## Cambridge International Examinations

Cambridge International General Certificate of Secondary Education

CANDIDATE NAME

## CENTRE NUMBER



## CAMBRIDGE INTERNATIONAL MATHEMATICS

0607/63
Paper 6 (Extended)
May/June 2014
1 hour 30 minutes
Candidates answer on the Question Paper
Additional Materials: Graphics Calculator

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO NOT WRITE IN ANY BARCODES.

Answer both parts $\mathbf{A}$ and $\mathbf{B}$.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40 .

## Answer both parts A and B.

## A INVESTIGATION

## TOTALS (20 marks)

You are advised to spend no more than 45 minutes on this part.

This investigation looks at possible totals when you make addition sums using only two different positive integers. The integers have no common factor unless otherwise stated.

## Example

Using only $\mathbf{3}$ and $\mathbf{4}$ you can make a total of 23 .

$$
\begin{array}{ll} 
& 23=\mathbf{3}+\mathbf{3}+\mathbf{3}+\mathbf{3}+\mathbf{3}+\mathbf{4}+\mathbf{4} . \\
\text { This is written as } & 23=5 \times \mathbf{3}+2 \times 4 .
\end{array}
$$

In the table below, when a total is possible, the calculation is shown.
When a total is not possible, the cell is crossed out.
Totals using only $\mathbf{3}$ and $\mathbf{4}$

| Total | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Calculation |  |  | $1 \times \mathbf{3}$ | $1 \times \mathbf{4}$ |  | $2 \times \mathbf{3}$ | $1 \times \mathbf{3}+1 \times \mathbf{4}$ | $2 \times \mathbf{4}$ |

You can make all the totals bigger than 8 by adding 3 to the possible totals $6,7,8, \ldots$ to get $9,10,11, \ldots$. The largest total that is not possible is 5 .

1 (a) Complete the tables below. One integer is always 2.
Totals using only $\mathbf{2}$ and $\mathbf{3}$

| Total | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Calculation |  | $1 \times \mathbf{2}$ | $1 \times \mathbf{3}$ | $2 \times \mathbf{2}$ |  | $2 \times \mathbf{3}$ | $2 \times \mathbf{2}+1 \times \mathbf{3}$ |  |

The largest total that is not possible is 1 .

Totals using only $\mathbf{2}$ and $\mathbf{5}$

| Total | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Calculation |  | $\mathbf{1} \times \mathbf{2}$ |  | $\mathbf{2} \times \mathbf{2}$ |  | $3 \times \mathbf{2}$ | $1 \times \mathbf{2}+1 \times \mathbf{5}$ | $4 \times \mathbf{2}$ |

The largest total that is not possible is $\qquad$

Totals using only $\mathbf{2}$ and $\mathbf{7}$

| Total | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Calculation |  |  |  | $\mathbf{3 \times 2}$ | $1 \times \mathbf{7}$ | $4 \times \mathbf{2}$ |  | $5 \times \mathbf{2}$ |

The largest total that is not possible is
(b) From your answers in part (a) complete the following statement.

The largest total, using only $\mathbf{2}$ and $\boldsymbol{y}$, that is not possible is $\qquad$

2 (a) Complete the tables below. One integer is always 3 .
Totals using only $\mathbf{3}$ and $\mathbf{5}$

| Total | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Calculation | $1 \times \mathbf{3}$ |  | $1 \times \mathbf{5}$ | $2 \times \mathbf{3}$ |  |  | $3 \times \mathbf{3}$ |  |

The largest total that is not possible is 7 .

Totals using only $\mathbf{3}$ and 7

| Total | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Calculation |  |  | $1 \times \mathbf{3}+1 \times \mathbf{7}$ |  | $4 \times \mathbf{3}$ |  | $2 \times \mathbf{7}$ | $5 \times \mathbf{3}$ |

The largest total that is not possible is 11 .

Totals using only $\mathbf{3}$ and $\mathbf{8}$

| Total | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Calculation |  |  | $4 \times \mathbf{3}$ |  | $2 \times \mathbf{3}+1 \times \mathbf{8}$ | $5 \times \mathbf{3}$ | $2 \times \mathbf{8}$ |

The largest total that is not possible is 13 .
(b) $\mathbf{3}$ and $\mathbf{6}$ have a common factor of 3 .

Explain why, using only $\mathbf{3}$ and $\mathbf{6}$, you cannot find the largest total that is not possible.

