Cambridge IGCSE

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

7757861635

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/63**

Paper 6 (Extended) **October/November 2018**

1 hour 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer both parts **A** (Questions 1 to 3) and **B** (Questions 4 to 6).
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and to communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40.

This document consists of **15** printed pages and **1** blank page.

DC (CE/SW) 153620/4

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Answer both parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 3)

NEAREST NEIGHBOURS (20 marks)

You are advised to spend no more than 45 minutes on this part.

This investigation is about pairs of dots on rectangular grids.

When two dots are next to each other, they are called **nearest neighbours**.
Here are four equally spaced dots in a row.

• • • •

The dots marked with crosses are nearest neighbours.
There are 3 pairs of nearest neighbours.

× × • •   • × × •   • • × ×

1 **(a)** Complete this table showing the number of pairs of nearest neighbours.

Number of dots in row	Number of pairs of nearest neighbours
2	1
3	
4	3
5	
6	
n	

(b) With four dots in a row there are a total of 6 different arrangements of two crosses.
In 3 of these arrangements the crosses mark dots that are nearest neighbours.

× × • •   × • × •   × • • ×

• × × •   • × • ×   • • × ×

Complete this table.

Number of dots in row	Number of pairs of nearest neighbours	Total number of different arrangements of two crosses
2	1	1
3		
4	3	6
5		
6		15
n		

(c) **(i)** Multiply the number of dots in a row by the number of pairs of nearest neighbours.

Write down a connection between your answer and the total number of different arrangements of two crosses.

....

....

(ii) In a row of n dots, write down an expression, in terms of n, for the total number of different arrangements of two crosses.

....

2 As well as pairs of nearest neighbours there are also pairs of 2nd nearest neighbours, 3rd nearest neighbours, 4th nearest neighbours and so on.
Here are some examples.

• × × •   × • × •   × • • ×

Nearest neighbours   2nd nearest neighbours   3rd nearest neighbours

(a) Complete this table.

Number of dots in row	Number of pairs of		
	Nearest neighbours	2nd nearest neighbours	3rd nearest neighbours
2	1		
3			
4	3		
5			
6			
n			

(b) Find an expression for the number of pairs of kth nearest neighbours in a row of n dots.

....

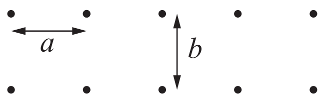
(c) **(i)** You can find the total number of different arrangements of two crosses by adding the number of pairs of nearest neighbours, 2nd nearest neighbours, 3rd nearest neighbours and so on.

Use this method to find the total number of different arrangements of two crosses in a row of 11 dots.

....

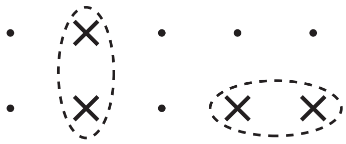
(ii) Use your expression in **question 1(c)(ii)** to show that your answer in **question 2(c)(i)** is correct.

3 Here is a 2 by 5 rectangle of dots.
It has 2 rows of 5 dots.



The distance between the dots in each column is a and the distance between each row is b.

(a) **When $a = b$**, here are two pairs of crosses which are nearest neighbours.

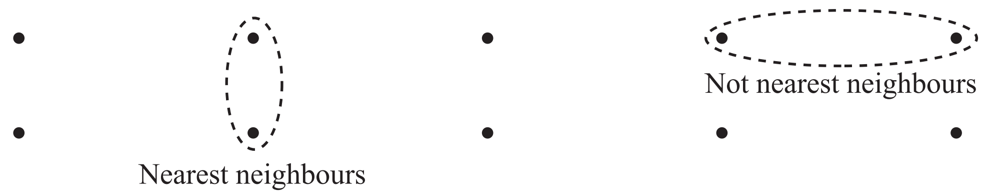


There are a total of 13 pairs of nearest neighbours for a 2 by 5 rectangle of dots.

For a 2 by w rectangle of dots, where $w \geqslant 2$, find an expression for the number of pairs of nearest neighbours.

....

(b) **When $a \neq b$**, the pairs of nearest neighbours are the dots that are closest to each other.



