CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Pre-U Certificate

MARK SCHEME for the May/June 2015 series

9795 FURTHER MATHEMATICS

9795/01 Paper 1 (Further Pure Mathematics), maximum raw mark 120

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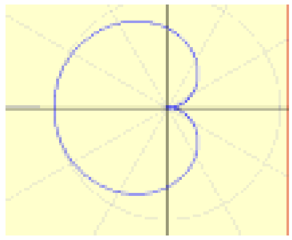
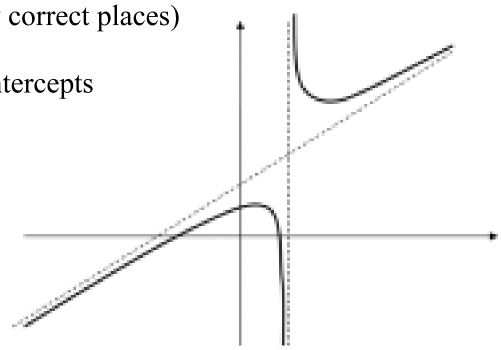
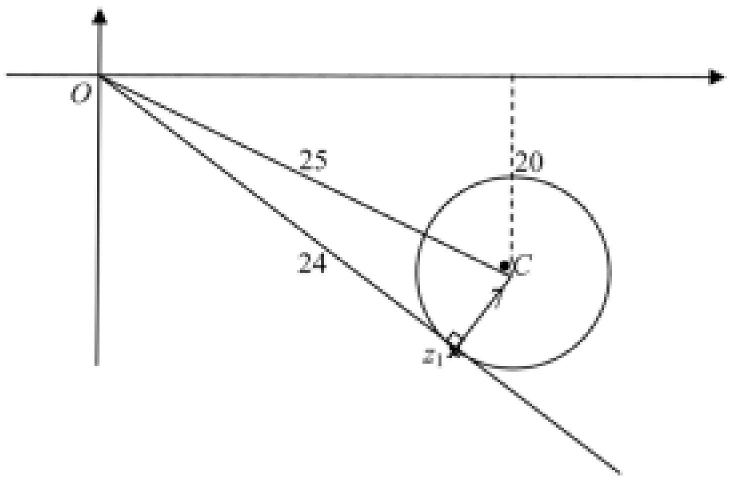
1	use of formula $V = \frac{1}{6}\left|\mathbf{a} \bullet \mathbf{b} \times \mathbf{c}\right|$ or equivalent *NB* $\mathbf{b} \times \mathbf{c} = -4\mathbf{i} + 10\mathbf{j} + 2\mathbf{k}$	M1
	attempt at relevant scalar triple product = $\begin{vmatrix} 2 & 3 & -2 \\ 2 & 0 & 4 \\ 6 & 1 & 7 \end{vmatrix} = 18$	M1
	(or a scalar and a vector product)	
	$V = 3$ **cso**	A1 [3]
2	$a_4 = \dfrac{y^{(4)}(1)}{4!} = \dfrac{(-2)^3}{1.3.5.7}$ from the Taylor series expansion	M1 A1
	$\Rightarrow y^{(4)}(1) = \dfrac{(-2)^3}{1.3.5.7} \times 4! = -\dfrac{64}{35}$	A1
	ALTERNATIVE	
	Diff^te. four times to get $\dfrac{\mathrm{d}^4 y}{\mathrm{d}x^4} = \sum_{n=4}^{\infty} \dfrac{(-2)^{n-1} n(n-1)(n-2)(n-3)(x-1)^{n-4}}{1.3.5....(2n-1)}$	M1 A1
	When $x = 1$, this is $\dfrac{(-2)^3 \times 4(3)(2)(1)}{1.3.5.7} + 0 = -\dfrac{64}{35}$	A1 [3]
3	$n = 1$, $\mathbf{M}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 2(1)+1 \\ 2(1)^2 + 2(1) \\ 2(1)^2 + 2(1) + 1 \end{pmatrix} \Rightarrow$ result true for $n = 1$ (both sides shown)	B1
	induction hypothesis that $\mathbf{M}^k \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2k+1 \\ 2k^2 + 2k \\ 2k^2 + 2k + 1 \end{pmatrix}$	M1
	attempt at $\mathbf{M}^{k+1}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \mathbf{M}\begin{pmatrix} 2k+1 \\ 2k^2 + 2k \\ 2k^2 + 2k + 1 \end{pmatrix} = \begin{pmatrix} 2k+1-4k^2-4k+4k^2+4k+2 \\ 4k+2-2k^2-2k+4k^2+4k+2 \\ 4k+2-4k^2-4k+6k^2+6k+3 \end{pmatrix}$	M1
	$= \begin{pmatrix} 2k+3 \\ 2k^2 + 6k + 4 \\ 2k^2 + 6k + 5 \end{pmatrix}$	A1
	$= \begin{pmatrix} 2(k+1)+1 \\ 2(k+1)^2 + 2(k+1) \\ 2(k+1)^2 + 2(k+1) + 1 \end{pmatrix}$	A1
	This is the result with k replaced by $(k + 1)$. Hence IF the result is true for $n = k$ THEN it also true for $n = k + 1$. Since the result is true for $n = 1$, it follows that it is true for all positive integers n.	E1 [6]

