UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
Cambridge International Level 3 Pre-U Certificate
Principal Subject

FURTHER MATHEMATICS **9795/01**

Paper 1 Further Pure Mathematics **October/November 2013**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

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UNIVERSITY *of* CAMBRIDGE
International Examinations

1 For real values of t, the non-singular matrices $\mathbf{A}$ and $\mathbf{B}$ are such that

$$\mathbf{A}^{-1} = \begin{pmatrix} t & 5 \\ 2 & 8 \end{pmatrix} \quad \text{and} \quad \mathbf{B}^{-1} = \begin{pmatrix} 2 & -t \\ 3 & -1 \end{pmatrix}.$$

(i) Determine the values which t cannot take. [2]

(ii) Without finding either $\mathbf{A}$ or $\mathbf{B}$, determine $(\mathbf{AB})^{-1}$ in terms of t. [2]

2 Use de Moivre's theorem to express $\cos 3\theta$ in terms of powers of $\cos\theta$ only, and deduce the identity $\cos 6x \equiv \cos 2x(2\cos 4x - 1)$. [5]

3 The curve C has equation $y = \dfrac{2x}{x^2 + 1}$.

(i) Write down the equation of the asymptote of C and the coordinates of any points where C meets the coordinate axes. [2]

(ii) Show that the curve meets the line $y = k$ if and only if $-1 \leqslant k \leqslant 1$. Deduce the coordinates of the turning points of the curve. [5]

[Note: You are NOT required to sketch C.]

4 Let $\mathrm{f}(n) = 2(5^{n-1} + 1)$ for integers $n = 1, 2, 3, \ldots$.

(i) Prove that, if $\mathrm{f}(n)$ is divisible by 8, then $\mathrm{f}(n+1)$ is also divisible by 8. [3]

(ii) Explain why this result does **not** imply that the statement

'$\mathrm{f}(n)$ is divisible by 8 for all positive integers n'

follows by mathematical induction. [1]

5 The curve S has polar equation $r = 1 + \sin\theta + \sin^2\theta$ for $0 \leqslant \theta < 2\pi$.

(i) Determine the polar coordinates of the points on S where $\dfrac{\mathrm{d}r}{\mathrm{d}\theta} = 0$. [5]

(ii) Sketch S. [3]

6 G is the set $\{2, 4, 6, 8\}$, H is the set $\{1, 5, 7, 11\}$ and $\times_n$ denotes the operation of multiplication modulo n.

(i) Construct the multiplication tables for $(G, \times_{10})$ and $(H, \times_{12})$. [2]

(ii) By verifying the four group axioms, show that G and H are groups under their respective binary operations, and determine whether G and H are isomorphic. [6]

[You may assume that $\times_n$ is associative.]

7 Relative to an origin O, the points P, Q and R have position vectors

$$\mathbf{p} = \mathbf{i} + 2\mathbf{j} - 7\mathbf{k}, \quad \mathbf{q} = -3\mathbf{i} + 4\mathbf{j} + \mathbf{k} \quad \text{and} \quad \mathbf{r} = 6\mathbf{i} + 4\mathbf{j} + \alpha\mathbf{k}$$

respectively.

(i) Determine $\mathbf{p} \times \mathbf{q}$. [2]

(ii) Deduce the value of α for which

(a) OR is normal to the plane OPQ, [1]

(b) the volume of tetrahedron $OPQR$ is 50, [3]

(c) R lies in the plane OPQ. [2]

8 **(i)** Determine x and y given that the complex number $z = x + \mathrm{i}y$ simultaneously satisfies

$$|z - 1| = 1 \quad \text{and} \quad \arg(z + 1) = \tfrac{1}{6}\pi. \qquad [4]$$

(ii) On an Argand diagram, shade the region whose points satisfy

$$1 \leqslant |z - 1| \leqslant 2 \quad \text{and} \quad \tfrac{1}{6}\pi \leqslant \arg(z + 1) \leqslant \tfrac{1}{3}\pi. \qquad [6]$$

9 **(i)** Show that there is exactly one value of k for which the system of equations

$$\begin{aligned} kx + 2y + kz &= 4 \\ 3x + 10y + 2z &= m \\ (k - 1)x - 4y + z &= k \end{aligned}$$

does not have a unique solution. [4]

(ii) Given that the system of equations is consistent for this value of k, find the value of m. [4]

(iii) Explain the geometrical significance of a non-unique solution to a 3×3 system of linear equations. [2]

10 The roots of the equation $x^4 - 2x^3 + 2x^2 + x - 3 = 0$ are α, β, γ and δ. Determine the values of

(i) $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$, [2]

(ii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} + \dfrac{1}{\delta}$, [2]

(iii) $\alpha^3 + \beta^3 + \gamma^3 + \delta^3$. [4]

[Questions 11, 12 and 13 are printed on the next page.]

11 **(i)** Given that $y = -4$ when $x = 0$ and that

$$\frac{dy}{dx} - y = e^{2x} + 3,$$

find the value of x for which $y = 0$. [7]

(ii) Find the general solution of

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + 3,$$

given that $y = cx^2e^{2x} + d$ is a suitable form of particular integral. [7]

12 **(i)** **(a)** Use the method of differences to prove that

$$\sum_{n=k}^{N} \frac{1}{n(n+1)} = \frac{1}{k} - \frac{1}{N+1}.$$ [4]

(b) Deduce the value of $\displaystyle\sum_{n=k}^{\infty} \frac{1}{n(n+1)}$ and show that $\displaystyle\sum_{n=k}^{\infty} \frac{1}{(n+1)^2} < \frac{1}{k}$. [3]

(ii) Let $S = \displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$. Show that $\frac{205}{144} < S < \frac{241}{144}$. [3]

13 **(a)** Let $I_n = \displaystyle\int_0^{\alpha} \cosh^n x \, dx$ for integers $n \geqslant 0$, where $\alpha = \ln 2$.

(i) Prove that, for $n \geqslant 2$, $nI_n = \dfrac{3 \times 5^{n-1}}{4^n} + (n-1)I_{n-2}$. [5]

(ii) A curve has parametric equations $x = 12\sinh t + 4\sinh^3 t$, $y = 3\cosh^4 t$, $0 \leqslant t \leqslant \ln 2$. Find the length of the arc of this curve, giving your answer in the form $a + b\ln 2$ for rational numbers a and b. [8]

(b) The circle with equation $x^2 + (y - R)^2 = r^2$, where $r < R$, is rotated through one revolution about the x-axis to form a solid of revolution called a *torus*. By using suitable parametric equations for the circle, determine, in terms of π, R and r, the surface area of this torus. [11]