## Cambridge IGCSE ${ }^{\text {TM }}$

CANDIDATE NAME

CENTRE NUMBER


## ADDITIONAL MATHEMATICS

Paper 1
May/June 2022
2 hours
You must answer on the question paper.
No additional materials are needed.

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.


## INFORMATION

- The total mark for this paper is 80 .
- The number of marks for each question or part question is shown in brackets [ ].


## Mathematical Formulae

## 1. ALGEBRA

## Quadratic Equation

For the equation $a x^{2}+b x+c=0$,

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

## Binomial Theorem

$$
(a+b)^{n}=a^{n}+\binom{n}{1} a^{n-1} b+\binom{n}{2} a^{n-2} b^{2}+\ldots+\binom{n}{r} a^{n-r} b^{r}+\ldots+b^{n}
$$

where $n$ is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}$

Arithmetic series

$$
u_{n}=a+(n-1) d
$$

$$
S_{n}=\frac{1}{2} n(a+l)=\frac{1}{2} n\{2 a+(n-1) d\}
$$

Geometric series

$$
\begin{aligned}
& u_{n}=a r^{n-1} \\
& S_{n}=\frac{a\left(1-r^{n}\right)}{1-r} \quad(r \neq 1) \\
& S_{\infty}=\frac{a}{1-r}(|r|<1)
\end{aligned}
$$

## 2. TRIGONOMETRY

Identities

$$
\begin{gathered}
\sin ^{2} A+\cos ^{2} A=1 \\
\sec ^{2} A=1+\tan ^{2} A \\
\operatorname{cosec}^{2} A=1+\cot ^{2} A
\end{gathered}
$$

Formulae for $\triangle A B C$

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
a^{2}=b^{2}+c^{2}-2 b c \cos A \\
\Delta=\frac{1}{2} b c \sin A
\end{gathered}
$$

1 (a) Find the rational numbers $a, b$ and $c$, such that the first three terms, in descending powers of $x$, in the expansion of $\left(3 x^{2}-\frac{1}{9 x}\right)^{5}$ can be written in the form $a x^{10}+b x^{7}+c x^{4}$.
(b) Hence find the coefficient of $x^{4}$ in the expansion of $\left(3 x^{2}-\frac{1}{9 x}\right)^{5}\left(1+\frac{1}{x^{3}}\right)^{2}$.

2 In this question, all lengths are in centimetres and all angles are in radians.


The diagram shows a circle, centre $O$, radius 8 . The points $A, B$ and $C$ lie on the circumference of the circle. The chord $A B$ has length 10.
(a) Show that angle $B O A$ is 1.35 correct to 2 decimal places.
(b) Given that the minor arc $B C$ has a length of 18 , find angle $B O C$.
(c) Find the area of the minor sector $A O C$.

3 (a) Find the exact solution of the equation $2 \mathrm{e}^{6 x}-3 \mathrm{e}^{3 x}-5=0$.
(b) Solve the following simultaneous equations.

$$
\begin{aligned}
\mathrm{e}^{4 x-7} \div \mathrm{e}^{5 x+7 y} & =\frac{1}{\mathrm{e}^{2}} \\
x y+18 & =0
\end{aligned}
$$

4 Variables $x$ and $y$ are such that when $\mathrm{e}^{4 y}$ is plotted against $x$, a straight line of gradient $\frac{2}{5}$, passing through ( 10,2 ), is obtained.
(a) Find $y$ in terms of $x$.
(b) Find the value of $y$ when $x=45$, giving your answer in the form $\ln p$.
(c) Find the values of $x$ for which $y$ can be defined.

5 The velocity, $v \mathrm{~ms}^{-1}$, of a particle moving in a straight line, $t$ seconds after passing through a fixed point $O$, is given by $v=6 \sin 3 t$.
(a) Find the time at which the acceleration of the particle is first equal to $-9 \mathrm{~ms}^{-2}$.
(b) Find the displacement of the particle from $O$ when $t=5.6$.

6 (a) It is given that

$$
\begin{aligned}
& \mathrm{f}: x \rightarrow 2 x^{2} \text { for } x \geqslant 0, \\
& \mathrm{~g}: x \rightarrow 2 x+1 \text { for } x \geqslant 0 .
\end{aligned}
$$

Each of the expressions in the table can be written as one of the following.

$$
\begin{array}{lllllllll}
\mathrm{f}^{\prime} & \mathrm{f}^{\prime \prime} & \mathrm{g}^{\prime} & \mathrm{g}^{\prime \prime} & \mathrm{fg} & \mathrm{gf} & \mathrm{f}^{2} & \mathrm{~g}^{2} & \mathrm{f}^{-1}
\end{array} \mathrm{~g}^{-1}
$$

Complete the table. The first row has been completed for you.

| Expression | Function notation |
| :---: | :---: |
| 2 | $\mathrm{~g}^{\prime}$ |
| 0 |  |
| $4 x$ |  |
| $8 x^{2}+8 x+2$ |  |
| $4 x+3$ |  |
| $\frac{x-1}{2}$ |  |

(b) It is given that $\mathrm{h}(x)=(x-1)^{2}+3$ for $x \geqslant a$. The value of $a$ is as small as possible such that $\mathrm{h}^{-1}$ exists.
(i) Write down the value of $a$.
(ii) Write down the range of h .
(iii) Find $\mathrm{h}^{-1}(x)$ and state its domain.

7 A curve has equation $y=\frac{(2 x+1)^{\frac{3}{2}}}{x+5}$ for $x \geqslant 0$.
(a) Show that $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{(2 x+1)^{\frac{1}{2}}}{(x+5)^{2}}(A x+B)$, where $A$ and $B$ are integers to be found.
(b) Show that there are no stationary points on this curve.
(c) Find the approximate change in $y$ when $x$ increases from 1 to $1+p$, where $p$ is small.
(d) Given that when $x=1$ the rate of change in $x$ is 2.5 units per second, find the corresponding rate of change in $y$.

8 (a) A 6-digit number is formed from the digits $0,1,2,5,6,7,8,9$. A number cannot start with 0 and each digit can be used at most once in any 6 -digit number.
(i) Find how many 6-digit numbers can be formed if there are no further restrictions.
(ii) Find how many of these 6 -digit numbers are divisible by 5 .
(iii) Find how many of these 6-digit numbers are greater than 850000 .
(b) A team of 8 people is to be chosen from 12 people. Three of the people are brothers who must not be separated. Find the number of different teams that can be chosen.

9 (a) Solve the equation $3 \operatorname{cosec}^{2}\left(2 \phi-\frac{\pi}{3}\right)=4$, for $0<\phi<\pi$. Give your solutions in terms of $\pi$.
(b) Given that $2 x-1=\operatorname{cosec}^{2} \theta$ and $y+1=\tan ^{2} \theta$, find $y$ in terms of $x$.

10 (a) Show that $\frac{6}{2+3 x}+\frac{4}{(x+1)^{2}}-\frac{2}{x+1}$ can be written as $\frac{14 x+10}{(2+3 x)(x+1)^{2}}$.
(b) Hence find the exact value of $\int_{0}^{2} \frac{14 x+10}{(2+3 x)(x+1)^{2}} \mathrm{~d} x$. Give your answer in the form $p+\ln q$, where $p$ and $q$ are rational numbers.

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