Cambridge IGCSE

Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

4821203557

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/63**

Paper 6 (Extended) **October/November 2019**

1 hour 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer both parts **A** (Questions 1 to 6) and **B** (Questions 7 to 9).
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and to communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40.

This document consists of **12** printed pages.

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The Investigation starts on the next page.

Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 6)

REMAINDERS (20 marks)

You are advised to spend no more than 45 minutes on this part.

This investigation is about the remainder when one positive integer is divided by another.

Example
$8 \div 38 = 0$, with a remainder of 8.
This can be written as $\mathrm{R}[8 \div 38] = 8$.

1 Find

(a) $\mathrm{R}[13 \div 5]$,

....

(b) $\mathrm{R}[5 \div 13]$.

....

2 Show that $\mathrm{R}[4000 \div 19] = 10$.

3 x is a factor of 20.

Show that $\mathrm{R}[20 \div x] = 0$.

4 For a positive integer, n, write down the largest and smallest values of $\mathrm{R}[n \div 100]$.

Largest ....

Smallest ....

5 **(a)** Complete this table of values of $\text{R}[a \div b]$.

		b					
		1	**2**	**3**	**4**	**5**	**6**
a	**1**	0	1	1	1	1	1
	2	0					
	3	0					
	4	0					
	5	0					
	6	0					

(b) $\text{R}[\mathbf{10} \div 9] + \text{R}[9 \div \mathbf{10}] = \mathbf{10}$

$\text{R}[\mathbf{11} \div 7] + \text{R}[7 \div \mathbf{11}] = \mathbf{11}$

These examples suggest that

$$\text{R}[\boldsymbol{x} \div y] + \text{R}[y \div \boldsymbol{x}] = \boldsymbol{x}.$$

Use values from the table to show one example that this is **not** always true.

(c) The remainder when a positive integer, n, is divided by 100 is $\text{R}[n \div 100]$.

Explain why dividing $\text{R}[n \div 100]$ by 100 gives the same remainder.

....

....

6 In this question x, y and z are positive integers.

(a) When $\mathrm{R}[x \div z] = a$ and $\mathrm{R}[y \div z] = b$,

then $\mathrm{R}[xy \div z] = \mathrm{R}[ab \div z]$.

(i) Check that this is true when $x = 2$, $y = 8$ and $z = 5$.

(ii) Using algebra, show that $\mathrm{R}[x^2 \div z] = \mathrm{R}[a^2 \div z]$.

(iii) Use the result in **part(a)(ii)** to show that $\mathrm{R}[76^2 \div 7] = 1$.

(b) From part **(a)** $R[xy \div z] = R[ab \div z]$
and $R[x^2 \div z] = R[a^2 \div z]$.

(i) Use patterns to help you complete the table.

$R[7919 \div 13] = 2$
$R[7919^2 \div 13] = R[2^2 \div 13]$ $= R[4 \div 13]$ $= 4$
$R[7919^4 \div 13] = R[4^2 \div 13]$ $= R[16 \div 13]$ $= 3$
$R[7919^8 \div 13] = R[3^2 \div 13]$ $= R[9 \div 13]$ $= 9$
R[ ] =

(ii) 7919^7 is over 20 digits long.
The following example shows how to calculate $R[7919^7 \div 13]$.

$$7919^7 = 7919^4 \times 7919^2 \times 7919^1$$
$$R[7919^7 \div 13] = R[(3 \times 4 \times 2) \div 13]$$
$$= R[24 \div 13]$$
$$= 11$$

Find $R[7919^{11} \div 13]$.

....

(iii) Is 13 a factor of 7919^9?
Show how you decide.

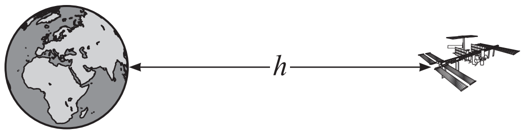
(c) Work out whether 7 is a factor of $7919^{64} + 5$.

B MODELLING (QUESTIONS 7 to 9)