4	(i)	Closed curve containing the pole, O Cusp at the pole Essentially all correct, including at least $(r, \theta) = (1, \pi)$ 	B1 B1 B1 [3]
	(ii)	$A = \left(\frac{1}{2}\right)\int_0^{2\pi} \sin^2 \frac{1}{2}\theta \, d\theta$ Condone missing $\frac{1}{2}$ until final A1	M1
		use of double-angle identity $A = \frac{1}{2}\int_0^{2\pi}\left(\frac{1}{2} - \frac{1}{2}\cos\theta\right) d\theta$	M1
		correct integration. $= \frac{1}{4}[\theta - \sin\theta]$	A1
		$= \frac{1}{2}\pi$	A1 [4]
5	(i)	VA $x = 3$	B1
		$y = \frac{2x^2 + 5x - 25}{x - 3} = \frac{2x(x-3) + 11(x-3) \quad [+8]}{x - 3}$	M1
		$y = 2x + 11$ oblique asymptote	A1 [3]
	(ii)	for $\frac{dy}{dx} = \frac{(x-3)(4x+5) - (2x^2 + 5x - 25)}{(x-3)^2}$ (good attempt at the *Quotient Rule*)	M1
		correct unsimplified **OR** $\frac{d}{dx}\left(2x + 11 + \frac{8}{x-3}\right) = 2 - \frac{8}{(x-3)^2}$	A1
		solving quadratic equation $4x^2 - 7x - 15 = 2x^2 + 5x - 25$ i.e. $2x^2 - 12x + 10 = 0$ **OR** $(x - 3)^2 = 4$	M1
		(1, 9) and (5, 25) (Give one A1 for both x's correct without either y)	A1 A1 [5]
	(iii)	General shape (with asymptotes and turning points in approximately correct places) 	B1
		$2x^2 + 5x - 25 = 0$ solved to find x-intercepts	M1
		$x = -5$ or $2\frac{1}{2}$	A1 [3]

6	(i)	$z^n = \cos n\theta + \mathrm{i}\sin n\theta$ and $z^{-n} = \cos n\theta - \mathrm{i}\sin n\theta \Rightarrow z^n + \frac{1}{z^n} \equiv 2\cos n\theta$	B1 [1]
	(ii)	$\left(z+\frac{1}{z}\right)^5 \equiv \left(z^5+\frac{1}{z^5}\right)+5\left(z^3+\frac{1}{z^3}\right)+10\left(z+\frac{1}{z}\right)$	M1 A1
		Repeated use of result $\Rightarrow 32\cos^5\theta \equiv 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$	M1
		$\Rightarrow 16\cos^5\theta \equiv \cos 5\theta + 5\cos 3\theta + 10\cos\theta$ **Answer Given**	A1 [4]
	(iii)	$16\cos^5\theta = \pm\cos\theta$ Condone sign error here to allow for first A1	M1
		$\cos\theta = 0 \Rightarrow \theta = \frac{1}{2}\pi$ or $\frac{3}{2}\pi$ (both)	A1
		$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{1}{3}\pi$ or $\frac{5}{3}\pi$ (both)	A1
		$\cos\theta = -\frac{1}{2} \Rightarrow \theta = \frac{2}{3}\pi$ or $\frac{4}{3}\pi$ (both)	A1
		but allow one A1 for $\theta = \frac{1}{3}\pi$ and $\theta = \frac{2}{3}\pi$ if both 2nd answers missing	[4]
7	(i)	Circle with centre (20, –15)	B1
		Circle of radius = 7 stated or deducible from diagram	B1
		Allow B1 for a circle entirely in the 4th Quad.	
		Interior of circle shaded (Ignore boundary in/out)	B1 [3]
	(ii)		
		for z_1 in correct place	B1
		for (sufficient) distances	B1
		for $\arg(z_1) = -\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{7}{24}\right)$ or equivalent, using other inverse trig. functions	M1
		[NB – this is $\left(-\tan^{-1}\frac{4}{3}\right)$ using a result in Q13]	
		for – 0.927 (or 2π – 0.927 = 5.356)	A1 [4]

