Cambridge International Examinations
Cambridge Pre-U Certificate

FURTHER MATHEMATICS (PRINCIPAL) **9795/02**

Paper 2 Further Applications of Mathematics **May/June 2016**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

The syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document consists of **5** printed pages and **3** blank pages.

JC16 06_9795_02/3R

Section A: Probability (60 marks)

1 An investigation was carried out of the lengths of commuters' journeys. For a random sample of 500 commuters, the mean journey time was 75 minutes, and the standard deviation was 40 minutes.

(i) Calculate a 95% confidence interval for the mean journey time. [4]

(ii) Explain whether you need to assume that journey times are normally distributed. [1]

2 The mass in grams of a pre-cut piece of Brie cheese is a random variable with the distribution $\mathrm{N}(150, 1200)$. Brie costs 80p per 100 g.

(i) Find the probability that a randomly chosen piece of Brie costs more than £1.40. [4]

The mass in grams of a pre-cut piece of Stilton cheese is an independent random variable with the distribution $\mathrm{N}(180, 1500)$.

(ii) Find the probability that the total mass of four randomly chosen pieces of Brie is less than the total mass of three randomly chosen pieces of Stilton. [4]

3 **(i)** Show that the probability generating function of a random variable with the distribution $\mathrm{B}(n, p)$ is $(1 - p + pt)^n$. [3]

(ii) R and S are independent random variables with the distributions $\mathrm{B}\left(8, \frac{1}{4}\right)$ and $\mathrm{B}\left(8, \frac{3}{4}\right)$ respectively. Show that the probability generating function of $R + S$ can be expressed as

$$\left(\tfrac{3}{16} + \tfrac{1}{16}t(10 + 3t)\right)^8$$

and use this result to find $\mathrm{P}(R + S = 1)$. [5]

4 In a Football League match, the number of goals scored by the home team can be modelled by the distribution $\mathrm{Po}(2.4)$. The number of goals scored by the away team can be modelled by the distribution $\mathrm{Po}(1.8)$.

(i) State a necessary assumption for the total number of goals scored in one match to be modelled by the distribution $\mathrm{Po}(4.2)$. [1]

(ii) Assume now that this assumption holds.

(a) Write down an expression for the probability that the total number of goals scored in n randomly chosen games is less than 4. [3]

(b) Find the probability that the result of a randomly chosen game is either 0–0 or 1–1. [3]

5 The random variable R has the distribution $B(n, p)$.

(i) State two conditions that n and p must satisfy if the distribution of R can be well approximated by a normal distribution. [2]

Assume now that these conditions hold. Using the normal approximation, it is given that $P(R < 25) = 0.8282$ and $P(R \geqslant 28) = 0.0393$, correct to 4 decimal places.

(ii) Find the mean and standard deviation of the approximating normal distribution. [5]

(iii) Hence find the value of p and the value of n. [3]

6 A continuous random variable X has probability density function

$$f(x) = \begin{cases} 4xe^{-2x} & x \geqslant 0, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that the moment generating function $M_X(t)$ of X is $\dfrac{4}{(2-t)^2}$. You may assume that $xe^{-kx} \to 0$ as $x \to +\infty$. [5]

(ii) What condition on t is needed in finding $M_X(t)$? [1]

(iii) Y is the sum of three independent observations of X. Find the moment generating function of Y, and use your answer to find $\text{Var}(Y)$. [6]

7 A continuous random variable X has probability density function

$$f(x) = \begin{cases} \dfrac{3x^2}{k^3} & 0 \leqslant x \leqslant k, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a parameter.

(i) Find $E(X)$. Hence show that $\frac{4}{3}X$ is an unbiased estimator of k. [3]

Three independent observations of X are denoted by X_1, X_2 and X_3, and the largest value of X_1, X_2 and X_3 is denoted by M.

(ii) Write down an expression for $P(M \leqslant x)$, and hence show that the probability density function of M is

$$f_M(x) = \begin{cases} \dfrac{9x^8}{k^9} & 0 \leqslant x \leqslant k, \\ 0 & \text{otherwise.} \end{cases}$$

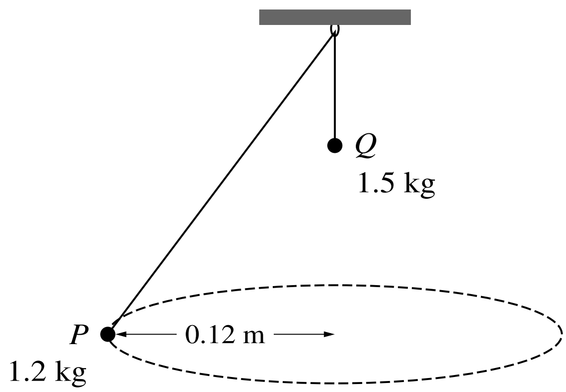
[4]