For a 2 by w rectangle of dots, where $w \geqslant 2$, find an expression for the number of pairs of nearest neighbours,

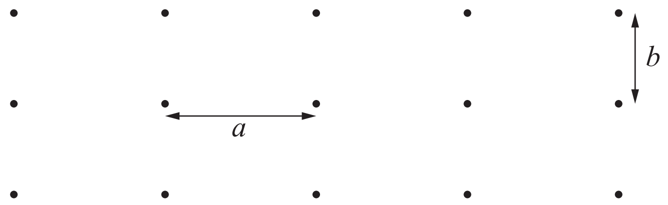
(i) when $a > b$,

....

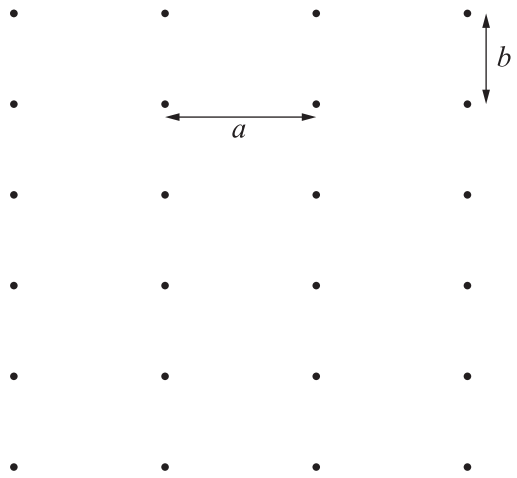
(ii) when $a < b$.

....

(c) Here are two rectangles of dots.



3 by 5 rectangle of dots



6 by 4 rectangle of dots

(i) For an h by w rectangle of dots, here are some expressions for the number of pairs of nearest neighbours, T.
In all cases $a > b$ and $w \geqslant 2$.

h	Number of pairs of nearest neighbours (T)
3	$2w$
4	$3w$
5	$4w$
6	$5w$

Find the formula for T in terms of h and w.

....

(ii) For $a > b$, there are 1000 pairs of nearest neighbours in a 101 by w rectangle of dots.

Find the value of w.

....

(d) For an h by w rectangle of dots, find an expression for T when $\boldsymbol{a} = \boldsymbol{b}$ and $w \geqslant 2$.

THE MODELLING STARTS ON PAGE 11.

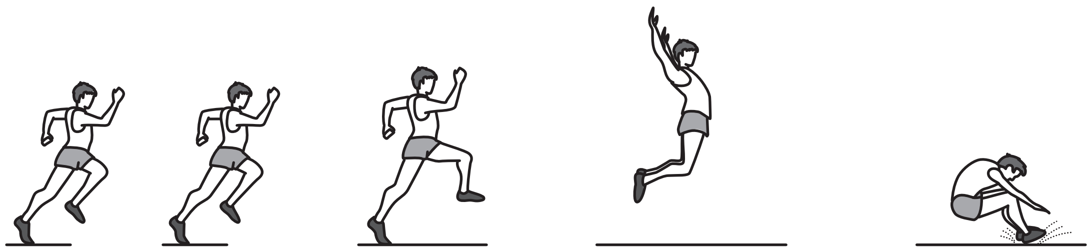
B MODELLING (QUESTIONS 4 to 6)

LONG JUMP (20 marks)

You are advised to spend no more than 45 minutes on this part.

This task is about modelling long jumping.

Athletes run along a track and then jump as far as they can in one leap.
The horizontal distance jumped is called their long jump distance.



Run-up Take-off Flight through the air Landing

At take-off, athletes jump up giving them a vertical speed.
Their horizontal speed carries them forward in their flight through the air.
The vertical speed and the horizontal speed give the take-off speed.

4 Here is a formula connecting the **vertical** speed, v m/s, and the maximum height, j metres, jumped by the athlete.

$$v^2 = 20j$$

An athlete has a vertical speed of 3.5 m/s.

Find the maximum height jumped by this athlete.