8	**(i)**	(group table below)	B4, 3, 2, 1, 0 [4]

	e	a	b	c	ab	bc	ca	abc
e	e	a	b	c	ab	bc	ca	abc
a	a	e	ab	ca	b	abc	c	bc
b	b	ab	e	bc	a	c	abc	ca
c	c	ca	bc	e	abc	b	a	ab
ab	ab	b	a	abc	e	ca	bc	c
bc	bc	abc	c	b	ca	e	ab	a
ca	ca	c	abc	a	bc	ab	e	b
abc	abc	bc	ca	ab	c	a	b	e

(ii) Subgroups of order 2: $\{e, a\}, \{e, b\}, \{e, c\}, \{e, ab\}, \{e, bc\}, \{e, ca\}, \{e, abc\}$

6 distinct given — B1

all 7, no repeats — B1

Subgroups of order 4: $\{e, a, b, ab\}, \{e, b, c, bc\}, \{e, c, a, ca\}$
$\{e, a, bc, abc\}, \{e, b, ca, abc\}, \{e, c, ab, abc\}$
$\{e, ab, bc, ca\}$

Either (all 3 of 1st type) + **B1** (all3 of 2nd type) + **B1** (last one) — B2

Or (5 distinct) + **B1** (6th) + **B1** (all 7 and no repeats) — [5]

9 **(i)** Comp. Fn. is $u = A\cos 2x + B\sin 2x$ Allow $Ae^{2ix} + Be^{-2ix}$ here — M1 A1

For Part. Int. try $u = ax + b$ ($u' = a$ and $u'' = 0$) and substd. into given d.e. — M1

$a = 2,\ b = \frac{1}{4}$ — A1

Gen. Soln. is $u = A\cos 2x + B\sin 2x + 2x + \frac{1}{4}$ **ft** — B1

Must have two arbitrary constants
Condone apparently complex coefficients here
Don't allow $Ae^{2ix} + Be^{-2ix}$ here

[5]

(ii) $xy = u \Rightarrow x\frac{dy}{dx} + y = \frac{du}{dx}$ and $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = \frac{d^2u}{dx^2}$ — M1 A1

$\Rightarrow$ (*) becomes $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4xy = 8x + 1$ — A1

$\Rightarrow y = \frac{A\cos 2x}{x} + \frac{B\sin 2x}{x} + 2 + \frac{1}{4x}$ **ft** — B1

Accept $xy = \ldots$
Must have two real arbitrary constants

[4]

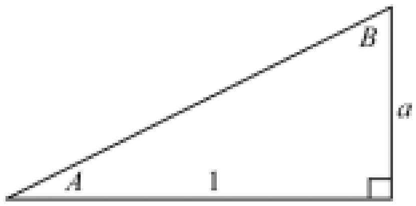
10	**(i)**	d.v. = $\begin{pmatrix}1\\7\\-6\end{pmatrix}\times\begin{pmatrix}3\\-5\\8\end{pmatrix}=\begin{pmatrix}26\\-26\\-26\end{pmatrix}\equiv\begin{pmatrix}-1\\1\\1\end{pmatrix}$	M1 A1
		finding one point on line: e.g. (0, 8, 11), (8, 0, 3), (11, –3, 0)	M1 A1
		ft Line equation in vector form $\mathbf{r}=\begin{pmatrix}8\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}$ (must have **r** = …, ***l*** = …)	B1 [5]
		ALTERNATIVE 1	
		finding two points on line: e.g. (0, 8, 11), (1, 7, 10), (8, 0, 3), (11, –3, 0) , …	M1 A1 A1
		ft d.v. $\mathbf{r}=\begin{pmatrix}-1\\1\\1\end{pmatrix}$ (e.g.)	B1
		ft Line equation in vector form $\mathbf{r}=\begin{pmatrix}8\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}$ (must have **r** = …)	B1 [5]
		ALTERNATIVE 2	
		Eliminating x (say): $z = y + 3$	M1 A1
		Setting $y = \lambda$ (or equivalent) and finding z and x in terms of the parameter	M1
		$z = \lambda + 3$ and $x = 8 - \lambda$	A1
		ft Line equation in vector form $\mathbf{r}=\begin{pmatrix}8\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}$ (must have **r** = …)	B1 [5]

