerical term. An expansion not in the appropriate form meant that candidates could not score in this question. Candidates should be clear what is meant by 'term independent of x'. This was sometimes misunderstood and a term in x was found. Two products had to be added but some candidates only identified one numerical term and gave that as their answer without looking further.

Question 5

Vectors are an area of weakness for candidates and few responses had all three parts fully correct. Candidates often did not respond to one or more parts and would benefit from practising vectors questions.

(i) It was essential to find the modulus to proceed with this part and not all candidates calculated it. Candidates who did find the modulus rarely made use of it in a division.

(ii) Some candidates seemed unaware how to approach this part and seemed unfamiliar with the form required for the position vector at time t. A significant number of responses did not include t.

(iii) Few responses included a position vector of P from the previous part that could be used to set up an equation in t. Successful responses correctly found a position vector for Q and equated it to the position vector of P and invariably arrived at $t = 4$. The position vector at the time 4s was omitted from some otherwise good responses.

Question 6

This question required candidates to structure a response using a variety of mathematical techniques. The first step of forming and solving an equation to find the x values at the points of intersection was nearly always done correctly. From here candidates should have a plan to proceed either by subtracting the two equations and integrating or by finding the area of the trapezium and the area under the curve and subtracting. Some candidates did not have either of these plans and found the area under the curve only. Working is always required for questions involving application of limits to an integral and correct integrals and substitution of limits had to be seen. Candidates often left themselves short of space at the end of the question, having unnecessarily spread out the first part, thus making their response difficult to read.

Question 7

(a) Many good solutions were seen with most responses correctly using a change of base. Successful responses went on to show a good understanding of the laws of logarithms and the relationship between logs and powers. Candidates were expected to solve the equation using their knowledge of logarithms and calculator trials were not appropriate.

(b) Many good solutions were seen. Most candidates used a first step of $2\log_4 (y-1) = \log_4 (y-1)^2$. Candidates using a second step of moving $\log_4 (y-1)^2$ to the left-hand side and applying the division rule were usually successful. Those attempting to combine the terms on the right hand side using the multiplication rule were less successful as this approach made it more difficult to deal with the $\frac{1}{2}$ and mistakes were made with the order of operations. Many candidates recognised and used $\frac{1}{2} = \log_4 2$ correctly. Candidates who multiplied the given equation by 2 and obtained a quartic equation added an unnecessary level of difficulty. Candidates who obtained a quadratic equation usually went on to solve it correctly, but candidates should be aware that in questions involving logs they should check the validity of their answer. On this occasion –6 led to an invalid answer. Candidates should be reminded of the importance of showing all their steps in this type of question and that correct statements in terms of logs should be shown before moving on to an equation in y.

Question 8

(i) To answer this part successfully, both a knowledge of the language relating to functions and an understanding of how exponential functions behaved were required. Few correct responses were seen.

(ii) This part seemed more familiar to candidates and some good responses were seen for the finding of the inverse. The domain was often left out or not a valid expression for a domain. Some responses showed that candidates were familiar with the idea that the domain of the inverse related to the range of the original function.

(iii) This part was well answered with responses that showed a good understanding of composite functions that applied the functions in the correct order.

(iv) The notation in this part, $g^2(x)$, was not always understood by candidates as meaning $gg(x)$ and there were difficulties in its application to obtain $(x^2 + 4)^2 + 4$. Candidates who obtained a correct quartic were often unsure how to solve it. They should be aware that it is a quadratic equation in x^2 and can be solved as such to find x^2 and hence x. The most successful approach was to use $(x^2 + 4)^2 = 36$ rather than expanding and factorising. Most candidates who solved the equation were aware that x^2 could not be negative.

Question 9

(i) Formulas relating to surface area and volume of a cylinder are not provided in the formula sheet as they are assumed knowledge, but most candidates knew the formula for the surface area and equated it correctly to 600π. The usual and most successful approach was to rearrange to find *h* in terms of *r*. Candidates are aware that their response has to be clear and unambiguous in a question where something has to be shown and the majority of responses were well set out with the manipulation well done. Candidates should be careful when cancelling as it not always clear. They should be encouraged to show as many successive lines as needed to make it clear what they are doing. If candidates need to start again, they should use an additional sheet rather than running their work into the next question.

(ii) The majority of candidates differentiated correctly and found *r*. Some did not reject the negative value. Some candidates overlooked some of the question and did not give a value for the maximum volume. Many candidates successfully evaluated the second derivative to show that it was a maximum.

Question 10

(i) Many responses correctly stated or implied that $\lg y = A + Bx^2$. Most candidates related the gradient of the line to *B* but many did not use a correct method for obtaining *A*, not realising that the given coordinates were of the form $(x^2, \lg y)$ and that squaring the first coordinate was not required when substituting in $\lg y = A + Bx^2$.

(ii) Most candidates correctly substituted for *x* in their expression for *y*. In some responses further manipulation revealed misunderstandings in the laws of indices.

(iii) Most candidates correctly equated their *y* to 2 or their $\lg y$ to lg2. Responses using a correct answer to part **(i)** often went on to obtain the correct answer, with some losing the final mark through premature rounding. Candidates whose answer to part **(i)** was incorrect often showed enough working to earn both method marks in this part.

Question 11

(i) Most candidates made a good attempt at the product rule for differentiation but some errors were made in the derivatives of $x^2 + 1$ and $(2x-3)^{\frac{1}{2}}$. Although it was possible to obtain a correct result using the quotient rule, most attempts to use it were inappropriate, possibly suggested by the form of the given result. Candidates should be encouraged to use the product rule for a product of terms. The simplification required to obtain the final result was difficult and working was often unconvincing. Throughout this question there was some carelessness in copying terms from step to step.

(ii) Most candidates correctly found the *y*-coordinate at $x = 2$. Many substituted 2 into their expression from the previous part but not all appreciated that a normal gradient was required. Those that did usually employed a correct method for finding the equation. Candidates should be aware that *a*, *b* and *c* had to be integers in the final form of the equation.

ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

It is important that candidates are familiar with the rubric on the front of the examination paper as it will remind them of important facts which tend to be forgotten during the examination itself. It is important that the degree of accuracy is noted as many candidates lose accuracy marks due to inaccurate working. It is also important that candidates ensure that they have fulfilled the demands of each question and have also set their work out clearly, showing sufficient steps in their working.

General comments

The majority of candidates appeared to have sufficient time to work through the paper and also sufficient space in which to answer their questions. It was pleasing to note that when extra space was needed for an answer, work was done on additional sheets which were invariably annotated with the appropriate question number.

It should also be noted that many candidates did not obtain some of the accuracy marks available as they did not appreciate the implication of the word ‘exact’ in the mathematical context. It should also be noted that when candidates are required not to use a calculator, this instruction extends to the whole question and not just the first two or three lines of a solution.

Comments on specific questions

Question 1

(a) The majority of candidates scored full marks by correctly shading both Venn diagrams. Those that made an error tended to make it on the shading of the Venn diagram for $A' \cap B' \cap C'$.

(b) This part proved to be more problematic for candidates, with many not providing all the elements from 0° to 360° for set *P*. It was common to see just the two elements in the range 0° to 180° rather than the four elements in the range 0° to 360°, although most candidates dealt with the double angle correctly. Some candidates gave just one element for each of sets *P* and *Q*.

It was acceptable for the elements of sets *P* and *Q* to be listed and most candidates interpreted the intersection of their sets correctly. However, correct set notation was required for the final answer as a set was asked for. Many candidates listed the elements of the intersection, although the elements 30° and 150° were often obtained fortuitously from an incorrect set *Q*. These candidates were usually only able to gain one mark.

Question 2

It was essential that candidates appreciated that the use of a calculator was not allowed in any part of this question.

The solutions offered fell mainly into 3 types:

The first type involved candidates equating the two equations and cancelling out $2x+3$ from each side of their equation. The remaining quadratic equation was then solved by factorisation, with the cancelled factor of $2x+3$ being ignored, thus restricting candidates to 3 marks at best.

The second type involved candidates equating the two equations as before and forming a single cubic equation, equated to zero. However many candidates then immediately produced the roots of this equation, showing no intermediate working, which was not the required (non-calculator) method. Such candidates were only able to score a mark for the initial equating of the two equations.

The third type involved a correct method with candidates producing a cubic equation equated to zero and then showing a method for testing for a root or factor. The cubic equation could then be fully factorised first producing a linear factor and a quadratic factor which then led to the production of three *x*-coordinates which usually (but not always) were used to find corresponding *y*-coordinates. Occasionally the coordinates were only found for two points, occurring when the candidates attempted to make use of the quadratic factor $4x^2-9$, producing only $x=\frac{3}{2}$. Sometimes the root from the first factor was ignored.

Very few candidates produced the solution that had been envisaged with the two equations being equated and then factorisation using the common factor of $2x+3$ taking place.

Question 3

(i) Nearly all candidates obtained the correct answer of 1000.

(ii) Most candidates were able to differentiate the exponential expression correctly, with just a very small number trying to subtract one from the powers of the exponential terms. These exponential terms were then equated to 1200 and correctly divided throughout by 400. Even though the final answer was given, a small number of candidates were not able to complete the solution by multiplying their correct result of $e^{2t}-4e^{-2t}-3=0$ throughout by e^{2t}.

(iii) Many completely correct solutions were seen, but some candidates did not check that they had completely answered the question. The most common error was for candidates to use the given substitution and solve the resulting quadratic equation to give two values for *u*, without completing to find *t* as required. The final answer was acceptable in either logarithmic or decimal form. However, a few candidates gave their final answer for *t* as 0.69 rather than the expected decimal answer of 0.693. Most candidates realised correctly that the equation $e^{2t}=-1$ does not provide a solution. Those that gave an erroneous solution for this equation did not gain the final accuracy mark as it depended on having a correct solution and no other.

Question 4

(a) The majority of candidates were able to find the correct value for each of the unknown indices. Very few candidates did not get any correct. Some candidates calculated the powers of *p*, *q* and *r* but did not always match them up correctly with the *a*, *b* and *c* required by the question. Such candidates were awarded the marks provided a correct simplification had been seen.

