

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Pre-U Certificate

MARK SCHEME for the May/June 2014 series

9795 FURTHER MATHEMATICS

9795/02

Paper 2 (Further Application of Mathematics),
maximum raw mark 120

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, Pre-U, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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1	(i)	$X - Y \sim N(10, 5)$ $z_1 = \frac{8-10}{\sqrt{5}} [= -0.894\dots]; z_2 = \frac{11-10}{\sqrt{5}} [= 0.447\dots]$ $P(8 < \text{Breadth} < 11) = \mathbf{0.487}$	Either. cc or σ^2 : M1A0 Both. FT on σ cwo	B1 M1A1 A1 [4]
	(ii)	$X_1 + X_2 + Y_1 + Y_2 \sim N(80, 10)$ $z = \frac{75-80}{\sqrt{10}} [= -1.58\dots]$ $P(\text{Perimeter} > 75) = \mathbf{0.943}$	As above [e.g. $N(80, 20)$, 0.8681: 2/4] cwo	B1 M1A1√ A1 [4]
2	(i)	$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ stated or implied $\frac{\mu + 0.5\sigma - \mu}{\sigma/\sqrt{n}} > 1.96$ $\Rightarrow 0.5\sqrt{n} > 1.96 \Rightarrow n > 15.355 \Rightarrow$ Least n is 16	1.96	B1 M1A1 M1A1 [5]
	(ii)	$z = \frac{\mu - 0.1\sigma - \mu}{\sigma/4} = -0.4$ $\Rightarrow P(\bar{X} > \mu - 0.1\sigma) = 0.655$	FT on their integer n , and also -0.392 : M1A1A0	M1 A1√ A1 [3]
3	(i)	Let N be the estimated number; $\frac{400}{N} = \frac{20}{400} \Rightarrow N = 8000$		B1 [1]
	(ii)	98% confidence: $z = \pm 2.326$ $p_s = 0.05$ $0.05 \pm 2.326 \sqrt{\frac{0.05 \times 0.95}{400}}$ $= \mathbf{(0.0246(5), 0.0753(5))}$	Need $\sqrt{\frac{pq}{n}}$ 0.05 and variance correct Both, correct to 3 SF	B1 B1 M1A1 A1 [5]
	(iii)	$400 \div 0.07535 = 5309$ and $400 \div 0.02465 = 16228$	$400 \div \text{CV}$ Integer answers, a.r.t. 5310, 16200	M1 A1 [2]

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4	(i)	$\left[-3e^{-x}\right]_0^k = 1 \Rightarrow -3e^{-k} + 3 = 1 \Rightarrow e^{-k} = \frac{2}{3} \text{ [AG]}$		M1A1 [2]
	(ii)	$M_X(t) = \int_0^k 3e^{(t-1)x} dx = \left[3 \frac{e^{(t-1)x}}{(t-1)}\right]_0^k$ $= 3 \frac{e^{(t-1)k}}{(t-1)} - \frac{3}{(t-1)} = \frac{3}{(t-1)} (1 - e^{-k} e^{kt})$ $= \frac{3}{(t-1)} \left(1 - \frac{2}{3} e^{kt}\right) \text{ [AG]}$		M1 A1 M1 A1 [4]
	(iii)	$M_X(t) = 3(1+t+t^2) \left(1 - \frac{2}{3} \left[1 + kt + \frac{1}{2} k^2 t^2\right]\right)$ $= 1 + (1-2k)t + (1-2k-k^2)t^2 \text{ [AG]}$	Both series correct Deals with $\frac{2}{3}$	B1 B1 B1 [3]
	(iv)	$E(X) = 1 - 2k = 1 - 2 \ln\left(\frac{3}{2}\right) \text{ [AG]}$	Allow longer methods if correct	B1 [1]
5	(i)	$G(t) = e^{\lambda(t-1)}$ $G'(t) = \lambda e^{\lambda(t-1)}$ $\Rightarrow \mu = G'(1) = \lambda$ $G''(t) = \lambda^2 e^{\lambda(t-1)}$ $\Rightarrow G''(1) = \lambda^2$ $\therefore \sigma^2 = E(X^2) + \mu - \mu^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$	In tables	M1 A1 M1 A1 A1 [5]
	(ii)	$z_1 = \frac{229.5 - 250}{\sqrt{250}} = -1.297$ $z_2 = \frac{260.5 - 250}{\sqrt{250}} = 0.664$ $P(230 \leq X \leq 260) = \mathbf{0.649}$	One cc correct Both ccs, $\sqrt{250}$ Correct handling of tails Answer in range [0.649, 0.650]	M1A1 A1 M1 A1 [5]
	(iii)	We are approximating to B(250, 0.1). Using N(25, 22.5) or Using Po(25) $z = \frac{30.5 - 25}{\sqrt{22.5}} (= 1.1595) \quad z = \frac{30.5 - 25}{\sqrt{25}} (= 1.1)$ P(>30) = 0.123 P(>30) = 0.136	Distribution stated or implied Correct CC and $\sqrt{\quad}$ Exact binomial 0.125: M0 Exact Poisson 0.136691: 3	M1 A1 A1 [3]