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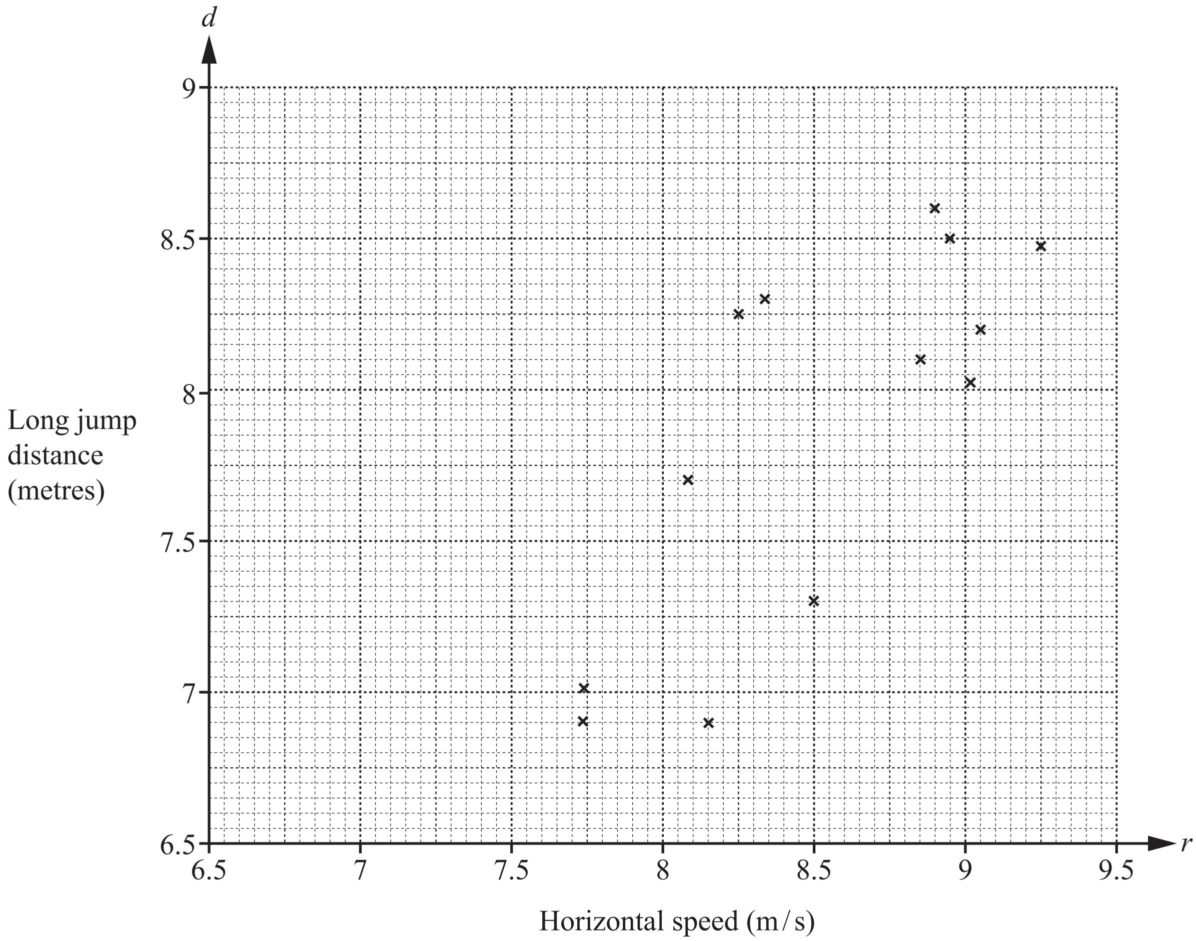
5 This scatter diagram shows the horizontal speed and the long jump distance for each of 13 athletes in a recent World Championship.



The mean horizontal speed, r, is 8.5 m/s.
The mean long jump distance, d, is 7.9 m.

(a) **(i)** On the scatter diagram plot the mean point and draw, by eye, a line of best fit.

(ii) Use your line of best fit to work out the equation that models the relationship between horizontal speed and long jump distance.

....

(iii) An athlete has a horizontal speed of 6.6 m/s.
This gives a long jump distance of 4.45 m.