(b) Many candidates were able to find a successful method for dealing with the given simultaneous equations, often by substituting a pair of letters for $\sqrt{x}$ and $\sqrt{y}$, and then solving in the usual way, Those candidates that chose to use *x* and *y* as the substitute pair often confused themselves later. One of the most common incorrect methods was to square each term in each equation producing the incorrect $9x - y = 16$ and $16x - 9y = 196$. Another common error was to forget to multiply every term in one of the equations in preparation for elimination of one variable. Quite often candidates found having the correct square roots as the unknowns had problems handling them, so, for example, $\sqrt{x} = 2$ would become $x = \sqrt{2}$. Similar misapplications during the process of eliminating a variable produced much unnecessary and incorrect work from some candidates.

Question 5

The majority of candidates used radians, as intended, throughout this question.

(i) Most candidates made use of the arc length formula to obtain a correct angle in radians. Very few incorrect results were seen.

(ii) The correct two areas needed to produce a correct solution were identified by most candidates with many going on to find these areas correctly. The better prepared candidates were able to obtain the areas needed by using either $\tan 0.8$ or $\cos 0.8$ as intended. Some candidates found the triangle area without presenting and/or working to the correct degree of accuracy for the length of either *OB* or *AB*. Candidates who rounded these lengths to 3 significant figures before finding the area of the appropriate triangle at this stage obtained a final area that was inaccurate, thus being unable to obtain the final accuracy mark.

Other methods, such as use of the sine rule to find the lengths needed were also acceptable. Unfortunately some candidates mistakenly assumed, from the diagram, that the triangle was isosceles. If this was the case, the only mark available was for the area of the appropriate sector. Candidates should be guided to some extent by the mark allocation for a question as to how much work is involved in the solution of that question.

Question 6

Most candidates were able to identify that part **(a)** of this question involved the use of permutations and part **(b)** of this question involved combinations.

(a) (i) The majority of candidates were able to find the correct solution.

(ii) Most candidates realised the number of arrangements of the mathematics books within themselves was 4!, but then did not always treat them subsequently as one unit. This meant that multiplying by 120 was not always done. Rearranging the remaining 4 books and then doubling was a common error so 4! x 4! x 2 was a frequent incorrect response. A few candidates attempted to use ${}^4C_4$ for the arrangement of the mathematics books without success.

(iii) There were similar errors to those in part **(ii)**. Many candidates realised that 4! x 3! was needed for the number of ways the mathematics books can be arranged amongst themselves and the number of ways the geography books can be arranged amongst themselves, Unfortunately, this was often multiplied by 3 or 4 suggesting that the number of arrangements was being counted instead of using 3! or the equivalent.

(b) (i) The majority of candidates were able to find the correct solution.

(ii) Many candidates were able to produce a correct solution by finding all the combinations that included 1, 2, 3 or 4 women in the team rather than the more efficient route of finding how many teams were available with only men and subtracting that from their solution for the previous part – this process would have been less prone to some of the arithmetical slips or omissions that were liable to occur in the longer method.

Question 7

The greatest problem that candidates had dealing with this question was being able to interpret the information in order to draw a suitable triangle to start to work with.

(i) Many candidates were unable to draw a correct vector triangle. The basic understanding of bearings was generally sound, with *AB* usually drawn correctly. However, many diagrams were simply right-angled triangles with *AB* as the hypotenuse. It was also a common error to label the path *AB* as being at 650 kmh^{-1}. Those candidates who drew correct diagrams usually realised that triangle side lengths are in proportion to speeds and that the west to east wind assists the plane. However, wind direction was often shown as east to west so that the plane's course was the longest side of an obtuse-angled triangle. This gave a correct angle at *B* of $6.08°$ but incorrect bearing and speed for the plane. Some candidates drew acute-angled triangles with no regard to relative speeds and lengths and, although they were able to get an angle of $6.08°$, were then unable to use their triangles to convert this angle to the correct bearing. Candidates were adept at applying both sine rule and cosine rule to their diagrams to find angles and speeds.

(ii) A variety of methods were used to find the resultant speed of the plane going *A* to *B*. As in part **(i)**, the majority of candidates used the cosine rule (or Pythagoras when they had right-angled triangles) for the resultant speed. Others misused vector arithmetic and produced a speed of 770 kmh^{-1} or 530 kmh^{-1}. Candidates very rarely confused speeds and distances in their diagrams, and were clear in their use of distance and speed to find the journey time. A few candidates calculated the actual distance travelled by the plane and divided by 650 for the time taken. From correct diagrams there were some errors in calculating the time taken due to inappropriate rounding of angles and/or speeds in the calculations, or simply due to careless rounding of the final answer.

There were, however, some exemplary solutions from capable candidates.

Question 8

(i) Many completely correct expressions were seen for e^y in terms of *x*. However, some candidates made errors in the final stage of their working when attempting to use logarithms. Many completely correct expressions for *y* were also seen. Most candidates realised that an equation of the form $e^y = \frac{m}{x} + c$ connected the variables *x* and *y*. Some candidates misused the given coordinates in the equation $e^y = \frac{m}{x} + c$ and thus obtained incorrect values for *m* and for *c*. Other candidates found the gradient of the straight line, *m*, correctly but then used a non-linear form to attempt to find *c*.

(ii) Although many candidates had a correct expression for *y* in part **(i)**, few were able to find the correct values of *x* for which *y* is defined. Of those that made a reasonable attempt, some made an error with the inequality sign or used $32 - \frac{6}{x} > 1$ rather than $32 - \frac{6}{x} > 0$.

(iii) Correct solutions were often seen from those candidates who had a correct expression for *y* in part **(i)**. Some of these candidates gave an exact answer which was then followed by the decimal equivalent. This was condoned as an exact answer had been seen. Those candidates who gave a decimal answer only, however, were unable to gain any credit.

(iv) Unfortunately, it was again fairly common to see solutions that used inexact values for e^2 and hence a decimal answer for *x* rather than an exact value of x as required by the question.

This question highlights the importance of candidates to understand and appreciate the meaning of the word 'exact' when used in a mathematical context.

Question 9

(i) Most candidates obtained the equation $2\cos 3x = 1$ and were thus able to find a value for $3x$. Some candidates then went astray in finding correct results for x. Many candidates then considered positive values only for x apparently not realising that P had a negative coordinate. The diagram had been given in order to help candidates decide on the appropriate values for x, but it was clear that many candidates had completely discounted the diagram. Many candidates offered solutions of $\frac{\pi}{9}$ and $\frac{5\pi}{9}$ which did earn them the first two available marks.

(ii) Most candidates attempted integration with many correct attempts seen. The most common error was to have the coefficient of $\sin 3x$ as 6. As the solution was required in exact form a method mark was available if the substitution of the candidate's limits was seen still in exact form, but many candidates went straight to a non-exact form. There were a number of candidates who did not make it clear which limits they were using in their work. Only a minority of candidates used the subtraction method which yielded the integrand $2\cos 3x - 1$. Most candidates considered the area of a rectangle either calculated using integration or by a simple application of length multiplied by width. Occasionally this area was omitted.

Question 10

(i) This question required candidates to have a clear understanding of the surface area and the volume of a solid. Most candidates made a good attempt at the required proof, realising that they needed to write formulas for both volume and surface area and eliminate h, the height, between the two formulae. Many candidates managed this very efficiently, but others were less careful with their algebra. Many considered the container to be closed when considering the surface area and, as a result, many ended up with a term of $8x^2$ rather than the required $4x^2$. This was often corrected properly, but too many just 'obtained' the given result. It was essential that candidates write down that they were considering the surface area as some just wrote down terms which were not equated or allocated to surface area.

(ii) The majority of candidates recognised this as a maximum/minimum question and knew the process needed. Most candidates were able to differentiate the given equation correctly and then find x when $\frac{dS}{dx} = 0$ with few errors, although there were some who were unable to deal with $\sqrt[3]{250}$ correctly. Too many candidates did not find the value of S for often a correct value of x, highlighting the importance of checking that the demands of the question have been met. The correct use of the second derivative and subsequent conclusion of a minimum was quite common; however, it was essential that any calculations used were correct. A few candidates appeared to consider the gradient on either side of the turning point, but rarely presented their evidence in sufficient detail to make a convincing argument. A few candidates were unclear about the whole process and attempted to find a minimum value by erroneously setting the second derivative to zero and finding the value of x for this condition.

Question 11

Most candidates were able to use their problem solving skills and formulate a correct approach and order of operations to produce a clear solution for this unstructured question. There were very few errors in the differentiation of the product involved which also involved the use of the chain rule. Again, an exact response was needed so it was expected that candidates work with exact values/fractions throughout. Those candidates that did resort to the use of decimals, were able to obtain method marks and were not overly penalised for not working with exact values. There were many completely correct solutions to this question. Some candidates were less clear about the process and after successfully differentiating the product, set the gradient to zero and attempted to find what would have been the value of x at a stationary point.

ADDITIONAL MATHEMATICS

Paper 0606/13
Paper 13

Key messages

It is important that candidates are familiar with the rubric on the front of the examination paper as it will remind them of important facts which tend to be forgotten during the examination itself. It is important that the degree of accuracy is noted, as many candidates lost accuracy marks due to inaccurate working. It is also important that candidates ensure that they have fulfilled the demands of each question, giving answers in a specific form if required. Work should be set out clearly, showing sufficient steps in each stage of their solution.

General comments

The majority of candidates appeared to have sufficient time to work through the paper and also sufficient space in which to answer their questions. It was pleasing to note that when extra space was needed for an answer, work was done on additional sheets which were invariably annotated with the appropriate question number.

It should also be noted that many candidates did not obtain some of the accuracy marks available as they did not appreciate the implication of the word 'exact' in the mathematical context.

It should also be noted that when candidates are required not to use a calculator, this instruction extends to the whole question and not just the first two or three lines of a solution.