(iii) Find $E(M)$ and use your answer to construct an unbiased estimator of k based on M. [3]

Section B: Mechanics (60 marks)

8 A rough plane is inclined at 20° to the horizontal. A particle of mass 0.4 kg is projected down the plane, along a line of greatest slope, at $0.5\,\mathrm{m\,s^{-1}}$. After it has travelled 3 m down the plane its speed is $2.5\,\mathrm{m\,s^{-1}}$. By considering the change in energy, find the magnitude of the frictional force, assumed constant. [5]

9



Particles P and Q, of masses 1.2 kg and 1.5 kg respectively, are attached to the ends of a light inextensible string. The string passes through a small smooth ring which is attached to the ceiling but which is free to rotate. P rotates at $\omega\,\mathrm{rad\,s^{-1}}$ in a horizontal circle of radius 0.12 m, and Q hangs vertically in equilibrium (see diagram). Determine

(i) the vertical distance below the ring at which P rotates, [4]

(ii) the value of ω. [4]

10 A uniform ladder AB of length 5 m and mass 8 kg is placed at an angle θ to the horizontal, with A on rough horizontal ground and B against a smooth vertical wall. The coefficient of friction between the ladder and the ground is 0.4.

(i) By taking moments, find the smallest value of θ for which the ladder is in equilibrium. [5]

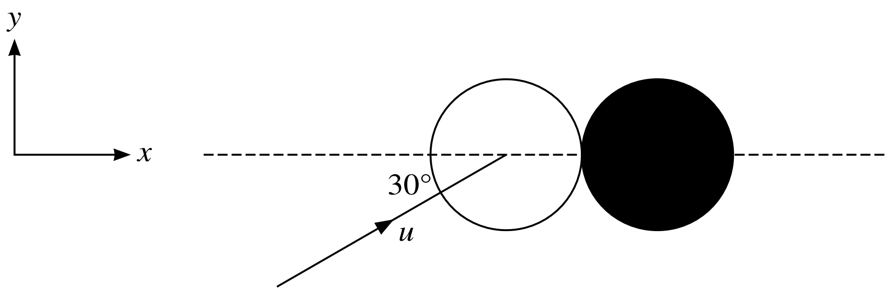
(ii) A man of mass 75 kg stands on the ladder when $\theta = 60°$. Find the greatest distance from A that he can stand without the ladder slipping. [3]

11 A car of mass 800 kg has a constant power output of 32 kW while travelling on a horizontal road. At time t s the car's speed is $v\,\mathrm{m\,s^{-1}}$ and the resistive force has magnitude $20v$ N.

(i) Show that v satisfies the differential equation $\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{1600 - v^2}{40v}$. [2]

(ii) Given that $v = 0$ when $t = 0$, solve this differential equation to find v in terms of t. State what the solution predicts as t becomes large. [6]

12



A white snooker ball of mass m moves with speed u towards a stationary black snooker ball of the same mass and radius. Taking the x-axis to be the line of centres of the two balls at the moment of collision, the direction of motion of the white ball before the collision makes an angle of 30° with the positive x-axis (see diagram).

(i) Given that the coefficient of restitution is 0.9, find the angle made with the x-axis by the velocity of the white ball after the collision. [5]

(ii) Show that after the collision the white ball cannot have a negative x-component of velocity whatever the value of the coefficient of restitution. [3]

13 A cricket ball is hit from a point P on a sloping field. The initial velocity of the ball is $30\,\text{m s}^{-1}$ at 40° above the field, which under the path of the ball slopes upwards at 10° to the horizontal. Air resistance is to be ignored.

(i) Find the vertical height of the ball above the field after 2.5 seconds. [5]

(ii) The ball lands on the field at the point X. Find the distance PX. [4]

14 One end of a light elastic string of natural length 0.5 m and modulus of elasticity 3 N is attached to a ceiling at a point P. A particle of mass 0.3 kg is attached to the other end of the string.

(i) Find the extension of the string when the particle hangs vertically in equilibrium. [2]

The particle is released from rest at P so that it falls vertically. Find

(ii) the maximum extension of the string, [4]

(iii) the equation of motion for the particle when the string is stretched, in terms of the displacement x m below the equilibrium position, [3]

(iv) the time between the string first becoming stretched and next becoming unstretched again. [5]

BLANK PAGE

BLANK PAGE

BLANK PAGE