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<p>6 (i)</p> $\frac{4}{\pi} [\tan^{-1} x]_0^{0.41} = 0.495\dots, \frac{4}{\pi} [\tan^{-1} x]_0^{0.42} = 0.506\dots$ <p>$\Rightarrow 0.41 < \text{median} < 0.42$ [AG]</p> <p>Or: $\frac{4}{\pi} [\tan^{-1} x]_0^m = 0.5$; $m = \frac{\pi}{8} = 0.414\dots$</p> <p>(ii)</p> $E(X) = \frac{4}{\pi} \int_0^1 \frac{x}{1+x^2} dx$ $= \frac{2}{\pi} \ln[1+x^2]_0^1 = \frac{2}{\pi} \ln 2$ [AG] <p>(iii)</p> $E(X^2) = \frac{4}{\pi} \int_0^1 \frac{x^2}{1+x^2} dx = \frac{4}{\pi} \int_0^1 1 - \frac{1}{1+x^2} dx$ $= \frac{4}{\pi} [x - \tan^{-1} x]_0^1 = \frac{4}{\pi} \left(1 - \frac{\pi}{4}\right) = \frac{4}{\pi} - 1$ $\text{Var}(X) = \left(\frac{4}{\pi} - 1\right) - \left(\frac{2}{\pi} \ln 2\right)^2 = 0.0785$ <p>(iv)</p> $\frac{4}{\pi} [\tan^{-1} x]_{\sqrt{2}-1}^1 = 1 - \frac{1}{2} = \frac{1}{2}$ $\frac{4}{\pi} [\tan^{-1} x]_{\sqrt{3}/3}^1 = 1 - \frac{2}{3} = \frac{1}{3}$ $P(X > 1/3 \sqrt{3} \mid X > \sqrt{2} - 1) = \frac{1}{3} \div \frac{1}{2} = \frac{2}{3} \text{ or } \frac{\frac{\pi}{4} - \frac{\pi}{6}}{\frac{\pi}{4} - \frac{\pi}{8}} = \frac{2}{3}$		<p>Correct integral</p> <p>One correct output</p> <p>Conclusion fully justified (need some comment if 0.41 and 0.42 used)</p> <p>$\int x f(x) dx$ attempted</p> <p>$\int x^2 f(x) dx$ seen</p> <p>Method for integration</p> <p>$\left(\frac{4}{\pi} - 1\right)$ or 0.2732</p> <p>Answers from calculator can get full marks</p> <p>Quotient of relevant probabilities</p> <p>One correct</p>	<p>M1</p> <p>A1</p> <p>A1 [3]</p> <p>M1</p> <p>A1 [2]</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 [5]</p> <p>M1</p> <p>A1</p> <p>A1 [3]</p>
<p>7</p> <p>Recognising (3, 4, 5) triangle</p> <p>Resolving vertically for P: $T = 0.5g$</p> <p>Resolving vertically for Q: $\frac{3}{5}T = mg$</p> <p>Resolving horizontally for Q: $\frac{4}{5}T = 4m\omega^2$</p> <p>$\Rightarrow m = 0.3$</p> <p>$\omega = \sqrt{\frac{g}{3}}$ or 1.83</p>		<p>[can award B1 if answer correctly obtained without use of $T = 5$]</p> <p>Needs resolving with trig (if resolve QR must have \cos or $\sin \times m r \omega^2$)</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1 [8]</p>