Comments on specific questions

Question 1

The majority of candidates were able to describe the relationship between the sets shown in the first diagram offering one of the two acceptable answers.

Correct answers for the description of the relationship between the sets in the second Venn diagram were not as common. Most candidates were able to identify that the set Z was contained in the intersection of sets X and Y but often found it difficult to use the correct notation involving subsets.

Question 2

This question was an example of where candidates did not always check that they had given their answer in the form required. Most candidates were able to obtain at least one correct term in the expression $p^{-\frac{3}{2}}q^{-\frac{7}{3}}r^{-3}$, often giving their final answer in this form. The question required candidates to find the index of each of the terms when the expression was written in the form $\frac{1}{p^a q^b r^c}$.

Question 3

Most candidates recognised the correct process that was needed to start the question. The two given equations were equated and simplified to a quadratic form equated to zero. There were some errors in the simplification of the terms involving *m*. The use of the discriminant was most common, although some attempted to use the quadratic formula. For those candidates who made errors in the simplification, it was not usually possible to make a conclusion that *m* could take any value from their work. This should have alerted them to the fact that there were errors in their work and that perhaps a check of their work may identify their errors.

Many correct results of $m^2 - 10m + 25 \geqslant 0$ were seen. However, few candidates were able to make a correct deduction from this result. It was expected that $m^2 - 10m + 25 \geqslant 0$ be written in the form $(m - 5)^2 \geqslant 0$ from which it could be deduced that whatever value *m* takes, as the term involving *m* is squared, the result will always be zero or greater than zero, implying either intersection of the line and the curve, or touching of the line and curve. It is important that candidates learn to write meaningful conclusions in questions of this type.

Question 4

(i) Most candidates recognised that differentiation of a quotient was involved and attempted such. The most common error appeared to occur in the differentiation of $\ln(2x^3 + 5)$, with candidates not using the result $\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)}$. Most errors involved the numerator of this fraction. Other errors involved getting the terms in the incorrect order, incorrect signs and omitting to square the term in the denominator. Occasionally, errors were made in the evaluation of a correct derivative for the given value of *x*.

(ii) That the use of the answer to part **(i)** was needed to find the answer to part **(ii)** was recognised by most candidates. However, some candidates appeared to start the question again or attempt to make substitutions of $2 + p$. Candidates should be guided by the mark allocation for a question. The fact that there is only one mark for this part of the question should indicate to a candidate that little work is involved.

Question 5

(i) Most candidates were able to produce a reasonable sketch of the given function. The values of *x* and *y* where the curve either touched or intersected the axes should have been either marked on the graph itself or stated next to the graph. It is important that candidates realise that it is the lower part of a quadratic curve that is being 'reflected' in the *x*-axis, so there will be cusps on the *x*-axis, not minima and the outer parts of the curve above the *x*-axis do not curve back towards each other.

(ii) It was required that candidates recognise that they needed to find the value of *y* at the maximum point on their curve from part **(i)**. This could be done relatively simply by use of symmetry or by differentiation.

Question 6

(a) (i) It is important to show each step of working in a question of this kind. The demand is to show a particular result and even if the step appears to be trivial, such as $\frac{\cos^2\theta}{\cos\theta} = \cos\theta$, it should be included.

(ii) It was intended that the result from part **(i)** be used in part **(ii)** so that the equation being solved could be written as $\cos 2\theta = \frac{\sqrt{3}}{2}$. Most candidates did just this, recognising that a double angle was involved and dealing with it correctly.

(b) It was essential that the correct order of operations be recognised so that the trigonometric equation that ultimately needed to be solved was $\sin\left(\phi + \frac{\pi}{3}\right) = \pm\frac{1}{\sqrt{2}}$. Some candidates did not consider the negative root and, as a result, did not obtain all the possible solutions, while others only found intermediate values of $\phi + \frac{\pi}{3}$ in the first and second quadrants. Other candidates did not have a correct order of operations and were thus unable to obtain any marks.

Question 7

The use of a calculator was not allowed in this question, so it was essential that every step in a calculation be shown, even if considered trivial.

(i) A simple use of Pythagoras' equation was expected and most candidates did just this. It was essential that the expansions of both terms and subsequent simplification be shown in full. Many candidates did this and gained the appropriate marks.

(ii) This part of the question was designed to test the candidates' understanding of rationalisation. Again it was necessary to show each step of the rationalisation and subsequent simplification in full. Many candidates did this and gained the appropriate marks.

(iii) Use of the answer to part **(ii)** together with the trigonometric identity $\sec^2 ACB = \tan^2 ACB + 1$ was intended. Again, it was necessary to show each step of the calculation and simplification in full. Many candidates did this and gained the appropriate marks.

Question 8

(i) Many correct responses were seen. It was essential that the correct notation was used. For example, a statement of $x \geqslant 1$ would have received no marks.

(ii) Provided the correct order of operations was used, most candidates were able to show the expected result. Again, it was essential that each step in this solution was shown clearly.

(iii) Provided the notation was recognised and the appropriate differentiation used, most candidates were able to reach the correct answer.

(iv) Some correct responses were seen. It was important that the idea of symmetry about the line $y = x$ be either implied by a clear sketch and the coordinates (0, 1) and (1, 0) or by the actual inclusion of the line $y = x$. Unfortunately, many candidates did not remember the fact that the domain of g was $x \geqslant 0$, so many candidates drew a full quadratic curve for both functions.

Question 9

Most candidates recognised that part **(a)** concerned permutations and part **(b)** concerned combinations.

(a) **(i)** Most candidates obtained the correct result, but it was essential that 7! be evaluated.

(ii) Most candidates were able to obtain credit for realising that the number of ways of arranging the football trophies amongst themselves was 4!. Fewer though realised that the number of ways of arranging the remaining trophies amongst themselves was 4! and multiplying both together.

(iii) Many correct solutions were seen as candidates realised that they just needed the product of the number of ways that each of the cricket and football trophies could be arranged amongst themselves, together with the number of ways that three distinct items (treating each set of trophies as an item) can be arranged.

(b) **(i)** Most candidates obtained the correct result, but it was essential that ${}^{14}C_6$ be evaluated.

(ii) Most candidates obtained the correct result, but it was essential that ${}^{8}C_2$ be evaluated.

(iii) It was intended that candidates use the answer to part **(i)** with 1 subtracted, so that they were in effect considering the total number of possible teams with the number of teams containing just the boys subtracted. Unfortunately, many candidates chose to look at each possible situation with girls in the team. This often led to omissions and calculation errors. Candidates should be guided by the mark allocation for a question as to how much work is expected in a solution.

Question 10

(i) Provided candidates could integrate the function $(2x+3)^{-\frac{1}{2}}$ to obtain a form $k(2x+3)^{\frac{1}{2}}$ then most method marks were available. Some candidates omitted the arbitrary constant when attempting to find an expression for $\frac{dy}{dx}$. A second integration was attempted by most and provided candidates could integrate the function $(2x+3)^{\frac{1}{2}}$ to obtain a form $k(2x+3)^{\frac{3}{2}}$ then most method marks were available. Some candidates again omitted the arbitrary constant. Many correct solutions were seen.

(ii) It was essential that candidates realise that they had already been given the value of the gradient at the given point in the stem of the question. However, if they chose to use their $\frac{dy}{dx}$ from part **(i)**, they were not penalised, except in the final mark, if an incorrect value had been used. This was another example of the importance of checking that the answer for a solution is given in the form demanded. Several candidates omitted to give their equation with integer coefficients.

Question 11

The greatest problem that candidates had dealing with this question was being able to interpret the information in order to draw a suitable triangle to start to work with.

(i) Many candidates were unable to draw a correct vector triangle. The basic understanding of bearings was generally sound, with *AB* usually drawn correctly. However, many diagrams were simply right-angled triangles with *AB* as hypotenuse. It was also a common error to label the path *AB* as being 600 kmh^{-1}. Those candidates who drew correct diagrams usually realised that triangle side lengths are in proportion to speeds and that the north to south wind would reduce the resultant speed of the plane. Some candidates drew acute-angled triangles with no regard to relative speeds and lengths and although they were able to get an angle of 8.81°, were then unable to use their triangles to convert this angle to the correct bearing. Candidates were adept at applying both sine rule and cosine rule to their diagrams to find angles and speeds.

(ii) A variety of methods were used to find the resultant speed of the plane going from *A* to *B*. As in part **(i)**, the majority of candidates used the cosine rule (or Pythagoras when they had right-angled triangles) for the resultant speed. Others misused vector arithmetic to produce a speed of 720 kmh^{-1} or 480 kmh^{-1}. Candidates very rarely confused speeds and distances in their diagrams, and were clear in their use of distance and speed to find the journey time. A few candidates calculated the actual distance travelled by the plane and divided by 600 for the time taken.

From correct diagrams there were some errors in calculating the time taken due to inappropriate rounding of angles and/or speeds in the calculations, or simply due to careless rounding of the final answer.

There were, however, some exemplary solutions from capable candidates.

ADDITIONAL MATHEMATICS

Paper 0606/21
Paper 21

Key messages

In order to do well in this paper, candidates need to show full and clear methods in order that marks can be awarded. In questions where the answer is given, candidates are required to show that it is correct and fully explained solutions with all method steps shown are needed. Repeating information given in the question cannot be credited. In such questions candidates are encouraged to use consistent notation such as using the same variable throughout a solution and should avoid replacing a function of a variable with the variable itself. In questions that state that a calculator should not be used, omitting method steps often results in full credit not being given for a solution. Combining the steps directly into a simplified form, such as that produced by a calculator, cannot be credited. In questions that require a solution of several steps, clearly structured and logical solutions are more likely to gain credit. Candidates should be encouraged to write down any general formula they are using as this reduces errors and is likely to improve the accuracy of their solutions. In particular, method marks cannot be given for solving an incorrect equation when the solutions are taken directly from a calculator, without showing any working. When a diagram or graph is required then they should be completed in full and as accurately as possible taking note of specific features which are requested in the question.