(ii)	$\begin{vmatrix} 1 & 7 & -6 \\ 3 & -5 & 8 \\ k & 2 & 3 \end{vmatrix} = 26k - 130$	M1 A1
	$= 0$ when $k = 5$ **ft** from a linear eqn.	A1
	e.g. $\begin{matrix} x+7y-6z=-10 \\ 3x-5y+8z=48 \\ 5x+2y+3z=16 \end{matrix} \rightarrow \begin{matrix} -33y+33z=66 \\ -26y+26z=78 \end{matrix} \rightarrow \begin{matrix} -y+z=2 \\ -y+z=3 \end{matrix}$	
	eliminating one variable (twice); correctly Inconsistency noted/explained (allow valid **ft**)	M1 A1 A1 [5]
	ALTERNATIVE	
	Substituting $x = 8 - \lambda$, $y = \lambda$ and $z = 3 + \lambda$ into $kx + 2y + 3z = 16$ $\Rightarrow \lambda(5 - k) + (11k - 22) = 0$ λ terms; other terms $k = 5$ gives non-unique soln. Then $33 = 0$ **ft** and system inconsistent (allow valid **ft**)	M1 A1 A1 A1 A1 B1 [5]
11 (a)	Roots $\{qr, rp, pq\} = \left\{\frac{pqr}{p}, \frac{pqr}{q}, \frac{pqr}{r}\right\}$	M1
	$= \left\{\frac{4}{p}, \frac{4}{q}, \frac{4}{r}\right\}$ from $pqr = 4$, so subst. $y = \frac{4}{x}$	A1
	Setting $x = \frac{4}{y} \Rightarrow \frac{64}{y^3} + 2.\frac{16}{y^2} + 3.\frac{4}{y} - 4 = 0$	M1
	$\Rightarrow 64 + 32y + 12y^2 - 4y^3 = 0$	M1 A1
	$\Rightarrow y^3 - 3y^2 - 8y - 16 = 0$	A1 [6]
	ALTERNATIVE (for last four-mark section)	
	$\sum \alpha' = qr + rp + pq = 3$	B1
	$\sum \alpha'\beta' = pqr(p + q + r) = 4 \times (-2) = -8$	B1
	$\sum \alpha'\beta'\gamma' = (pqr)^2 = 16$	B1
	New eqn. is $x^3 - 3x^2 - 8x - 16 = 0$ (must have "= 0")	B1 [6]
(b) (i)	β and γ are complex conjugates (since the coefficients are real)	B1 [1]
(ii)	$\alpha\beta\gamma = 4 \Rightarrow \beta\gamma = \frac{4}{\alpha}$	
	and $\lvert\beta\rvert = \lvert\gamma\rvert$ since conjugates $\Rightarrow \lvert\beta\gamma\rvert = \lvert\beta\rvert.\lvert\gamma\rvert = \lvert\beta\rvert^2 = \frac{4}{\alpha}$	M1

	and $\lvert \beta \rvert = \dfrac{2}{\sqrt{\alpha}}$ (since $\alpha > 0$ given) **Answer Given**	A1 [2]
(iii)	p(2.695) = –0.003… < 0 and p(2.705) = 0.049… > 0 so $2.695 < \alpha < 2.705$ (by the "*Change-of-Sign Rule*") $\Rightarrow \alpha = 2.70$ to 3s.f. (Allow numerical methods, e.g. *Newton-Raphson* – to find $\alpha = 2.70$ to 3s.f.)	B1
	$\alpha + \beta + \gamma = 4 \Rightarrow \beta + \gamma = 4 - \alpha$ Then $2.695 < \alpha < 2.705 \Rightarrow 1.295 < 4 - \alpha < 1.305$	M1
	Now, if $\beta = u + \mathrm{i}v$, then $\gamma = u - \mathrm{i}v$ and $\beta + \gamma = 2u$, i.e. 2 Re(β) Thus $1.295 < 2\,\mathrm{Re}(\beta) < 1.305 \Rightarrow 0.6475 < \mathrm{Re}(\beta) < 0.6525$	M1
	and Re(β) = 0.65 to 2 s.f. (A0 for failure to justify 2s.f. accuracy properly)	A1 [4]