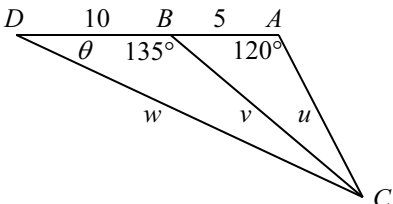
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8	<p>Components of speed \parallel to AB are 12 and 3 ms^{-1}. CLM: $2 \times 12 - 3 \times 3 = 2v_A + 3v_B$ NEL: $-\frac{2}{3}(12+3) = v_A - v_B$</p> <p>$v_A = -3, v_B = 7$</p> <p>Components of velocity \perp to AB are 5 and 4 ms^{-1}. Speed $A = \sqrt{3^2 + 5^2} = \sqrt{34}$ or 5.83 ms^{-1}, Speed $B = \sqrt{7^2 + 4^2} = \sqrt{65}$ or 8.06 ms^{-1} (OE)</p>	<p>Any signs on RHS for A1 Consistent signs on RHS for A1 (both $\sqrt{\quad}$ on x-components) Both either Both answers, correct to 3SF if necessary</p>	<p>B1 M1A1$\sqrt{\quad}$ M1A1$\sqrt{\quad}$ A1 B1 M1 A1 [9]</p>
9	<p>(i) $750 = \frac{1}{2}(5 + 15)t \Rightarrow t = 75$</p> <p>(ii) Let P denote the constant power of the engine. Newton II: $\frac{P}{v} = m \frac{dv}{dt} \Rightarrow \frac{P}{m} \int_0^t dt = \int_5^{15} v dv$ $\Rightarrow \frac{Pt}{m} = \left[\frac{v^2}{2} \right]_5^{15} (=100)$ Also $\frac{P}{v} = m v \frac{dv}{dx} \Rightarrow \frac{P}{m} \int_0^{750} dx = \int_5^{15} v^2 dv$ $\Rightarrow \frac{750P}{m} = \left[\frac{v^3}{3} \right]_5^{15} \left(= \frac{3250}{3} \right)$ $\Rightarrow \frac{P}{m} = \frac{13}{9}t$ and $t = \frac{900}{13}$ or 69.2 seconds</p>	<p>If $a [= \frac{2}{15}]$ found, need second equation for M1 Or: Energy: $750F = \frac{1}{2}m(15^2 - 5^2)$ $15 = 5 + \frac{F}{m}t \Rightarrow t = \frac{10m}{F} = 75$</p> <p>For $F = \frac{P}{v}$ seen If ΔKE used, must equate to Pt</p>	<p>M1 A1 (B1) (B1) [2] B1 M1 A1 A1 A1 A1 A1 A1 [9]</p>

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<p>10 (i)</p> <p>(ii)</p> <p>(iii)</p> <p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	<p>Hooke's Law: $\frac{0.05\lambda}{0.5} = 2.5g \Rightarrow \lambda = 250 \text{ N}$ [AG]</p> <p>Let e m be the extension in the new equilibrium position.</p> <p>$4g = \frac{250e}{0.5} \Rightarrow e = 0.08 \text{ m}$</p> <p>Newton II: $4\ddot{x} = 4g - \frac{250(0.08+x)}{0.5} \Rightarrow \ddot{x} = -125x$</p> <p>Amplitude of new motion is $0.08 - 0.05 = 0.03 \text{ m}$</p> <p>Maximum speed = $\sqrt{125} \times 0.03 = 0.335 \text{ ms}^{-1}$.</p> <p>$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{125}} = 0.562 \text{ seconds}$.</p> <p>$\frac{1}{\sqrt{125}} \left(\frac{\pi}{2} + \sin^{-1} \frac{2}{3} \right) = 0.206 \text{ seconds}$</p>	<p>Find new e</p> <p>Needs $0.08\sqrt{\quad}$</p> <p>SR: $\ddot{x} = 10 - 125x$ and then $x = y - 0.08$: B2</p> <p>cwo</p> <p>$\sqrt{\quad}$ on a or ω</p> <p>$\sqrt{\quad}$ on ω</p> <p>$\sin^{-1}(\frac{2}{3})/\omega$ or $\cos^{-1}(\frac{2}{3})/\omega$</p> <p>Deal with quadrants</p>	<p>M1A1 [2]</p> <p>M1 A1 M1</p> <p>A1 [4]</p> <p>B1 [1]</p> <p>B1$\sqrt{\quad}$ [1]</p> <p>B1$\sqrt{\quad}$ [1]</p> <p>M1 M1 A1 [3]</p>
<p>11 (i)</p> <p>(ii) (a)</p> <p>(b)</p>	<p>$gx^2 \tan^2 \alpha - 2V^2x \tan \alpha + (gx^2 + 2V^2y) = 0$ or equiv For boundary of accessible points $B^2 - 4AC = 0$ $\Rightarrow 4V^4x^2 - 4gx^2(gx^2 + 2V^2y) = 0$ (*) $\Rightarrow 2gV^2y = V^4 - g^2x^2 \Rightarrow y = \frac{1}{2gV^2}(V^4 - g^2x^2)$ [AG]</p> <p>Or: $\frac{dy}{d\alpha} = x \sec^2 \alpha - \frac{gx^2}{2V^2} \tan \alpha \sec^2 \alpha$ $= 0 \Rightarrow x = \frac{V^2}{g \tan \alpha} \Rightarrow y = \frac{1}{2gV^2}(V^4 - g^2x^2)$</p> <p>Putting $y = -h$ and $V^2 = gh \Rightarrow -h = \frac{1}{2g^2h}(g^2h^2 - g^2x^2)$ $\Rightarrow x = \sqrt{3h}$</p> <p>Substituting for x in (*) gives $3gh^2 \tan^2 \alpha - 2\sqrt{3}gh^2 \tan \alpha + 3gh^2 - 2gh^2 = 0$ $\Rightarrow 3 \tan^2 \alpha - 2\sqrt{3} \tan \alpha + 1 = 0$ $\Rightarrow \tan \alpha = \frac{1}{\sqrt{3}} \Rightarrow \alpha = 30^\circ$ or $\frac{\pi}{6}$</p>	<p>y must be part of quadratic Allow $\Delta \geq 0$</p> <p>“Quadratic equation” required in question, so B0</p> <p>$y = h$: M1A0</p> <p>Or quote $\tan \alpha = \frac{-B}{2A}$</p>	<p>B1 M1 A1</p> <p>A1 [4]</p> <p>B0M1 A1 A1 [max 3]</p> <p>M1 A1 A1 [3]</p> <p>M1 A1 A1 [3]</p>