General comments

Some candidates produced high quality work displaying wide-ranging mathematical skills, with well-presented, clearly organised answers. This meant that solutions were generally clear to follow. Other candidates produced solutions with a lot of unlinked working, often resulting in little or no credit being given. More credit was likely to be given when a clear sequence of steps was evident. Quoting a formula which referred to only part of the previous line then applying it on the next line led to candidates confusing themselves.

Questions which required the knowledge of standard methods were done well. Candidates had the opportunity to demonstrate their ability with these methods in many questions. Most candidates showed some knowledge and application of technique. The majority of candidates attempted most questions, demonstrating a full range of abilities.

Some candidates need to improve their reading of questions and keep their working relevant in order to improve. Candidates should also read the question carefully to ensure that, when a question requests the answer in a particular form, they give the answer in that form, particularly when the question states that an exact answer is required. Candidates should ensure also that each part of a question is answered and the answer clearly identified. When a candidate uses the blank page or an additional booklet they should make it clear which question their work relates to. It is not possible in most cases to connect work otherwise to a specific question which can lead to the loss of potential credit. When a question demands that a specific method is used, candidates must realise that little or no credit will be given for the use of a different method. They should also be aware of the need to use the appropriate form of angle measure within a question. When a question indicates that a calculator should not be used, candidates must realise that clear and complete method steps should be shown and that the sight of values clearly found from a calculator will result in the loss of marks.

Where an answer was given and a proof was required, candidates needed to fully explain their reasoning. Omitting method steps in such questions resulted in a loss of marks. Working from both sides and so treating an identity like an equation is not a valid way to prove a given result. Candidates should work from the left hand side to arrive at the result stated on the right hand side or begin with a quotable formula and rearrange it correctly.

Candidates should take care with the accuracy of their answers. Centres are advised to remind candidates of the rubric printed on the front page of the examination paper, which clearly states the requirements for this paper. Candidates need to ensure that their working values are of a greater accuracy than is required in their final answer.

Candidates should be advised that any work they wish to delete should be crossed through with a single line so that it can still be read. There are occasions when such work may be marked and it can only be marked when it is readable.

Comments on specific questions

Question 1

The majority of candidates were able to successfully expand the given inequality to form a quadratic. Solving the quadratic to find the correct critical values was then completed effectively by nearly all candidates. It was more often the case that the two inequalities required were then not expressed correctly. Candidates seemed to be determined to express the required region as a single inequality which was either the inside region and hence incorrect or a mathematically incorrect statement. Other candidates maintained the given inequality for both critical values or stated the critical values with no further work. Diagrams to aid the solution were rarely seen.

Question 2

(i) Good knowledge of the quotient rule was shown by many candidates. Clear bracketing in the numerator was an issue for some especially those who went straight to a final answer without clarifying either the factorisation or cancelling required as an intermediate step. Candidates need to be aware that, in 'show that' questions such as this, it is essential that the work presented is absolutely clear; it is difficult for an examiner to give full credit if it is not possible to tell whether multiple crossings out represent corrections or cancelling in working. This proved to be less of an issue for those who chose to use the product rule and this method was usually fully successful. As always, candidates are advised to quote the relevant formula before inserting appropriate expressions. This is particularly important to gain partial credit when an error is made in differentiating one of the terms although very few could not differentiate ln$x$ correctly.

(ii) This part was less well attempted. Frequently candidates omitted either to evaluate the derivative at x = e or to multiply by the factor h. Of those who substituted x = e, it was rare to see the exact form $\frac{2h}{e^4}$ rather the approximate decimal form, which was often rounded to 2 significant figures rather than the 3 significant figures required and clearly stated in the instructions on the front of the examination paper.

Question 3

(i) Many candidates were able to produce a sketch of the correct shape. Even the most limited answers seemed to show appreciation of the fact that a modulus graph can have no part below the x-axis. These sometimes appeared as curves or W shapes. The question required that sketches show the coordinates of the points where the graph meets the coordinate axes. Frequently, well-drawn graphs were unable to gain full credit as one or both of these points were not clearly labelled. It would also be advisable for candidates to make it clear which lines on their graph have been drawn as 'working' and which lines are their final answer – full lines for the answer and dotted lines for working were the most frequent, where this occurred, but an 'X' shape, with the centre on the x-axis, was seen several times. As the correct section was not identified, it could not be credited.

(ii) Most candidates could calculate the value of $\frac{5}{6}$ by treating $5x - 3$ as positive. There were also many correct solutions of $\frac{1}{4}$ by initially multiplying either expression by –1. A considerable number of candidates incorrectly changed the sign of one term only. A small proportion of candidates, having formed one or more correct equations, made errors in rearranging leading to an incorrect value.

Question 4

This question stated quite clearly that a calculator should not be used. Similar questions have been set on many recent papers and candidates have been advised that full working must be shown at each stage in order to acquire full marks. The vast majority appreciated this and as a result this question was very well answered, although at times it was necessary to search the answer space for the required expansions. Almost universal familiarity was demonstrated of the rationalisation process.

Question 5

(i) Most candidates were aware that the gradient of the graph would give the acceleration, but many did not appreciate that the gradient was negative and that, as a result, their answer should also be negative. Some candidates even changed a negative answer to a positive one as a last step.

(ii) There were many correct solutions to this part although a small minority subtracted 4 from 23 rather than adding it.

(iii) Candidates who obtained the correct result in part **(ii)** were almost always correct in this part. They either found the area as a trapezium or as the sum of two triangles and a rectangle. Candidates who had found an incorrect *k* were still able to show a correct method although it was important that they realised that the 'top' side of the trapezium/rectangle, 23, was given in the question.

Question 6

(a) Some candidates did not appreciate the need to use the determinant and so made no progress. Others who correctly found $x^2 = 3$, only gave the positive root or gave two roots but in decimal form and ignored the need to find exact values.

(b) **(i)** This was well done by the majority. There were a significant number who reversed the order and several listed the given digits for **B** in order.

(ii) This part proved more challenging for candidates. Some chose to evaluate the matrix **CB** explicitly and were usually successful; others tried to use the shorter route of looking at or stating the orders of the matrices **BC** and **CB**. Some candidates effectively answered the question twice by doing both of these. Some candidates recalculated **BC** even though it was given and some stated the orders the wrong way around. Explanations in words were often difficult to follow although they often contained enough reference to the order of the matrices to gain some credit. Answers stating that matrix multiplication was not commutative gained no credit as this was just a restatement of the question.

Question 7

(i) This part was answered well by many candidates. There were also many incorrect or incomplete answers with derivations rather than statements. This is a standard rule and the expectation was that the result should be stated.

(ii) Good answers to this question showed clear understanding of the derivatives required and how the derivatives are connected through the chain rule. There were few such answers, with the result that this was possibly the least well answered question on the paper. Often the variable *t* appeared, presumably because it was thought that 'rate of change' automatically meant time was involved. Incorrect relationships between derivatives involving *x*, *y*, *u* and *t* were often presented and the only credit gained in many cases was for expressing *u* in terms of *x*.

Question 8

(i) This question highlighted the need for candidates to work with values which were as accurate as possible during their working before arriving at their final answer in order to give the final answer to 3 significant figures, as required in the instructions on the front of the examination paper. It is advisable to work with at least 4 significant figures or to use the memory on a calculator to achieve this. This was an unstructured question and required candidates to develop a strategy for finding the required area. This strategy was sometimes difficult to follow but was often fully correct. Candidates need to be aware that they are much less likely to make an error if they clearly identify the values they are trying to find. This can be achieved by clearly referencing the diagram and/or making a statement of what a calculation represents. Many fully correct methods concluded with an insufficiently accurate value. The range of such answers was extremely wide usually as a result of having rounded some or all of *AC* or *BC* or the angle *ECD* at an early stage. A final answer of 1.9 was not uncommon but, without a more accurate value preceding it, it was not accepted.

(ii) Accuracy was less of an issue in this part, possibly as the final answer rounded to 12.0. Candidates who did not find the correct answer often confused themselves because of a lack of clear labelling of calculations. Most candidates could find the length of the arc *BD* and the majority of these identified the appropriate method. There was scarcely any confusion between areas and arc lengths in the two parts of the question.

Question 9

(a) (i) Nearly all candidates were able to find the correct value for this part.

(ii) This part was done well but was not as successful as part **(i)**. Common incorrect answers were either from doubling the correct answer or using 11!.

(b) (i) Many candidates were successful with this part although it was not uncommon to see permutations used or combinations based on 11 rather than 5.

(ii) Candidates scored less well on this part although there was a correlation between the success rates on the two parts of part **(b)**. Omitting one of the two cases or including a third were the most common incorrect methods.

Question 10

(i) Almost all candidates scored full marks in this part. The standard method was to use the gradient formula and equate it to $\frac{1}{3}$ then solve to find *p*. There were very few errors in solution and even fewer who applied the formula incorrectly, usually by inverting it.

(ii) Good mathematics was seen frequently in this part but it was often incomplete for the proof required. It was necessary to find the mid-point of *AB* and to show this point was on the line *L*. It was also necessary to show that the two lines were perpendicular. The most commonly omitted of the three required elements was the proof of perpendicularity. On the other hand some candidates seemed to think that this was the only element to be considered, and the mid-point of *AB* was never found. As both gradients were essentially given it was insufficient to state that these were perpendicular or that one was the negative inverse of the other unlike questions which required the perpendicular bisector to be found. As candidates were asked to show this the connection needed to be clearly made.

(iii) This was generally well done with very few errors in solution.

(iv) The most common solution used here involved the 'shoe-lace'/'determinant' method which was usually applied well with few omissions of either the $\frac{1}{2}$ or the repeated point. More elaborate solutions often were very lengthy and rarely led to the correct answer. These usually involved combinations of rectangles and triangles but were often attempted without a sketch which would have been helpful. A common mistake was to assume that the triangle was right-angled at either *A* or *B* which, due to the shape of the isosceles triangle, often gave an answer which was very close to the correct one.