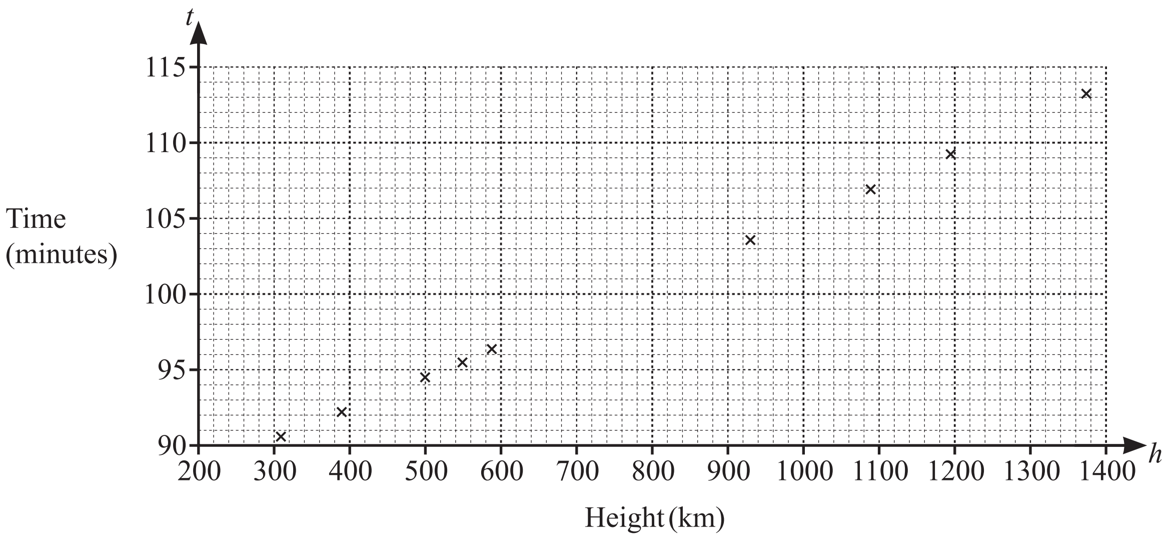
ORBITING SATELLITES (20 marks)

You are advised to spend no more than 45 minutes on this part.

This investigation is about satellites orbiting the Earth at a height, h kilometres, above the ground.



7 The scatter diagram shows the heights and the orbit times, t minutes, for 9 satellites making circular orbits around the Earth.



(a) **(i)** The mean of these satellite heights is 770 km and the mean orbit time is 100 minutes.

Plot this point.

(ii) Draw a line of best fit.

(iii) Use your line of best fit to find a straight line model (Model A) connecting t and h.

Give your answer in the form $t = mh + c$.

(b) Another model (Model B) connecting t and h is

$$t = 1.659 \times 10^{-4} \times \sqrt{(h+6370)^3}.$$

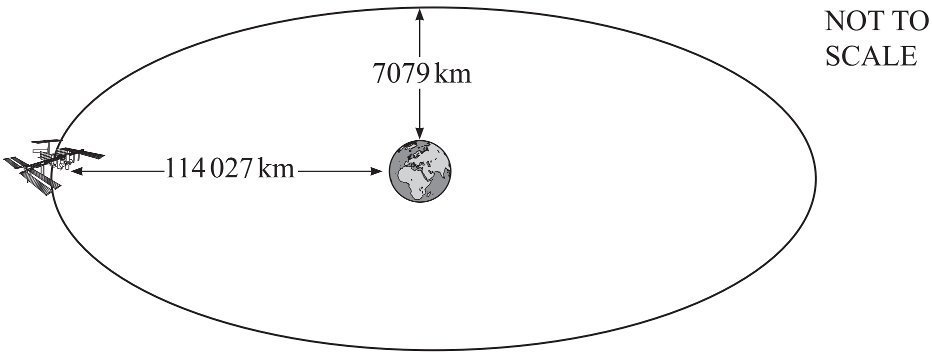
The satellite *NORAD 40730* has a circular orbit.
It takes 728.9 minutes to orbit the Earth at a height of 20 450 km.

Find which of the two models gives an orbit time closer to the actual orbit time.

(c) Communication satellites need an orbit time of 1440 minutes.

Use Model B to find the height which gives an orbit time of 1440 minutes.

8 Some satellites do not have circular orbits.



The height of satellite *NORAD 25989* is between 7079 km and 114 027 km.
It has an orbit time of 2872 minutes.

Use h as the mean of these two heights to calculate the orbit time using Model B.

$$t = 1.659 \times 10^{-4} \times \sqrt{(h + 6370)^3}$$

How does this time compare with the actual orbit time?

9 Some satellites eventually fall back to the Earth.

A model for the descent time of a satellite, D days, with an initial height, h kilometres, is

$$D = \left(5 \times 10^{-24}\right) h^{9.64}.$$

(a) Sketch the graph of this model for $400 \leqslant h \leqslant 600$.

(b) The first satellite, *Sputnik 1*, had an initial height of 577 km and a descent time of 92 days.

Compare this descent time with the descent time given by the model.

Question 9(c) is printed on the next page.

(c) Radiation from the Sun affects the descent time, D days.
This radiation, s, is measured in solar flux units (SFU).
The table shows the descent times, in days, for different values of h and s.

		Radiation (s SFU)							
		20	40	60	80	100	120	140	160
Initial height (h km)	500	553	350	233	163	118	88	68	53
	475	298	195	134	96	71	54	43	34
	450	158	107	76	56	43	33	27	22
	425	82	58	42	32	25	20	16	14
	400	42	31	23	18	15	12	10	9

(i) Write down the descent time of a satellite with an initial height of 450 km when the radiation is 100 SFU.

....

(ii) What does the table show about the effect of radiation on the descent time of a satellite?

....

....

(iii) A satellite has an initial height of 500 km.

$D = ks^{1.1}$ $D = k - s$ $D = ks^{0.9}$ $D = ks^{-1}$ where k is a constant.

Which one of these equations is the best model connecting descent time and radiation?
Give a reason for your answer.

....

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