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	Notation: Speed of wind relative to: ground, w ; cyclist at 15 ms^{-1} , u ; cyclist at 10 ms^{-1} , v Angle of wind velocity with due west, θ (see diagram in method α)		
12	<p>Method α:</p>  <p> $\angle ABC = 45^\circ$ $\angle BAC = 120^\circ$ $\angle BCA = 15^\circ$ </p> $\frac{v}{\sin 120^\circ} = \frac{5}{\sin 15^\circ} \Rightarrow v = \frac{5 \sin 120^\circ}{\sin 15^\circ} = 16.73$ $w^2 = 10^2 + 16.73^2 - 2 \times 10 \times 16.73 \times \cos 135^\circ$ $\Rightarrow w = 24.8(3)$ $\frac{\sin \theta}{16.73} = \frac{\sin 135^\circ}{24.83}$ $\Rightarrow \theta = 28.45^\circ.$ $\Rightarrow \text{Wind blows from bearing } 118^\circ.$ <p>Method β: $u \begin{pmatrix} -\sin 30^\circ \\ \cos 30^\circ \end{pmatrix} + \begin{pmatrix} -15 \\ 0 \end{pmatrix} = w \begin{pmatrix} -\sin 45^\circ \\ \cos 45^\circ \end{pmatrix} + \begin{pmatrix} -10 \\ 0 \end{pmatrix}$</p> $\frac{u}{2} + 15 = \frac{w}{\sqrt{2}} + 10$ $\frac{\sqrt{3}u}{2} = \frac{w}{\sqrt{2}}$ $u = 5 + 5\sqrt{3} = 13.66 \quad \text{or} \quad v = \frac{5}{2}\sqrt{2}(3 + \sqrt{3}) = 9.66$ <p>Either side = $\begin{pmatrix} -\frac{5}{2}(7 + \sqrt{3}) \\ \frac{5}{2}(3 + \sqrt{3}) \end{pmatrix} = \begin{pmatrix} -21.83 \\ 11.83 \end{pmatrix}$</p> <p>Method γ: $15 + \frac{w \sin \theta}{\sqrt{3}} = w \cos \theta$</p> $10 + w \sin \theta = w \cos \theta$ $\tan \theta = \frac{9 + 2\sqrt{3}}{23} \Rightarrow \theta = 28.45^\circ.$ <p>Wind blows from bearing 118°. $v = 24.8(3)$. Speed of wind is 24.8 km h^{-1}.</p>	<p>Diagram, with all three</p> <p>Or: $\frac{u}{\sin 135^\circ} = \frac{5}{\sin 15^\circ}$</p> $\Rightarrow u = \frac{5 \sin 135^\circ}{\sin 15^\circ} = 13.66 \text{ or } 5 + 5\sqrt{3}$ <p>(N.B. Accept this vector answer from any method)</p> <p>[Note: If a and b for $w \sin \theta$ and $w \cos \theta$ in the above equations, each equation gets M1A1, followed by M1A1A1 for v and M1A1A1 for θ.]</p>	<p>B1</p> <p>M1A1 A1</p> <p>M1A1 A1</p> <p>M1 A1 A1 [10]</p> <p>M1A1 M1A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1A1 A1 [10]</p> <p>M1A1A1 M1A1A1</p> <p>M1A1 M1A1[10]</p>