Question 11

(a) (i) Most candidates were able to start correctly replacing cosecθ and cotθ so that all terms involved sinθ and cosθ only. There were then many concise solutions which rearranged to a single denominator and used the appropriate identity. Factorising the denominator and clear cancelling were more challenging with the temptation to cancel just a cosθ and assume that the sign would change a fairly common final step. Some solutions, on the other hand, were far from concise and some candidates took many steps to gain a term with a single denominator or used tanθ for several lines before replacing it. It was less common to see candidates combining the two sides of the identity which is mathematically incorrect as it assumes the result to be correct in order to prove it. Candidates are advised to maintain a clear flow to their solutions and to avoid side working so that their proof can be followed. It is also advisable to include the variable in all terms throughout and to make it clear when they are cancelling as opposed to crossing out unwanted work.

(ii) Candidates were instructed to use the previous part and the vast majority did so and were successful. Not all gained full credit as additional answers within the required range were given or the answer was not given to one decimal place, as indicated in the instructions on the front of the examination paper. It is good practice for candidates to state a more accurate value before rounding an answer to the required accuracy. Some candidates omitted the correct answer and gave solutions in incorrect quadrants only.

(b) Most candidates were able to make initial progress with this part by using the inverse tangent appropriately. Both of the solutions which lay within the required range were derived from values of the inverse tangent which were negative. As a result most candidates omitted one of the solutions, usually 0.132, or gave answers outside the range. On this occasion, there were very few candidates who made an initial order of operations error, writing the given tangent term as the difference of two tangent terms.

Question 12

(a) Many candidates integrated to a term involving e^{2x} although coefficients of 1 and 2 were almost as common as the correct $\frac{1}{2}$. A coefficient of 2 made it unclear whether a candidate had actually differentiated despite the notation used. Terms involving e^{2x+1} and e^{3x} were also seen. Substitution of limits was usually applied correctly. The rearrangement to find *a* was not always accurate with sign errors fairly common. The question asked for an exact value with all working shown. It was therefore important that both the application of logs was explicit and that the final answer was given in an exact form. Many candidates spoilt an otherwise perfect solution by changing their exact form to a rounded decimal equivalent.

(b) (i) Candidates found this challenging with only a minority managing to get a fully correct answer. Many candidates treated this as an exercise in working out a tangent which gained no credit. There was frequent confusion when attempting to integrate the cosine function with both the factor of 5 and the negative sign causing problems. Terms involving $\cos^2 5x$ were also fairly common. The constant of integration was omitted in some cases or calculated as an inaccurate decimal rather than the exact value, π.

(ii) Success here correlated greatly with a correct or good attempt at the previous part. Errors in integration were often repeated here and some candidates even repeated the same integration. Those who carried out the two required integrations correctly usually arrived at an accurate final value. It is advisable when substituting limits into an integral to show this explicitly before showing decimal equivalents for each limit or the overall answer particularly when the integral is incorrect as it is not possible to award method marks otherwise.

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

Candidates should read each question carefully and identify any key words or phrases. Candidates also need to show enough method so that marks can be awarded. Candidates need to be aware of instructions in questions such as 'Showing all your working…' or 'Show that…'. Such instructions mean that when a solution is incomplete, often through calculator use, a significant loss of marks will result. Candidates should ensure that their answers are given to a greater degree of accuracy than that demanded in a question before they round as required. When no particular accuracy is required, candidates should ensure that they follow the instructions printed on the front page of the examination paper. This is especially the case for angles measured in degrees. Candidates need to take care to ensure that their calculator is in the appropriate mode when working with trigonometric expressions. When finding angles in radians, it is better to have the calculator in radian mode rather than to find the angle in degrees and then make a conversion.

General comments

Most candidates seemed to be well prepared for this examination and many excellent solutions were seen. Candidates were able to recall and use manipulative techniques when needed. Many candidates were also able to write problems using correct mathematical form. Some candidates may have improved if they had had a better understanding of the necessity to use correct bracketing to ensure correct mathematical form for functions with arguments, such as trigonometric functions and logarithms. This was seen in **Questions 1** and **7(b)(i)** in this examination.

Candidates usually presented their work in a clear and logical manner. Some candidates used additional paper for **Questions 7(b)(i)** and **8(a)**. This ensured that their work was readable and could be marked. Candidates who did this usually added a note in their script to indicate that their answer was written, or continued, elsewhere. This was very helpful.

Showing a clear and complete method for every step in a solution is essential if a question asks candidates to 'Show that…' a result is in a particular form. This instruction indicates that the answer has been given and that the marks will be awarded for the method. The need for this was highlighted in **Questions 3(i)**, **7(b)(i)** and **9(b)(i)** in this examination.

Candidates should also understand that, when a part of a question begins with the word 'Hence...', it is expected that they should use the previous part or parts of the question to answer this part. This will often be the most straightforward method of solution and will be assessing a specific skill. This was seen in **Questions 3(ii)**, **5(ii)**, **6(b)(ii)**, **7(b)(ii)** and **9(b)(ii)** in this examination.

Most candidates attempted to answer all questions. Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

A good number of candidates were able to differentiate sin$x$ correctly. Not as many candidates were able to differentiate lnx^2, with $\frac{1}{x^2}$ frequently seen. A good number of candidates were able to apply the correct form of the quotient rule, with the difference in the numerator correct and the denominator correctly squared. It was important to have correct bracketing in the final answer to ensure that the expression given was unambiguous. Candidates would have improved by writing $\frac{(\ln x^2)(\cos x)-(\sin x)\times\left(\frac{2}{x}\right)}{(\ln x^2)^2}$ as their first line, for example, rather than $\frac{\ln x^2 \cos x - \sin x \times \left(\frac{2}{x}\right)}{(\ln x^2)^2}$, which had ambiguities in both terms of the numerator and did not earn the accuracy mark. A few candidates incorrectly gave the denominator as lnx^4, confusing what was being squared. A small number of candidates attempted to use the product rule. This was much less successful as, often, the rearrangement to a product was incorrect and also the derivative of $(\ln x^2)^{-1}$ was more complex.

Question 2

A good number of fully correct solutions were seen. Most candidates started correctly and formed a correct expression for the discriminant. Those who were sufficiently careful arrived at a two-term quadratic expression which was simple to factorise by extracting the common factor, k. A few candidates, even after writing $c = -k$, used $c = 1$ or $c = k$ in their expression. Some candidates were unable to deal with the significant number of minus signs in the expression and errors in simplification were not uncommon. Those who made simplification errors usually had to apply the quadratic formula to find their critical values. Candidates who drew sketches usually obtained a pair of inequalities of the correct form for their final answer.

Question 3

(i) Most candidates used the factor theorem correctly, as required, with many earning full marks. The most efficient process was to solve $a(2^3) - 12(2^2) + 5(2) + 6 = 0$. As the answer was given, sufficient method needed to be shown to ensure that the accuracy mark could be awarded. An interim equation with simplified values, such as $8a - 48 + 10 + 6 = 0$, was required. A few candidates used synthetic division with algebraic expressions, which was allowed as this process uses the root and hence the factor theorem had been applied. A few candidates attempted long division. This was not permitted as, whilst the process itself was not incorrect, it did not answer the question. A few other candidates benefitted from the special case mark available for using $a = 4$ and showing that the result was 0. This was not given full credit as the process had been eased.

(ii) Many candidates were successful here and offered fully correct and complete solutions. Some candidates omitted to state the full factorisation of the cubic expression. These candidates commonly factorised the quadratic factor and then stated all three roots. As the cubic expression was required in factorised form, this was not credited. Occasionally candidates only stated the two roots which arose from the quadratic factor. Some candidates did not factorise but found the roots applying the quadratic formula to the quadratic factor equated to 0. This was not credited as the instruction in the question was clear. The roots were to be found by factorising the cubic expression and then solving. Some candidates ignored the given factor $x - 2$ and used one of the other factors. For this to be credited, candidates needed to justify that the factor they had used was indeed a factor. This was not always seen and some candidates are still too reliant on their calculator for solving cubic and quadratic equations. This was very evident from candidates who composed the incorrect factorisation $(x-2)\left(x-\frac{3}{2}\right)\left(x+\frac{1}{2}\right)$, which was not credited.

Question 4

Many candidates found this question quite challenging. Better candidates determined that the radius was $\frac{x}{2}$ and worked correctly with this, obtaining the correct, exact instantaneous rate of change, which was required as their final answer. These candidates used the correct notation, stated a correct chain rule and worked in terms of π throughout. A few candidates decimalised their answer or worked with decimals, resulting only in the correct decimal form of the answer. This did not earn the accuracy mark as an exact form was required. A few candidates worked with δx and δA rather than $\frac{dx}{dt}$ and $\frac{dA}{dt}$. Many of these candidates were often able to form a correct chain rule and obtain the correct answer, although others confused themselves because of their incorrect descriptions. Some candidates confused x with r and $A = \pi x^2$ as an incorrect starting point was common. A few candidates were able to form the correct expression for the area in terms of x but were unable to differentiate it correctly, with $\frac{2\pi x}{2} = \pi x$ being very common. A few candidates used $\frac{1}{2}r^2\theta$ for the area. This introduced an unnecessary complication and many of these candidates did not use $\theta = 2\pi$. There were a few attempts to find the average rate of change rather than the instantaneous rate of change. This did not answer the question.

Question 5

(i) Many candidates answered this correctly. A few candidates needed to take more care with brackets as, on occasion, the value of r was incorrect and this was commonly as a result of a bracketing error. A few candidates did not give an expression of the required form. Other candidates did little more than factor 5 out of the first two terms.

(ii) This part of the question assessed the ability of candidates to interpret their expression once they had completed the square. Candidates, therefore, needed to use their answer to part **(i)** to answer this part. Some candidates were able to do this successfully and simply wrote down that the least value was $\frac{1}{5}$ of –10.25, i.e. –2.05, and this occurred when $x = 1.5$. Other candidates used calculus to find the value of x at which the expression was a minimum and substitute. This was only accepted if the values obtained corresponded to the values which followed from part **(i)**. Calculus could have been used as a check, of course, and this may have helped some candidates make corrections to part **(i)**, where needed. Some candidates were unable to correctly identify the values they had found and stated that the least value was 1.5 and it occurred at –2.05, for example. A few candidates restarted in this part and completed the square again, rather than observing that the expression in this part was $\frac{1}{5}$ of the expression in part **(i)**. This was allowed as the required interpretation still needed to be carried out. A few candidates divided 1.5 by 5 as well as dividing –10.25 by 5. This was incorrect as the value of x was unaffected by the transformation made. These candidates could have *checked* this using calculus if they had wished. Weaker candidates tended to find only the roots of the given expression set equal to zero.

