

# Cambridge IGCSE<sup>™</sup>

CANDIDATE NAME		
CENTRE NUMBER		CANDIDATE NUMBER
CAMBRIDGE INTERNATIONAL MATHEMATICS		0607/52
Paper 5 Investigation (Core)		October/November 2022
		1 hour 10 minutes
You must answe		
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No additional materials are needed.

#### **INSTRUCTIONS**

- Answer all questions. •
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs. •
- Write your name, centre number and candidate number in the boxes at the top of the page. •
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid. •
- Do not write on any bar codes. •
- You should use a graphic display calculator where appropriate. •
- You may use tracing paper. •
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods. •
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working • to communicate your mathematics clearly and precisely.

#### **INFORMATION**

- The total mark for this paper is 36.
- The number of marks for each question or part question is shown in brackets [].

### Answer **all** the questions.

### INVESTIGATION

# TWO-STEP SEQUENCES

This investigation looks at *two-step sequences*. These are sequences which use two steps to get from one term to the next.

The first term in every sequence is 1.

The two steps are:

- multiply by a given number
- then add a given number.
- 1 In this question the two steps are:
  - multiply by 2
  - then add 1.

1 st term = 1

2nd term = 1st term  $\times$  2 + 1 = 1  $\times$  2 + 1 = 3 3rd term = 2nd term  $\times$  2 + 1 = 3  $\times$  2 + 1 = 7

4th term = 3rd term  $\times$  2 + 1 = 7  $\times$  2 + 1 = 15

(a) Work out the 5th term of this sequence.

(b) The 3rd term of this sequence is 7. You can write 7 as  $2^3 - 1$ .

Complete the table.

1st term	1	$2^{1}-1$
2nd term	3	$2^2 - 1$
3rd term	7	$2^3 - 1$
4th term	15	
5th term		

[1]

(c) Calculate the 20th term of this sequence. Write down all the digits shown on your calculator.

.....[2]

(d) (i) Use the last column in the table to write down an expression for the *n*th term of this sequence.

......[1]

(ii) Show that your expression gives the correct value for the 6th term of this sequence.

[2]

- 2 In this question the two steps are:
  - multiply by 3
  - then add 4.

The first term is 1.

(a) Calculate the 2nd, 3rd and 4th terms of this sequence.

(b) Complete the table.

1st term	1	$3^1 - 2$
2nd term		$3^2 - 2$
3rd term		
4th term		
5th term	241	3 <sup>5</sup> -

[2]

(c) Write down an expression for the *n*th term of this sequence.

......[1]

- 3 In this question the two steps are:
  - multiply by 4
  - then add 9.

The first term is 1.

Show that the expression for the *n*th term,  $4^n - 3$ , gives the correct value for the 3rd term of this sequence.

[4]

## 4 (a) Copy your results from Question 1(d)(i) and Question 2(c) into the table.

Use any patterns you notice to complete the table.

Steps to get the next term		Expression for the <i>n</i> th term
Multiply by 2,	then add 1	
Multiply by 3,	then add 4	
Multiply by 4,	then add 9	$4^{n} - 3$
Multiply by,	then add 16	$5^{n} - \dots$
Multiply by 6,	then add	5
Multiply by 7,	then add 36	
Multiply by,	then add	8 <sup><i>n</i></sup> – 7
	Multiply by 2, Multiply by 3, Multiply by 4, Multiply by, Multiply by 6, Multiply by 7,	Multiply by 2, then add 1   Multiply by 3, then add 4   Multiply by 4, then add 9   Multiply by, then add 16   Multiply by 6, then add

(b) A sequence has the two steps that you found in the last row of the table.

Show that the expression for the *n*th term gives the correct value for the 3rd term of this sequence.

[4]

(c) The *n*th term of a two-step sequence is  $22^n - 21$ .

Find the two steps.

•

(d) In a two-step sequence the steps are:

- multiply by 11
- then add 100.

The first term is 1.

(i) Find the value of the term nearest to 2000000. Write down all the digits shown on your calculator.

......[2]

(ii) Which term in the sequence is your answer to part (i)?

......[1]

Question 5 is printed on the next page.

5	In this question	the steps in Question	1 are in the reverse order.
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The two steps are now:

- add 1
- then multiply by 2.
- (a) The first term is 1. The second term is 4.

Calculate the 3rd, 4th and 5th terms.

(b) This two-step sequence has *n*th term equal to  $a \times 2^n - 2$ .

(i) The first term is 1.

Use this to find the value of *a*.

(ii) Use **part** (i) to show that the expression for the *n*th term gives the correct value for the 3rd term of this sequence.

[2]

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