12	**(i)**	**(a)** $I_1 = \int_0^2 x\sqrt{1+2x^2}\,\mathrm{d}x = \left[\frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}}\right]_0^2 = \frac{13}{3}$	M1 A1 A1 [3]
		(b) $I_n = \int_0^2 x^{n-1}.\,x\sqrt{1+2x^2}\,\mathrm{d}x$ Correct splitting and use of parts	M1
		$= \left[x^{n-1}.\,\frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}}\right]_0^2 - \int_0^2 (n-1)x^{n-2}.\,\frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}}\,\mathrm{d}x$	A1 A1
		$= 2^{n-1}.\,\frac{27}{6} - 0 \quad - \frac{1}{6}(n-1)\int_0^2 x^{n-2}.\,\left(1+2x^2\right)\sqrt{1+2x^2}\,\mathrm{d}x$	M1
		$= 2^{n-1}.\,\frac{27}{6} - \frac{1}{6}(n-1)\{2I_n + I_{n-2}\}$	A1
		$\Rightarrow 6\,I_n = 27\times 2^{n-1} - 2(n-1)I_n - (n-1)I_{n-2}$	
		$\Rightarrow (2n+4)\,I_n = 27\times 2^{n-1} - (n-1)I_{n-2}$ **Answer Given**	A1 [6]
		(c) $I_0 = \int_0^2 \sqrt{1+2x^2}\,\mathrm{d}x$	
		Let $x\sqrt{2} = \sinh\theta$ $\quad (\sqrt{2}\,\mathrm{d}x = \cosh\theta\,\mathrm{d}\theta,\ \sqrt{1+2x^2} = \cosh\theta)$	M1
		full substitution $I_0 = \frac{1}{\sqrt{2}}\int \cosh^2\theta\,\mathrm{d}\theta$ (Ignore limits for now)	M1 A1
		trig. identity $= \frac{1}{2\sqrt{2}}\int\left(1+\cosh 2\theta\right)\mathrm{d}\theta$	M1
		$= \frac{1}{2\sqrt{2}}\left[\theta + \frac{1}{2}\sinh 2\theta\right]$ **ft** ∫n. of $a + b\cosh 2\theta$ only	A1
		$(0, 2) \rightarrow (0, \sinh^{-1} 2\sqrt{2})$ Limits properly dealt with	M1
		$= \frac{1}{2\sqrt{2}}\left[\sinh^{-1} 2\sqrt{2} + 6\sqrt{2}\right]$	
		$\sinh^{-1} 2\sqrt{2} = \ln\left(3+2\sqrt{2}\right)$ to at least here $= \ln\left(1+\sqrt{2}\right)^2 = 2\ln\left(1+\sqrt{2}\right)$	M1
		$I_0 = 3 + \frac{1}{\sqrt{2}}\ln\left(1+\sqrt{2}\right)$ legitimately **Answer Given**	A1 [8]