Question 6

(a) This part was almost always correctly answered. A few candidates stated 4 by 2 and some simply described the number of rows and columns which was not accepted.

(b) (i) A very good number of fully correct answers were seen. Occasionally, sign slips resulted in the determinant of **A** being incorrect with –2 and 10 both seen on occasion. The adjoint matrix was almost always correct. Occasionally candidates changed the signs of the elements in the leading diagonal and changed the positions of the terms in the other diagonal.

(ii) Again, a good number of fully correct answers were seen. Most candidates used the correct strategy of squaring the inverse matrix found in part **(b)(i)** and earned three marks here very efficiently. Other candidates found the matrix $\mathbf{A}^2$ and then found the inverse of that. This was allowed for full credit on this occasion. Some candidates did not recall the matrix **I** correctly. The matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ was seen and used several times, as was $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, for example. Solutions attempted using simultaneous equations were more complex than necessary and much more prone to error than solutions using the product of inverse matrices, which was expected.

Question 7

(a) A good number of candidates gave fully correct solutions. Many candidates were able to make a correct first step in the method, commonly $x^2 - 3 = 10^0$. Some candidates omitted the negative solution while other candidates stated it and then disregarded it because it was negative. This was not appropriate for this question and candidates should look carefully at what is given before disregarding solutions. A few candidates incorrectly wrote $x^2 - 3 = e^0$, which was not accepted. A few candidates did not interpret the logarithm correctly and wrote $\lg x^2 - \lg 3 = 0$, indicating a total misunderstanding. Other candidates wrote $\frac{\lg x^2}{\lg 3} = 0$, misapplying the subtraction of logs/division of arguments rule.

(b) **(i)** A reasonable number of fully correct answers were seen. As the answer had, in effect, been stated, full method for each step needed to be seen for marks to be awarded. There were many approaches available to candidates in manipulating the expression given to the required form. Candidates whose solutions were fully correct showed full method for each step and were careful with brackets, ensuring their statements were mathematically correct, as required. Some candidates were able to write the answer down because the required form had been given but showed insufficient working to be credited. Weaker candidates tended to 'cancel' ln$a$ and state the answer as $\sin(2x + 5) + 1$, for example. Many candidates omitted brackets in this question and expressions such as '$\sin(2x + 5) - 1\ln a$' were not accepted. Some candidates attempted to verify that the expression could be written in the required form for particular values of a, such as 1 or e. This did not answer the question and so was not credited. A few, very weak, candidates attempted to differentiate using the quotient rule. These candidates needed to read the question more carefully.

(ii) Most candidates were able to access this part of the question. It should have been clear to candidates from part **(b)(i)** that they needed to integrate an expression of the form $\sin(2x + 5) + k$ in this part. All three marks could be earned provided this was the case and many candidates were able to do so. Those who did not often earned two marks for integrating the sine term correctly or one mark for a reasonable attempt at integrating the sine term.

Question 8

(a) Many complete, neat and fully correct solutions were seen. Those who identified the correct terms from the general term offered concise and simple solutions with very few errors. Candidates who wrote out the full expansion and then selected terms, occasionally made slips in earlier terms that impacted on the accuracy of the terms needed to answer the question. A few candidates worked accurately until the final step and, at this point, either only stated the positive solution or stated both solutions and then disregarded the negative one, incorrectly. It was clear in the question that more than one value was expected and these candidates may have improved by rereading the question before determining their final answer. A good number of candidates were able to find the correct two terms. However, many then went on to form an incorrect equation, usually by multiplying the coefficient of x^3 by 120. These candidates needed to take more care when reading the key information given in the question. Commonly, weaker candidates forgot to include any powers of two in their coefficients. Some candidates did not understand the difference between 'term' and 'coefficient' and worked with x all through, including it in their final answer. There were occasional misreads or misinterpretations of the question with candidates choosing, for example, the terms in a^3 and a^5 or the 3rd and 5th terms.

(b) (i) This part of the question was well answered with many candidates able to find the correct four terms. A few candidates stopped at the term in x^2. It may be that these candidates misinterpreted the question as being 'as far as the 3rd term'. Almost all expansions were simplified and many were fully correct.

(ii) Some candidates struggled to understand what was needed here. Better candidates understood the need to substitute $x = -0.01$ into their answer to part **(b)(i)**. Many of these found the accurate decimal arising from this process, 0.66688, and then concluded that this rounded to 0.67. The decimal 0.66688 needed to be seen for the accuracy mark to be awarded. Candidates should take care not to offer values they have already rounded or truncated which they then round again to draw a conclusion, as some did. Many candidates used their calculator to find 0.98^{20}, wrote down the long decimal from it and then rounded. This did not answer the question and was not credited. Others found that x should be –0.01 but then wrote $(1 + 2(-0.01))^{20} = 0.667$, which again did not answer the question. Other candidates substituted $x = 0.98$, incorrectly.

Question 9

(a) This part of the question was generally well answered. The majority of candidates used a correct initial strategy and correctly transformed the equation to one in terms of $\cos x$ only. A few candidates used $\cos^2 x - 1$ and these may have improved if they had written $\cos^2 x + \sin^2 x = 1$ and rearranged it, rather than trying to recall the expression for $\sin^2 x$ in terms of $\cos^2 x$ directly. A few candidates needed to take a little more care with rearrangement, as sign errors were occasionally seen. Many candidates, though, were able to form a correct quadratic equation in $\cos x$ and solve it to find the correct values $\frac{1}{3}$ and $-\frac{5}{2}$. Usually, these candidates were able to state the correct pair of solutions for x to an acceptable degree of accuracy. A few candidates needed to take more care to observe the instructions on the front of the examination paper. Angles in degrees should be rounded to one decimal place. Angles which were truncated to one decimal place or rounded to 3 significant figures were, therefore, not accepted. This, in particular, affected the solution in the 4th quadrant. Weaker candidates sometimes wrote $-13\cos x = 1 - 6\sin^2 x$ which then, incorrectly, became $-13\cos x = 6\cos^2 x$.

(b) (i) This was done very well by many candidates. As this question required candidates to 'Show that' a result of a particular form should be obtained, it was very important for any step in the solution to be fully justified. Simply listing relationships on the side of the page did not meet this requirement, unless it was perfectly clear at what stage a relationship had been used and the relationship stated was explicit for the step required, not implicit. The simplest way to ensure that each step was fully justified was to substitute using the correct relationships. Some candidates gave very neat, concise and accurate solutions, replacing $1 + \tan^2 y$ with $\sec^2 y$ and $\tan y$ with $\frac{\sin y}{\cos y}$. This almost always resulted in full marks. A few candidates needed to do a little more work in the denominator as they used $\tan^2 y = \frac{\sin^2 y}{\cos^2 y}$ and then had some rearranging to do before they reached an equivalent point in the solution. This was a little more prone to error. Some candidates decided to square the expression before substituting. This altered the question and was therefore not credited. A few candidates needed to take more care with writing $\frac{\sin y}{\cos y}$ as $\frac{\sin}{\cos} y$ was not uncommon and not acceptable. Many candidates seemed to know that $\tan y \cos y$ is $\sin y$. It was unfortunate that these candidates often did not justify the second step by showing why this was the case. These candidates needed to know that in a 'Show that' question, all steps need to be justified for full credit to be given. Some candidates attempted to 'rationalise' the denominator. This rarely produced a solution of any value. Some other candidates made the error $4\tan y = \frac{4\sin y}{4\cos y}$. Again, a few, very weak, candidates attempted to differentiate using the quotient rule. These candidates needed to read the question more carefully.

(ii) By contrast, this question was poorly answered. A few fully correct solutions were seen, usually from those using their calculator to find the angle in the 4th quadrant directly and then understanding that this was the only possible solution. Many candidates confused themselves by using $\sin^{-1}(0.75)$ to find the angle in the 1st quadrant and included this positive value as an answer. This was clearly incorrect and this confusion could have been avoided. Candidates who insist upon using this method should use a different letter, as some did, for their base angle and should make it perfectly clear what values they are offering for their answer. Some candidates offered answers accurate only to 2 decimal places, with no longer, more accurate, decimal being stated. These candidates needed to take more notice of the instructions, printed on the front of the examination paper that inexact answers, with the exception of angles in degrees, should be given to 3 significant figures. These candidates may have improved if they had written the longer decimal value down before they rounded. Some candidates worked in degrees and then converted to radians. These were rarely sufficiently accurate to score.

Question 10

(a) A very good proportion of candidates found an acceptable form of the correct unit vector. Those few who were incorrect sometimes multiplied by the magnitude, rather than dividing by it, not fully understanding the meaning of the term unit vector. Other candidates were unable to apply Pythagoras correctly to find the magnitude, or only found the magnitude. This, however, was uncommon. A few candidates needed to take a little more care as, on occasion, sign slips were made when composing their answer.