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	<p>Method δ:</p> $\frac{w}{\sin 120^\circ} = \frac{15}{\sin(60^\circ - \theta)} \Rightarrow w = \frac{15 \sin 120^\circ}{\sin 60^\circ \cos \theta - \cos 60^\circ \sin \theta}$ $\Rightarrow w = \frac{15\sqrt{3}}{\sqrt{3} \cos \theta - \sin \theta}$ $\frac{w}{\sin 135^\circ} = \frac{10}{\sin(45^\circ - \theta)} \Rightarrow w = \frac{10 \sin 135^\circ}{\sin 45^\circ \cos \theta - \cos 45^\circ \sin \theta}$ $\Rightarrow w = \frac{10}{\cos \theta - \sin \theta}$ <p>Equate: $15\sqrt{3} \cos \theta - 15\sqrt{3} \sin \theta = 10\sqrt{3} \cos \theta - 10 \sin \theta$</p> $\Rightarrow \tan \theta = \frac{5\sqrt{3}}{15\sqrt{3} - 10} = \frac{9 + 2\sqrt{3}}{23} \quad [= 0.5419]$ $\Rightarrow \theta = 28.45 \Rightarrow \text{Wind blows from bearing } 118^\circ$ <p>and $w = 24.83$</p> <p>Method ε: Components:</p> $w_x = u_x + 15 = v_x + 10$ $w_y = u_y = v_y$ $\frac{w_y}{w_x - 15} = \tan(60^\circ) = \sqrt{3}, \quad \frac{w_y}{w_x - 10} = \tan(45^\circ) = 1$ $w_x - 10 = \sqrt{3}(w_x - 15) \Rightarrow w_x = \frac{5}{2}(7 + \sqrt{3}),$ $w_y = \frac{5}{2}(3 + \sqrt{3})$ $\Rightarrow \text{speed} = 24.8(3) \text{ ms}^{-1}$ <p>Angle is $\tan^{-1}(0.5419) = 28.45^\circ$</p> <p>$\Rightarrow$ wind blows from bearing 118°.</p>	<p>Two equations for θ and w</p> <p>Compound angle formulae used</p> <p>Correct values of sin and cos</p> <p>Solve</p> <p>Correct θ, correct bearing</p> <p>Correct w</p> <p>Any two</p> <p>All four</p>	<p>M1A1</p> <p>M1A1</p> <p>M1A1</p> <p>M1 A1A1 A1 [10]</p> <p>M1 A1 M1 A1 A1</p> <p>M1A1</p> <p>A1 M1 A1 [10]</p>
	<p>Summary of answers:</p> $\mathbf{w} = \begin{pmatrix} -\frac{5}{2}(7 + \sqrt{3}) \\ \frac{5}{2}(3 + \sqrt{3}) \end{pmatrix} = \begin{pmatrix} -21.83 \\ 11.83 \end{pmatrix}, w = 2\sqrt{16 + 5\sqrt{3}} = 24.83$ $\theta = \tan^{-1}\left(\frac{3 + \sqrt{3}}{7 + \sqrt{3}}\right) \text{ or } \tan^{-1}\left(\frac{18 + 4\sqrt{3}}{46}\right) = 28.45^\circ \Rightarrow \text{wind blows from bearing } 118(.45)^\circ$ $\mathbf{u} = \begin{pmatrix} -\frac{5}{2}(1 + \sqrt{3}) \\ \frac{5}{2}(3 + \sqrt{3}) \end{pmatrix} = \begin{pmatrix} -6.83 \\ 11.83 \end{pmatrix}, u = 13.66 \quad \mathbf{v} = \begin{pmatrix} -\frac{5}{2}(1 + \sqrt{3}) \\ \frac{5}{2}(3 + \sqrt{3}) \end{pmatrix} = \begin{pmatrix} -11.83 \\ 11.83 \end{pmatrix}, v = 16.73$		