	ALTERNATIVE (partial)	
	Let $x\sqrt{2} = \tan\theta \ \left(\sqrt{2}\mathrm{d}x = \sec^2\theta\,\mathrm{d}\theta\right)$	M1
	full substitution $I_0 = \frac{1}{\sqrt{2}}\int \sec^3\theta\,\mathrm{d}\theta$ (Ignore limits for now)	M1 A1
	use of ∫n. by parts on $\int \sec\theta\sec^2\theta\,\mathrm{d}\theta$ and use of $\tan^2\theta = \sec^2\theta - 1$	M1
	$2\sqrt{2}I_0 = \sec\theta\tan\theta + \ln\lvert\sec\theta + \tan\theta\rvert$	A1
	Further progress (limits, etc.) essentially impossible	
	ALTERNATIVE	
	$I_0 = \int_0^2 \sqrt{1+2x^2} \times 1\,\mathrm{d}x = x\sqrt{1+2x^2} - \int \tfrac{1}{2}\left(1+2x^2\right)^{-\frac{1}{2}} . 4x . x\,\mathrm{d}x$ ∫n. by parts	M1 A1
	$= x\sqrt{1+2x^2} - \int \frac{\left(2x^2+1\right)-1}{\sqrt{1+2x^2}}\mathrm{d}x$	M1
	$= x\sqrt{1+2x^2} - I_0 + \int \frac{1}{\sqrt{1+2x^2}}\mathrm{d}x$	M1
	$\Rightarrow 2I_0 = x\sqrt{1+2x^2} + \int \frac{1}{\sqrt{1+2x^2}}\mathrm{d}x$	
	$\Rightarrow I_0 = \frac{1}{2}\left(x\sqrt{1+2x^2} + \frac{1}{\sqrt{2}}\sinh^{-1}\left(x\sqrt{2}\right)\right)$ (from MF20)	M1 A1
	Use of limits (0, 2) $\Rightarrow I_0 = 3 + \frac{1}{2\sqrt{2}}\sinh^{-1}(2\sqrt{2})$	
	$\sinh^{-1} 2\sqrt{2} = \ln\left(3+2\sqrt{2}\right) = \ln\left(1+\sqrt{2}\right)^2 = 2\ln\left(1+\sqrt{2}\right)$	M1
	$I_0 = 3 + \frac{1}{\sqrt{2}}\ln\left(1+\sqrt{2}\right)$ legitimately **Answer Given**	A1 [8]
(ii)	$y = \frac{1}{\sqrt{2}}x^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = x\sqrt{2}$ and $S = 2\pi\int_0^2 \frac{x^2}{\sqrt{2}}.\sqrt{1+2x^2}\,\mathrm{d}x$	M1
	$= \pi\sqrt{2}\,(I_2)$	A1
	use of R.F. for $n = 2$: $8I_2 = 54 - I_0$	M1
	use of *their* **(i)** result $= 51 - \frac{1}{\sqrt{2}}\ln\left(1+\sqrt{2}\right)$	M1
	$\Rightarrow S = \frac{\pi\sqrt{2}}{8}\left(51 - \frac{1}{\sqrt{2}}\ln\left(1+\sqrt{2}\right)\right)$ or $\frac{\pi}{8}\left(51\sqrt{2} - \ln\left(1+\sqrt{2}\right)\right)$	A1 [5]

13 (i)	 $\tan A = \frac{a}{1}$, $\tan B = \frac{1}{a}$ $\Rightarrow \tan^{-1} a + \tan^{-1}\frac{1}{a} = A + B = \frac{\pi}{2}$	B1 [1]
(ii)	$\tan\left(\tan^{-1} a \pm \tan^{-1} b\right) = \frac{\tan\left(\tan^{-1} a\right) \pm \tan\left(\tan^{-1} b\right)}{1 \mp \tan\left(\tan^{-1} a\right)\tan\left(\tan^{-1} b\right)} = \frac{a \pm b}{1 \mp ab}$	M1 A1 [2]
(iii)	$\tan^{-1}\left(\frac{1}{n-1}\right) - \tan^{-1}\left(\frac{1}{n+1}\right) = \tan^{-1}\left(\frac{2}{n^2}\right)$ noted at any stage	M1 A1
	$\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right) = \tan^{-1}(2) + \tan^{-1}\left(\frac{1}{2}\right) + \sum_{n=3}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right)$ Splitting off 1st 2 terms	
	$= \frac{\pi}{2} + \sum_{n=3}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right)$ using (i)'s result	B1
	use of difference method (finite or infinite series)	M1
	$\Rightarrow \sum_{n=1}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right) = \frac{\pi}{2} +$ $\left(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{6} + \tan^{-1}\frac{1}{7} \ldots\right)$ $-\left(\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{6} + \tan^{-1}\frac{1}{7} + \ldots\right)$	
	Cancelling of terms made clear	M1
	$= \frac{\pi}{2} + \left(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}\right)$	A1
	and $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}\right) = \tan^{-1} 1 = \frac{\pi}{4}$ using (ii)'s result	B1
	leading to … **Answer Given**	[7]

	ALTERNATIVE	
	$\tan^{-1}\left(\frac{1}{n-1}\right) - \tan^{-1}\left(\frac{1}{n+1}\right) = \tan^{-1}\left(\frac{2}{n^2}\right)$ noted at any stage	M1 A1
	Use of difference method (finite or infinite series)	M1
	$\sum_{n=1}^{N} \tan^{-1}\left(\frac{2}{n^2}\right) = \left(\tan^{-1}\frac{1}{0} + \tan^{-1}1 + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} + \ldots\right)$ $-\left(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} + \ldots\right)$	A1
	Cancelling of remaining terms made clear	M1
	$\tan^{-1}\infty + \tan^{-1}1 = \frac{\pi}{2} + \frac{\pi}{4}$	A1
	$= \frac{3\pi}{4}$ **Given Answer**	A1
		[7]