(b) (i) A good number of accurate and concise solutions were seen for this part. The use of a reasonable diagram was helpful to many candidates. Many candidates found $\overrightarrow{AB}$ and then used a correct vector route, either adding $\frac{2}{3}\overrightarrow{AB}$ to $\overrightarrow{OA}$ or adding $-\frac{1}{3}\overrightarrow{AB}$ to $\overrightarrow{OB}$, to find $\overrightarrow{OC}$. Candidates had to interpret the need to find $\overrightarrow{OC}$. A few candidates were unable to do this and gave the vector $\overrightarrow{CO}$, or occasionally $\overrightarrow{AC}$, as their final answer, which was not credited. A few candidates successfully used $\begin{pmatrix} x \\ y \end{pmatrix}$ for $\overrightarrow{OC}$ and then formed a correct equation using $\overrightarrow{AC} = 2\overrightarrow{CB}$. Other candidates attempted this but made no progress as their starting equation was $2\overrightarrow{AC} = \overrightarrow{CB}$, which was an incorrect interpretation of the given ratio. A few candidates attempted to find the midpoint of $\overrightarrow{AB}$, again misinterpreting the information given. A few other candidates chose algebraic methods involving the magnitude of vectors which they used to form various quadratic equations. Success using these, often very complicated, approaches was very limited and rarely did they result in a correct vector.

(ii) A small number of fully correct solutions were seen. Many candidates were able to find a correct, or correct follow through, vector $\overrightarrow{OD}$. Few of these were able to use this to find the correct value of λ. Those candidates who formed the proportions $\frac{OD}{OB} = \frac{1}{\lambda}$ and then used $\lambda\overrightarrow{OD} = \overrightarrow{OB}$ were most successful. Those who rearranged to $\overrightarrow{OD} = \frac{1}{\lambda}\overrightarrow{OB}$ often stated that $\lambda = \frac{1}{4}$. Some candidates misinterpreted the ratio given, commonly as $OD : DB$ is $1 : \lambda$. These candidates usually stated the answer $\lambda = 3$. A few candidates successfully used the magnitudes of $\overrightarrow{OB}$ and $\overrightarrow{OD}$ to find λ. This was given full credit if exact values were used. Candidates using this approach and choosing to use inexact decimal values were not credited.

Question 11

(i) Many candidates seemed to have an understanding of what was required but were unable to give a sufficiently rigorous explanation to score. It was necessary to indicate that the velocity of this particle could not be zero. This could be done by stating exactly that or commenting that the velocity was always greater than 0, for example. Stating that the velocity could never be negative, only, was insufficient as it did not indicate that the velocity could not be zero. Many comments were based around time always being positive and often these also then mentioned that the velocity was always positive, which was credited. However, some did not explicitly mention velocity or the implication of it was far too vague for marks to be awarded. It is important to make explanations explicit and clear and, should justification be needed, to then state that afterwards. Some candidates seemed to be trying to justify the velocity always being positive but, as they never actually stated this, could not be credited. A very common incorrect answer was that the particle was travelling in a straight line. This showed a lack of understanding as candidates seemed to think this meant it was not possible for the particle to change direction. Another common incorrect answer was 'the velocity is always constant'. There were many comments, with their basis in physics, concerning there being a lack of other forces acting on the particle. These comments were not based upon the information given in the question and were not credited.

(ii) The most efficient method of finding the acceleration was to differentiate the displacement, with respect to t, using the chain rule. Candidates who did this usually earned both marks. A few candidates chose to use the quotient rule. These solutions were slightly more prone to error as some candidates differentiated the constant 4 as 1 rather than 0. A few other candidates were unable to recall the correct form of the quotient rule correctly, which also resulted in an incorrect expression. Many candidates attempted to substitute $t = 5$ into the correct expression. However, some candidates changed the sign of their final answer, either thinking that acceleration was not a signed quantity or thinking that acceleration was always positive, perhaps. A few, weaker, candidates did not differentiate and some tried to use *suvat* equations here. A few other candidates needed to take a little more care as it was not uncommon for candidates to write $(t + 3)$ instead of $(t + 1)$ and often this was simple carelessness and not a misread of the question, as the expression was written correctly elsewhere.

(iii) A good number of candidates understood the need to integrate and did so correctly. Some of these candidates omitted to find the value of the constant of integration or stated that it was zero or made sign errors when attempting to find it. A small number of candidates attempted to integrate the correct expression but made a sign error or multiplied by –2, rather than dividing by –2, for example. A few candidates incorrectly thought that $\int 4(t+1)^{-3}\mathrm{d}t = \int 4\mathrm{d}t \times \int (t+1)^{-3}\mathrm{d}t$. Weaker candidates stated that $d = vt$, often stating 'distance = speed × time' also, and then gave their answer as $\frac{4t}{(t+1)^3}$.

(iv) Candidates needed to interpret the distance travelled in the fourth second as being the difference in the displacements when $t = 3$ and $t = 4$. A few candidates did this and were sufficiently accurate to earn full marks. Many candidates misinterpreted the question as being the displacement travelled when $t = 4$ only. Another misinterpretation was that the required distance was the difference in displacements when $t = 4$ and $t = 5$. These partially correct approaches earned a method mark as long as sufficient evidence was seen. Some candidates integrated the expression for v again, even though they had already done this work in part **(iii)**. A few candidates completely ignored their answer to part **(iii)** and used $d = vt$, with $t = 4$, in this part. A few other candidates calculated the displacements at $t = 1$, 2, 3 and 4 and summed all 4 quantities.

Question 12

(a) **(i)** This part of the question was well answered, with many candidates stating the correct range in an acceptable form. A few candidates seemed to misinterpret $x > 0$ as $x \geqslant 1$ and the answer $g \geqslant -5$ was not uncommon from those who were incorrect. Other candidates who were incorrect stated $g > -5$ and it may be that these confused the domains of f and g or that they misevaluated $4(0)^2 - 9$. A few other candidates stated $g \geqslant -9$ or $x > -9$, which was not accepted as x was not appropriate for the range of this function.

(ii) (iii) Please note that due to an issue with part **(iii)**, full marks have been awarded to all candidates for parts **(ii)** and **(iii)** in order that no candidate will be disadvantaged. The published question paper has been amended to remove the issue.

(b) **(i)** Many candidates were able to state that the functions were reflections of each other in the line $y = x$. A few candidates mentioned reflection but omitted to state the equation of the line or stated that the line was one of the axes or $y = -x$, for example. Some candidates only made comments about the domain of one being equal to the range of the other and did not comment on the geometrical relationship. Other candidates commented that the functions were one-one or that the x-coordinate of one graph was the y-coordinate of the other, but this again was not a description of the geometrical relationship. Some candidates described the relative positional relationship of the two graphs, but these comments were not sufficiently rigorous to be credited. It was common for candidates to state, for example, the graph of the inverse function will be the opposite of the graph of the function.

(ii) A few candidates were able to utilise the domain of h and stated the negative square root of the correct expression as their answer, earning full credit. A very high proportion of candidates were able to earn two marks, most commonly for finding the positive square root rather than the negative, or for not discarding the positive square root when both signs had been considered, or for leaving their final answer in terms of y. A few candidates made a circular argument and ended up with the same function they had started with. These candidates had usually confused themselves after the point where they swopped the variables. Weaker candidates sometimes square-rooted the expression $x^2 - 1$ term by term as a first incorrect step and were unable to recover.

ADDITIONAL MATHEMATICS

Paper 0606/23
Paper 23

Key messages

Candidates should read each question carefully and identify any key words or phrases. Candidates also need to show enough method so that marks can be awarded. Candidates need to be aware of instructions in questions such as 'You must show all your working', or 'do not use a calculator in this question'. Such instructions mean that when a solution is incomplete, often through calculator use, a significant loss of marks will result. Candidates should ensure that their answers are given to a greater degree of accuracy than that demanded in a question before they round as required. When no particular accuracy is required, candidates should ensure that they follow the instructions printed on the front page of the examination paper. This is especially the case for angles measured in degrees. Candidates need to take care to ensure that their calculator is in the appropriate mode when working with trigonometric expressions, particularly in calculus questions.

General comments

Most candidates seemed to be well prepared for this examination and many excellent solutions were seen. Candidates were able to recall and use manipulative technique when needed. Many candidates were also able to write problems using correct mathematical form. Some candidates may have improved if they had had a better understanding of the necessity to use correct bracketing to ensure correct mathematical form for functions with arguments, such as trigonometric functions. This was seen in **Question 2** in this examination.

Candidates usually presented their work in a clear and logical manner. Some candidates used additional paper for **Question 8(b)**. This ensured that their work was readable and could be marked. It would have been helpful if candidates who did this added a note in their script to indicate that their answer was written, or continued, elsewhere.

Candidates should also understand that, when a part of a question begins with the word 'Hence...', it is expected that they should use what they have just done to answer the next part of the question. This will often be the most straightforward method of solution and usually assesses a specific skill. This was seen in **Question 8(b)** in this examination.

Most candidates attempted to answer all questions. Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

This proved to be a good start to the paper for most candidates. The majority of candidates rearranged to a correct quadratic expression and found the correct critical values. A few candidates chose the pair of inequalities which represented $9x^2 + 17x - 2 > 0$. These may have benefitted by rereading the question or double checking their own working before writing down their final answer. Those who drew diagrams usually found them to be helpful in determining the correct form of the final answer.

Question 2

A reasonable number of candidates were able to differentiate each of the trigonometric functions correctly and use the product rule. A few candidates earned all four marks available as they used correct bracketing to ensure that their final answer was unambiguous in meaning. Candidates who gave, as their final answer, $-\frac{1}{2}\sin\frac{x}{2}\tan 3x + 3\sec^2 3x\cos\frac{x}{2}$ would have improved if they had bracketed each term, for example, $\left(-\frac{1}{2}\sin\frac{x}{2}\right)(\tan 3x) + (3\sec^2 3x)\left(\cos\frac{x}{2}\right)$. Many candidates bracketed the second term in each product, but not the first. This did not remove the ambiguity.

Question 3

(i) A good number of fully correct solutions were seen. A very good number of candidates found the correct gradient for *L* using the gradient of *AB*. They were often able to form a correct unsimplified equation for *L*. A few candidates decimalised their value of *c* and this impacted on the accuracy of their final answer. Some candidates were simply unable to carry out the algebraic manipulation required to obtain the answer in the required form while other candidates made no attempt to do so. A few candidates needed to read the question more carefully as they used a point other than *C* when forming their equation.

(ii) An answer in the required form $ax + by = c$ should have made the solution in this part much simpler. Regardless of this, many candidates were able to find the correct length and state the answer correct to 3 significant figures. A few candidates were using an incorrect equation from part **(i)**, but still earned the method mark in this part. Only weaker candidates tended to make no attempt to answer.

