Cambridge Assessment International Education

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CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/62**

Paper 6 Investigation and Modelling (Extended) **October/November 2022**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 6) and part **B** (Questions 7 to 10).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Any blank pages are indicated.

[Turn over

Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 6)

TWO-STEP SEQUENCES (30 marks)

You are advised to spend no more than 50 minutes on this part.

This investigation looks at *two-step sequences*.
These are sequences which use two steps to get from one term to the next.

> The first term in every sequence is 1.
>
> The two steps are:
> - multiply by a given number
> - then add a given number.

1 In this question the two steps are:
- multiply by 2
- then add 1.

1st term = 1

2nd term = 1st term × 2 + 1 = 1 × 2 + 1 = 3

3rd term = 2nd term × 2 + 1 = 3 × 2 + 1 = 7

4th term = 3rd term × 2 + 1 = 7 × 2 + 1 = 15

(a) Work out the 5th term of this sequence.

1, 3, 7, 15, ... [2]

(b) The nth term of another sequence is 2^n.

Calculate the 2nd, 3rd and 4th terms of this sequence.

2, ... , ... , ... , 32 [1]

(c) Look at your answers to **part (a)** and **part (b)**.

Write down an expression, in terms of n, for the nth term of the sequence in **part (a)**.

..... [1]

2 In this question the two steps are:
- multiply by 2
- then add 3.

The first term is 1.

(a) Work out the 2nd, 3rd and 4th terms of this sequence.

1 , ... , ... , ... , 61 [2]

(b) The nth term of this sequence is $a \times 2^n + b$.

(i) Substituting $n = 1$, to get the first term of the sequence, gives the equation $2a + b = 1$.

Substitute another value for n to make another equation in terms of a and b.

..... [1]

(ii) Solve the simultaneous equations in **part (i)** to show that the nth term of the sequence is

$$2 \times 2^n - 3.$$

[2]

3 In this question the two steps are:
- multiply by 2
- then add 5.

The first term is 1.
The expression for the nth term is $3 \times 2^n - 5$.

Show that this expression gives the correct value for the 4th term of this sequence.

[3]

4 In this question the two steps are always:
- multiply by 2
- then add k.

The first term is 1.

(a) Complete the table.
Use your answer to **Question 1(c)** and any patterns you notice.

Steps to get the next term	Expression for the nth term
Multiply by 2, then add 1	.....
Multiply by 2, then add 3	$2\times 2^n - 3$
Multiply by 2, then add 5	$3\times 2^n - 5$
Multiply by 2, then add 7	.....
Multiply by 2, then add	..... $- 9$

[2]

(b) An expression for the nth term of this sequence is $a\times 2^n + b$.

Find expressions for a and b in terms of k.

Write down the expression for the nth term of the sequence.

$a =$ .....

$b =$ .....

nth term = ... [3]

(c) The 5th term of a sequence using the nth term in **part (b)** is 286.

Complete the two steps.

- multiply by 2
- then add ..... [3]

5 In this question the two steps are:
- multiply by 3
- then add 2.

The expression for the nth term is $a \times 3^{(n-1)} + b$.

(a) The first term is 1.

(i) Find the value of the second term of the sequence.

..... [1]

(ii) Use the first two terms to write two equations in terms of a and b.

.....

..... [2]

(b) Find the value of a and the value of b.

$a =$ .....

$b =$ ..... [3]

6 **(a)** Complete the table.
Use your answer to **Question 5(b)** and any patterns you notice.

Steps to get the next term	Expression for the nth term
Multiply by 2, then add 1	$2 \times 2^{(n-1)} - 1$
Multiply by 3, then add 2	
Multiply by 4, then add 3	
Multiply by 5, then add 4	
Multiply by 6, then add 5	$2 \times 6^{(n-1)} - 1$

[1]

(b) For the sequence in the last row of the table,
the first term has the value 1 and the second term has the value 11.

Find which term has its value closest to 20 000 000.

..... [3]

B MODELLING (QUESTIONS 7 to 10)

DRIVING TO MY PLACE OF WORK (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task looks at a model for the time that I take to drive from my home to my place of work.

I live 20 km from my place of work.
When I leave my home at 7.00 am, I drive at an average speed of 50 km/h.

7 **(a)** Calculate the time, in minutes, to drive to work when I leave home at 7.00 am.

..... [3]

(b) The time that it takes me to drive to work is m minutes.

Find, in its simplest form, a model for m when my average speed is v km/h.

..... [1]

8 When I leave home after 7.00 am, there is more traffic, and my average speed is less than 50 km/h.

My average speed decreases steadily by 1 km/h for every 2 minutes after 7.00 am that I leave home. For example, when I leave at 6 minutes after 7.00 am, my average speed is 3 km/h less, which is 47 km/h.

(a) I leave home at 7.40 am.

(i) Find my average speed.

..... [2]

(ii) Show that the time to drive to work is 40 minutes.

[1]

(b) I leave home x minutes after 7.00 am.

Show that a model for the time, T minutes, to drive to work is $T = \frac{2400}{100 - x}$.

[2]

(c) Sketch the graph of the model $T = \frac{2400}{100 - x}$ for $0 \leqslant x \leqslant 90$.

[2]

(d) I do not want to drive for more than 30 minutes.

Find the latest time that I should leave home.

..... [2]

(e) I must be at work by 9.00 am.
One day I oversleep and leave home at 8.35 am.

(i) Use the model to find how late I will be for work.
Give your answer in hours and minutes.

..... [3]

(ii) Make a statement about the suitability of the model.

..... [1]

9 I leave home x minutes after 7.00 am.

(a) Explain why a model for A, the number of minutes after 7.00 am when I arrive at work, is

$$A = x + \frac{2400}{100 - x}.$$

..... [1]

(b) I must be at work by 9.00 am, which is two hours after 7.00 am.
So my maximum value of A is 120.

(i) Show that, for this maximum value of A, x is a solution to the equation

$$x^2 - 220x + 9600 = 0 \ .$$

[3]

(ii) Find this value of x.

..... [3]

(iii) Find the latest time that I can leave home to arrive at work on time.

..... [1]

10 I move to a new home and now live d km from my work.

When I leave my new home at 7.00 am, my average speed is v km/h.
As before, my average speed decreases steadily by 1 km/h for every 2 minutes after 7.00 am that I leave home.

(a) I leave my new home x minutes after 7.00 am.

Show that a model for the time, T minutes, to drive to work is $T = \dfrac{120d}{2v - x}$.

[2]

(b) I want to leave my new home at 7.30 am and arrive at work at 9.00 am.

Find a model for v in terms of d.

..... [3]

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