3 (a) Some answers for $\mathbf{5}$ and $\boldsymbol{y}$ are shown in the table.

|  | $y$ | Largest total that is not possible |
| :---: | :---: | :---: |
| 5 | 6 | 19 |
| 5 | 7 | 23 |
| 5 | 8 | 27 |
| 5 | 9 | 31 |

Find an expression, in terms of $y$, for the largest total that is not possible using only $\mathbf{5}$ and $\boldsymbol{y}$.
(b) Some answers for $\mathbf{7}$ and $\boldsymbol{y}$ are shown in the table.

|  | $y$ | Largest total that is not possible |
| :---: | :---: | :---: |
| 7 | 2 | 5 |
| 7 | 3 | 11 |
| 7 | 4 | 17 |
| 7 | 5 | 23 |
| 7 | 6 | 29 |

Find an expression, in terms of $y$, for the largest total that is not possible using only 7 and $\boldsymbol{y}$.

4 Some answers for $\boldsymbol{x}$ and $\boldsymbol{y}$ are shown in the table.

| $x$ | The expression for the largest total that is not possible |
| :---: | :---: |
| 2 |  |
| 3 | $2 y-3$ |
| 5 |  |
| 7 | $10 y-11$ |
| 11 |  |

(a) Complete the table by copying the results from questions $\mathbf{1}(\mathbf{b})$ and $\mathbf{3}$.

Write down an expression, in terms of $y$, for the largest total that is not possible when $x=13$.
(b) Write down an expression, in terms of $x$ and $y$, for the largest total that is not possible.

5 (a) Using only 24 and 25 , calculate the largest total that is not possible.
(b) Using only 24 and $\mathbf{2 5}$, show how it is possible to get a total of 320 .

6 The Example on page 2 used only $\mathbf{3}$ and $\mathbf{4}$.
Totals using $\mathbf{3}$ and $\mathbf{4}$

| Total | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Calculation |  |  | $1 \times \mathbf{3}$ | $\mathbf{1} \times \mathbf{4}$ |  | $2 \times \mathbf{3}$ | $1 \times \mathbf{3}+1 \times \mathbf{4}$ | $2 \times \mathbf{4}$ |

The largest total that is not possible is 5 .
The smallest possible total, after which all totals are possible, is 6 .
So $6,7,8,9, \ldots$ are all possible.
(a) Use your answer to question 4(b) to write down an expression, in terms of $x$ and $y$, for the smallest possible total after which all totals are possible.
(b) Show that your expression in part (a) is equal to $(x-1)(y-1)$.
(c) Find all the pairs of integers so that the smallest possible total, after which all totals are possible, is 24 .

## B MODELLING

DESIGNING AN OPEN BOX (20 marks)
You are advised to spend no more than 45 minutes on this part.


Engineers use an expensive metal to make a box for a machine.
The method to make a net is:

- take a square piece of the metal of side 25 cm
- cut a square, of side $x \mathrm{~cm}$, from each corner

1 Complete the inequality for $x$.
$\qquad$
$<x<$
2 Find a model for the area, $A \mathrm{~cm}^{2}$, of the net by writing $A$ in terms of $x$.

$$
A=
$$

$\qquad$

3 The engineers fold the net to make a box.


The diagram shows the 8 edges of the open box that are strengthened with a seal.
Find a model for the length of the seal, $L \mathrm{~cm}$, by writing $L$ in terms of $x$.
Write your answer in its simplest form.

$$
L=
$$

4 (a) Show that a model for the volume of the box, $V \mathrm{~cm}^{3}$, is

$$
V=625 x-100 x^{2}+4 x^{3}
$$

(b) Assume the measurements on the net are given accurately.

Give a reason why the actual volume of the box may be different to $V$.
(c) Sketch the graph of $V=625 x-100 x^{2}+4 x^{3}$.

(d) Find the maximum volume of the box.

5 For the machine to work efficiently, the box must have a capacity of at least one litre.
(a) Complete this new inequality for $x$.
$\qquad$
$\qquad$
(b) Because of cost, the engineers want the area, $A \mathrm{~cm}^{2}$, to be less than $450 \mathrm{~cm}^{2}$.

Use your answer to part (a) and question 2 to show that this is not possible.
(c) The engineers decide to allow an area of metal which is less than $500 \mathrm{~cm}^{2}$. Complete this inequality for $x$.
$\qquad$
$<x<$

6 The cost of making the box is made up of three items.

- $\$ 2$ per square centimetre for the area of metal $\left(A \mathrm{~cm}^{2}\right)$
- \$1 per centimetre for the length of the seal $(L \mathrm{~cm})$
- $\$ 500$ for the labour
(a) Find, in terms of $x$, a model for the cost, $\$ C$.
$C=$ $\qquad$
(b) The company wants to sell the box at a profit of $20 \%$.

Write down a model for the selling price, $\$ S$.
$S=$
(c) Find the lowest selling price for a box with a capacity of one litre.
$\qquad$