Question 4

(i) This was a standard sketch and a good number of candidates earned all 3 marks. Almost all candidates drew graphs with 2 cycles with a midline at $y = 4$. Some candidates drew graphs that were not sinusoidal, for example, points were occasionally joined with line segments. Some candidates expected the maximum and minimum points to be at $x = 60°$, 120°, 240° and 300°. This resulted in some very skewed graphs and also caused the amplitude to be slightly inaccurate. Generally, these efforts earned a single mark only for having the correct number of cycles.

(ii) (iii) Many correct answers were seen to both of these parts. Occasionally the period was stated as 90° or 360°. Occasionally the amplitude was stated as 4, rather than 3.

Question 5

(a) (i) This was almost universally correct. Those candidates who were incorrect usually stated '2 by 3' or gave the answer in an incorrect form such as 3 to 2.

(ii) Again, many correct answers were seen. A few candidates stated the 2 by 2 identity matrix or stated a zero matrix of an incorrect order.

(b) A good number of candidates found the correct matrix product to be the 2 by 2 identity matrix. Many were then able to deduce that this meant **B** was $\mathbf{C}^{-1}$ or vice versa. Some candidates simply stated that the product was the identity matrix without interpreting what this meant.

(c) Again, a good number of candidates were able to find the correct inverse matrix. A few candidates were challenged by finding the algebraic determinant of **D** and those who were incorrect had usually made sign errors. The adjoint matrix was generally stated correctly. Very few candidates made no response or made no real progress.

Question 6

(i) The majority of candidates efficiently and neatly applied the chain rule twice and found correct expressions for the first and second derivatives, as required. Less able candidates sometimes opted to multiply out the expressions. This was more prone to error and much less efficient. A few candidates attempted to apply the product rule using $u = (3x - 5)^3$ and $v = 2x$ or $-2x$ to find the first derivative and consequently the second. This was a complete misinterpretation and could not be credited. A few candidates needed to reread the question as they omitted to find the second derivative in this part.

(ii) Candidates needed to equate their first derivative to 0 and solve in this part. A good number of candidates were able to equate to zero but fewer were able to solve correctly or exactly. The answers were required in exact form and so conversion to a decimal after an exact value was seen was penalised. Weaker candidates equated the second derivative to 0 or made no attempt to answer.

(iii) This was very well answered by a good number of candidates who recalled and applied the second derivative test correctly. A few candidates were confused by the values they had found and reversed the nature of the two points. Weaker candidates made no attempt to answer or made no real progress in this part.

Question 7

(i) Candidates needed to devise a strategy to find and sum all the values required to answer this part of the question. They needed to carefully consider the ratio given to find the lengths of *AD* and *DO*. They also needed to use a trigonometric approach to find the length of *DC*. Many candidates were able to do this and also to find the arc length, *AB*, and hence complete the solution. Some candidates introduced an extra source of error by making an unnecessary conversion to degrees. A few candidates were unable to use the ratio 7 : 10 correctly and $DO = \frac{7}{17} \times 50 = \frac{350}{17}$ was common if this was the case, similarly for *AD*. These candidates were often otherwise correct in their calculations. A few candidates were unable to determine a complete plan to find the perimeter but were usually able to find the length of the lines *DC* and *AD*.

(ii) Again, many candidates earned full marks in this part and some neat and efficient solutions finding the difference between the area of the sector and the area of the triangle were seen. Some candidates attempted the sum of the area of the segment cut off by the chord *AB* and the trapezium which remained of the shaded area. This was much more work and much less successful. Some rounding errors were seen in working values. Candidates should know that, to state a final answer that is accurate to 3 significant figures, they should use working values to more than 3 significant figures.

Question 8

(a) (i) Many candidates were able to find all three values correctly. Occasional sign or arithmetic errors spoiled otherwise correct answers. A good number of candidates were able to find *p* correctly and use this to find either the correct value of *q* or *r*. Some candidates earned a method mark for forming a correct expansion with at least two of the three terms needed correct. Weaker candidates made no real progress or no attempt to answer this question.

(ii) Candidates struggled to find a sensible explanation in this part. Better candidates showed that the power of *x* in the general term could not be zero for an integer value of *n*. The simplest explanation, that the powers were decreasing by 3 each time and then showing that 0 could not therefore be a power of *x* in this expansion, was rarely attempted and usually not sufficiently well explained to score. The majority of candidates simply stated what the term 'independent of *x*' meant without justifying it in this case.

(b) Candidates found this question challenging and only better candidates made any real progress. Candidates needed to use the coefficient of the third term of the expansion and equate it to 30. A few were able to do this, but very few were able to form the required equation in *n* and solve it. Most candidates wrote $\frac{1}{4}\times {}^{n}C_2 = 30$ and then carried out trials on ${}^{n}C_2 = 120$ until they found the value of *n*. This was not credited as it did not answer the question. Candidates should understand that, whilst it is an excellent working tool, using a calculator in this way is no substitute for showing correct method. Weaker candidates often omitted this part of the question.

Question 9

Candidates, again, needed to devise a strategy to be able to answer this question correctly. Most candidates understood that they needed to integrate to find an area or areas. There were many correct approaches, but this was not a routine question and some candidates should have given it a little more thought before formulating their solution. Good candidates found the area under the curve between the limits of 3 and 4, doubled it, using symmetry, and then added the area of the rectangle in-between. This was quick and efficient. There were other, longer approaches that were in essence the same process as the method already described, such as finding the area between the limits of –4 and 4 and also –3 and 3, subtracting the two and then adding on the rectangle. A reasonable number of candidates did earn all 6 marks using one or other of these approaches. Some candidates miscalculated the area of the rectangle as $7 \times 8 = 56$. It was a requirement that all working be shown. This included evidence of using the limits in the integral. Some candidates omitted to show the difference between the expression at the upper limit and the expression at the lower limit and were penalised. Some candidates found the area under the curve between the limits –4 and 4 and then subtracted the area under the line between –4 and 4. This did not answer the question and few marks were available to these candidates.

Question 10

(i) A very good proportion of candidates found a correct expression for each vector. Occasional sign errors were seen, but these were rare. A few candidates misinterpreted $\overrightarrow{PB}$ as $3\mathbf{q} - \mathbf{p}$.

(ii) Many candidates were able to form a correct expression using their answers to part **(i)** and a correct vector route such as $\overrightarrow{PQ} = \overrightarrow{PR} + \overrightarrow{RQ}$. Those who did not have a correct answer often earned a mark for indicating a correct vector route such as $\overrightarrow{PQ} = \lambda\overrightarrow{PB} - \mu\overrightarrow{QA}$.

(iii) Fewer candidates attempted to answer this part of the question. Candidates needed to use the two expressions for $\overrightarrow{PQ}$ they had already derived and use them to form and solve a pair of simultaneous equations by equating scalars for **p** and **q**. A good number of candidates did this neatly and accurately. Few errors were made by candidates who understood what was required, although occasional sign errors were seen. A few candidates attempted to form proportional relationships. These were not valid as division of vectors is not appropriate.

Question 11

(i) This was well answered. Many candidates differentiated the expression for the displacement correctly and showed it was not possible for the resulting expression for the velocity to be equal to zero at any time. Very many candidates attempted to differentiate the expression for the displacement. A few candidates, after differentiating correctly, stated only that the lowest value of the velocity was 4. Whilst this implied that the velocity could not be zero, this had not been explicitly stated and therefore this statement was not credited as a key element in the interpretation of 'at rest' had been omitted. Some candidates, after differentiating correctly, substituted $t = 0$ instead of attempting to equate to zero. Weaker candidates often attempted to show that the displacement could not be zero or substituted $t = 0$ into the expression for the displacement or integrated instead of differentiating.

(ii) As the particle was never at rest, candidates needed to interpret the distance travelled as being the difference in the displacements between the two values of t given. A good number of candidates did this and were sufficiently accurate to earn full marks. A few candidates found the two displacements required but omitted to subtract them. Some candidates had their calculator in degree mode when it was necessary here to have it in radian mode. A few candidates misinterpreted what was needed and integrated the expression for the displacement between the values of t stated.

(iii) A good number of candidates differentiated their expression for the velocity to find an expression for the acceleration at time t seconds and then substituted $t = 4$. A few candidates made a sign error when differentiating $\cos t$, differentiated the constant incorrectly or found the value of the velocity or displacement when $t = 4$. Again, some candidates had their calculator in degree mode.

(iv) Many candidates made no attempt to answer this part of the question. Of those that did, candidates who used $\cos t = -1$ at the minimum point, giving the first minimum at $\cos^{-1}(-1)$, generally stated the correct answer, π. Candidates who solved $-\sin t = 0$ often omitted to take note that at $t = 0$, the velocity of the particle was first at its maximum and often 0 or 0 and π were stated as solutions. Candidates who made a simple sketch usually avoided this error.

Question 12

This question involved the application of many skills and assessed the candidate's ability to formulate problems into mathematical terms and select and apply appropriate techniques of solution. It was an unstructured question and candidates needed to, once again, devise their own strategy in order to solve the problem. It was well answered with a good number of candidates earning the majority of the marks. Many candidates equated the expressions for the line and curve, deriving the correct cubic equation. They then were able to deduce that $x - 2$ was a factor from the given coordinates of point C. The few who did not make this connection often used the factor theorem to show that $x - 2$ was a factor. Once this was established candidates were generally able to apply a correct technique to find and factorise the quadratic factor. The coordinates of A and B were quickly found and the mid-point was also generally correct. Occasional slips were seen in finding the mid-point as some candidates subtracted the two corresponding coordinates before halving. It was evident that, even though candidates had been instructed not to use a calculator, a small number had done so in order to find the roots as these were either stated without any working or the quadratic factor had been omitted and the linear factors clearly found by working back from the roots. This was not accepted, although marks could still be awarded for the forming of the cubic equation and the finding of the mid-point, if done correctly.