## Cambridge IGCSE ${ }^{\text {TM }}$

CANDIDATE NAME

CENTRE NUMBER


## ADDITIONAL MATHEMATICS

You must answer on the question paper.
No additional materials are needed.

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.


## INFORMATION

- The total mark for this paper is 80 .
- The number of marks for each question or part question is shown in brackets [ ].


## Mathematical Formulae

## 1. ALGEBRA

## Quadratic Equation

For the equation $a x^{2}+b x+c=0$,

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

Binomial Theorem

$$
(a+b)^{n}=a^{n}+\binom{n}{1} a^{n-1} b+\binom{n}{2} a^{n-2} b^{2}+\ldots+\binom{n}{r} a^{n-r} b^{r}+\ldots+b^{n}
$$

where $n$ is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}$

Arithmetic series

$$
\begin{aligned}
& u_{n}=a+(n-1) d \\
& S_{n}=\frac{1}{2} n(a+l)=\frac{1}{2} n\{2 a+(n-1) d\}
\end{aligned}
$$

Geometric series $\quad u_{n}=a r^{n-1}$

$$
\begin{aligned}
& S_{n}=\frac{a\left(1-r^{n}\right)}{1-r} \quad(r \neq 1) \\
& S_{\infty}=\frac{a}{1-r}(|r|<1)
\end{aligned}
$$

## 2. TRIGONOMETRY

Identities

$$
\begin{gathered}
\sin ^{2} A+\cos ^{2} A=1 \\
\sec ^{2} A=1+\tan ^{2} A \\
\operatorname{cosec}^{2} A=1+\cot ^{2} A
\end{gathered}
$$

Formulae for $\triangle A B C$

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
a^{2}=b^{2}+c^{2}-2 b c \cos A \\
\Delta=\frac{1}{2} b c \sin A
\end{gathered}
$$

1 Find the exact solutions of the equation $3(\ln 5 x)^{2}+2 \ln 5 x-1=0$.

2


The diagram shows the graph of $y=a \sin b x+c$ where $x$ is in radians and $-2 \pi \leqslant x \leqslant 2 \pi$, where $a, b$ and $c$ are positive constants. Find the value of each of $a, b$ and $c$.

3 The line $A B$ is such that the points $A$ and $B$ have coordinates $(-4,6)$ and $(2,14)$ respectively.
(a) The point $C$, with coordinates $(7, a)$ lies on the perpendicular bisector of $A B$. Find the value of $a$.
(b) Given that the point $D$ also lies on the perpendicular bisector of $A B$, find the coordinates of $D$ such that the line $A B$ bisects the line $C D$.

4 (a) Show that $2 x^{2}+5 x-3$ can be written in the form $a(x+b)^{2}+c$, where $a, b$ and $c$ are constants.
(b) Hence write down the coordinates of the stationary point on the curve with equation $y=2 x^{2}+5 x-3$.
(c) On the axes below, sketch the graph of $y=\left|2 x^{2}+5 x-3\right|$, stating the coordinates of the intercepts with the axes.

(d) Write down the value of $k$ for which the equation $\left|2 x^{2}+5 x-3\right|=k$ has exactly 3 distinct solutions.

5 In this question all lengths are in kilometres and time is in hours.
Boat $A$ sails, with constant velocity, from a point $O$ with position vector $\binom{0}{0}$. After 3 hours $A$ is at the point with position vector $\binom{-12}{9}$.
(a) Find the position vector, $\overrightarrow{O P}$, of $A$ at time $t$.

At the same time as $A$ sails from $O$, boat $B$ sails from a point with position vector $\binom{12}{6}$, with constant velocity $\binom{-5}{8}$.
(b) Find the position vector, $\overrightarrow{O Q}$, of $B$ at time $t$.
(c) Show that at time $t|\overrightarrow{P Q}|^{2}=26 t^{2}+36 t+180$.
(d) Hence show that $A$ and $B$ do not collide.

6 (a) A geometric progression has first term 10 and sum to infinity 6 .
(i) Find the common ratio of this progression.
(ii) Hence find the sum of the first 7 terms, giving your answer correct to 2 decimal places.
(b) The first three terms of an arithmetic progression are $\log _{x} 3, \log _{x}\left(3^{2}\right), \log _{x}\left(3^{3}\right)$.
(i) Find the common difference of this progression.
(ii) Find, in terms of $n$ and $\log _{x} 3$, the sum to $n$ terms of this progression. Simplify your answer.
(iii) Given that the sum to $n$ terms is $3081 \log _{x} 3$, find the value of $n$.
(iv) Hence, given that the sum to $n$ terms is also equal to 1027, find the value of $x$.

## 7 DO NOT USE A CALCULATOR IN THIS QUESTION

In this question all lengths are in centimetres.


The diagram shows a trapezium $A B C D E$ such that $A B$ is parallel to $E C$ and $A B C D$ is a rectangle. It is given that $B C=\sqrt{17}+1, E D=\sqrt{17}-1$ and $D C=\sqrt{17}+4$.
(a) Find the perimeter of the trapezium, giving your answer in the form $a+b \sqrt{17}$, where $a$ and $b$ are integers.
(b) Find the area of the trapezium, giving your answer in the form $c+d \sqrt{17}$, where $c$ and $d$ are integers.
(c) Find $\tan A E D$, giving your answer in the form $\frac{e+f \sqrt{17}}{8}$, where $e$ and $f$ are integers.
(d) Hence show that $\sec ^{2} A E D=\frac{81+9 \sqrt{17}}{32}$.

8 (a) (i) Show that $\sin x \tan x+\cos x=\sec x$.
(ii) Hence solve the equation $\sin \frac{\theta}{2} \tan \frac{\theta}{2}+\cos \frac{\theta}{2}=4$ for $0 \leqslant \theta \leqslant 4 \pi$, where $\theta$ is in radians. [4]
(b) Solve the equation $\cot \left(y+38^{\circ}\right)=\sqrt{3}$ for $0^{\circ} \leqslant y \leqslant 360^{\circ}$.

9 The polynomial $\mathrm{p}(x)=2 x^{3}-3 x^{2}-x+1$ has a factor $2 x-1$.
(a) Find $\mathrm{p}(x)$ in the form $(2 x-1) \mathrm{q}(x)$, where $\mathrm{q}(x)$ is a quadratic factor.


The diagram shows the graph of $y=\frac{1}{x}$ for $x>0$, and the graph of $y=-2 x^{2}+3 x+1$. The curves intersect at the points $A$ and $B$.
(b) Using your answer to part (a), find the exact $x$-coordinate of $A$ and of $B$.
(c) Find the exact area of the shaded region.

Question 10 is printed on the next page.

10 A curve has equation $y=\frac{\left(2 x^{2}+10\right)^{\frac{3}{2}}}{x-1}$ for $x>1$.
(a) Show that $\frac{\mathrm{d} y}{\mathrm{~d} x}$ can be written in the form $\frac{\left(2 x^{2}+10\right)^{\frac{1}{2}}}{(x-1)^{2}}\left(A x^{2}+B x+C\right)$, where $A, B$ and $C$ are
integers.
(b) Show that, for $x>1$, the curve has exactly one stationary point. Find the value of $x$ at this stationary point.

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