Compare this long jump distance with the distance given by your equation in **part (ii)**.

....

....

(b) Long jump distance (d) and horizontal speed (r) can also be modelled by the following quadratic equation.

$$d = -0.46r^2 + 8.7r - 32.8$$

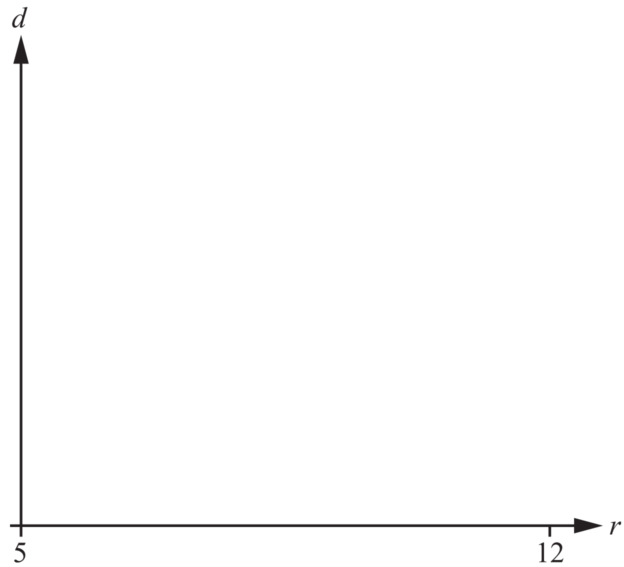
(i) The athlete in **part (a)(iii)** had a horizontal speed of 6.6 m/s and a long jump distance of 4.45 m.

Which model gives a better approximation for this athlete's long jump distance, the model you used in **part (a)(iii)** or this quadratic model?

....

....

(ii) Sketch the graph of $d = -0.46r^2 + 8.7r - 32.8$ for $5 \leqslant r \leqslant 12$.



(iii) Write down an inequality, in terms of r, for which the model is valid.

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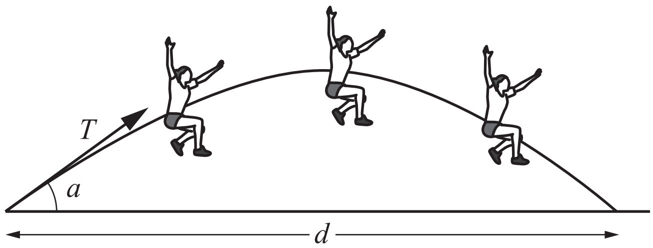
6 When athletes jump they take-off at an angle a to the horizontal.
The vertical speed and the horizontal speed give the take-off speed, T m/s.



A model for the long jump distance, d metres, is

$$d = \frac{T^2 \sin a \cos a}{5}.$$

(a) Sketch the graph of $y = \sin a \cos a$ for $0° \leqslant a \leqslant 90°$.

(b) Show that when $\sin a \cos a$ has its maximum value the greatest jump distance is

$$d = \frac{T^2}{10}.$$

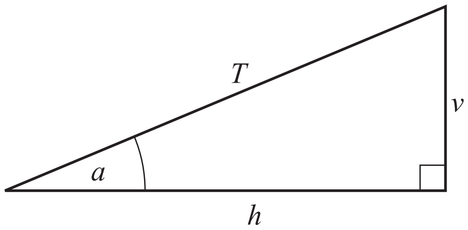
(c) An experienced long jumper has a take-off speed of 9.6 m/s.
This gives a long jump distance of 7.89 m.

Compare this long jump distance with the distance given by the formula in **part (b)**.

....

....

(d) Take-off speed (T m/s), horizontal speed (h m/s) and vertical speed (v m/s) are related by the right-angled triangle below.



(i) Use trigonometry to show that $d = \dfrac{T^2 \sin a \cos a}{5}$ can be written as $d = \dfrac{vh}{5}$.

(ii) An athlete has a take-off speed of 10 m/s.
This gives a greatest height of 1.25 m.

Find the horizontal speed, h m/s, and the angle a.
You may need to use the formula in **question 4**.

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