## Cambridge International Examinations

Cambridge International General Certificate of Secondary Education

## CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

## CAMBRIDGE INTERNATIONAL MATHEMATICS

0607/61
Paper 6 (Extended)
October/November 2016
1 hour 30 minutes
Candidates answer on the Question Paper.
Additional Materials: Graphics Calculator

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO NOT WRITE IN ANY BARCODES.
Answer both parts $\mathbf{A}$ and $\mathbf{B}$.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40.

## Answer both parts A and B.

## A INVESTIGATION

## SQUARES ON GRIDS (20 marks)

You are advised to spend no more than 45 minutes on this part.
This investigation looks at the number of squares drawn on a grid.
All squares are drawn using gridlines.

1 (a) Here is a 2 by 2 grid.


Explain why there are 5 squares on a 2 by 2 grid.
$\qquad$
$\qquad$
(b) Here is a 3 by 3 grid.


Complete these statements about the numbers of different sized squares on a 3 by 3 grid.

The number of 1 by 1 squares on a 3 by 3 grid is The number of 2 by 2 squares on a 3 by 3 grid is 4 The number of 3 by 3 squares on a 3 by 3 grid is $\qquad$

So the total number of squares on a 3 by 3 grid is $\qquad$
(c) Complete these statements about the numbers of different sized squares on a 4 by 4 grid. You may use the grids below to help you.

The number of 1 by 1 squares on a 4 by 4 grid is The number of 2 by 2 squares on a 4 by 4 grid is The number of 3 by 3 squares on a 4 by 4 grid is The number of 4 by 4 squares on a 4 by 4 grid is So the total number of squares on a 4 by 4 grid is


2 (a) Use your results from question 1 to help you complete this table.

|  | Number of $\ldots$ |  |  |  |  |  | Total <br> Size of grid |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 by 1 <br> squares | 2 by 2 <br> squares | 3 by 3 <br> squares | 4 by 4 <br> squares | 5 by 5 <br> squares | 6 by 6 <br> squares | number of <br> squares |
| 1 by 1 | 1 |  |  |  |  |  |  |
| 2 by 2 |  |  |  |  |  |  | 5 |
| 3 by 3 |  | 4 |  |  |  |  |  |
| 4 by 4 |  |  |  |  |  |  | 30 |
| 5 by 5 |  |  |  |  |  |  |  |
| 6 by 6 |  |  |  |  |  |  |  |

(b) What is the mathematical name for the numbers in the $\mathbf{1}$ by $\mathbf{1}$ squares column?
(c) Work out the total number of squares on an 8 by 8 grid.
(d) Write down an expression, in terms of $n$, for the number of 2 by 2 squares on an $n$ by $n$ grid.

3 Here is a formula for the total number of squares, $T$, on an $n$ by $n$ grid.

$$
T=\frac{n^{3}}{3}+\frac{n^{2}}{2}+c n+d
$$

(a) Find the values of $c$ and $d$.
(b) Show that your formula gives a total of 385 squares on a 10 by 10 grid.
(c) The total number of squares on an $n$ by $n$ grid is 1240 .

Find the value of $n$.

4 Here is a 1 by 2 grid.


There is a total of 2 squares on a 1 by 2 grid.
Write an expression, in terms of $n$, for the total number of squares on a 1 by $n$ grid.

5 Here is a 2 by 3 grid.


There is a total of 8 squares on a 2 by 3 grid.
(a) Find the total number of squares on a 2 by 4 grid.

(b) Complete this table.

|  | Number of ... |  | Total number |
| :---: | :---: | :---: | :---: |
| Size of grid | 1 by 1 <br> squares | 2 by 2 <br> squares | $(T)$ |
| 2 by 1 | 2 | 0 | 2 |
| 2 by 2 |  |  |  |
| 2 by 3 |  |  | 8 |
| 2 by 4 |  |  |  |
| 2 by 5 |  |  |  |
| 2 by $n$ |  |  |  |

6 Complete this table for 3 by $n$ grids.
You may use the grid below to help you.

| Size of grid | Number of $\ldots$ <br> squares |  |  | 2 by 2 <br> squares |
| :---: | :---: | :---: | :---: | :---: |
| 3 by 1 | 3 | 0 | 3 by 3 <br> squares | Total number <br> of squares <br> $(T)$ |
| 3 by 2 | 6 | 2 | 0 | 3 |
| 3 by 3 | 9 |  | 1 | 8 |
| 3 by 4 | 12 |  |  |  |
| 3 by 5 | 15 |  |  |  |
| 3 by $n$ | $3 n$ |  |  |  |



7 The expression for $T$ in question $\mathbf{6}$ does not work when $n=1$.
The expression for $T$ for a 4 by $n$ grid is $10 n-10$.
For what values of $n$ will the expression for $T$ for a 4 by $n$ grid not give the correct total?

## B MODELLING

## MEASURING ROD (20 marks)

You are advised to spend no more than 45 minutes on this part.


Jim has a tank for oil, shown in the picture.
He uses a measuring rod to put in the top of the tank to find the amount of oil.
This investigation is about how Jim marks the measuring rod.
Here is some information about the tank.

| Length | 1770 mm |
| :--- | :---: |
| Width | 1030 mm |
| Capacity | 1235 litres |



Jim decides to model the volume of oil in his tank.
1 (a) Select the shape that Jim should use for the model.
cuboid cylinder sphere cone pyramid circle
(b) Jim uses a capacity of 1200 litres and a width of 100 cm .

For his model, show that the length of the tank is 153 cm , correct to the nearest centimetre.

2 (a) Give a practical reason why the length of his measuring rod should be more than 100 cm .
$\qquad$
$\qquad$
(b) This is a cross-section of the tank showing the measuring rod and some oil.

(i) The tank contains 600 litres of oil.

Find how many centimetres from the bottom of the measuring rod Jim should mark " 600 litres".
(ii) Jim now wants to mark " 300 litres" on the measuring rod.

Explain why he should not mark this point halfway between the bottom of the measuring rod and the " 600 litres" mark.
$\qquad$
$\qquad$

3 Jim works out the distance, $d \mathrm{~cm}$, where " 300 litres" should be marked on the measuring rod.

$O$ is the centre of the cross section and $O A$ and $O B$ are radii.
$x^{\circ}$ is the angle between $O A$ and $O B$.
He uses this method.
Shaded area $=$ Area of sector $O A B-$ area of triangle $O A B$
(a) Show that an expression for the area of triangle $O A B$ is $1250 \sin x^{\circ} \mathrm{cm}^{2}$.
(b) Show that an expression for the area of sector $O A B$ is approximately $21.8 x \mathrm{~cm}^{2}$.
(c) Write down an expression, in terms of $x$, for the area of the shaded segment.
(d) Using your result from question 1(b) rounded to the nearest centimetre and your result from question 3(c), show that a model for the volume of oil, $V \mathrm{~cm}^{3}$, is approximately

$$
V=3340 x-191000 \sin x^{\circ} .
$$

(e) On the axes, sketch the graph of this model.

(f) When the tank contains exactly 300 litres of oil, use the model to find
(i) the value of $x$,
(ii) the value of $d$, the distance from the bottom of the measuring rod.
(g) Write down the distance from the bottom of the measuring rod to the "900 litres" mark.

4 Jim buys oil when the tank contains 100 litres of oil.
Work out the distance from the bottom of the measuring rod to where " 100 litres" should be marked.

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