Cambridge Pre-U

Cambridge Assessment International Education
Cambridge Pre-U Certificate

FURTHER MATHEMATICS (PRINCIPAL) **9795/01**

Paper 1 Further Pure Mathematics **May/June 2019**

3 hours

* 8 9 2 4 3 6 5 2 3 2 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

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1 Prove by induction that $\sum_{r=1}^{n} r(3r-1) = n^2(n+1)$ for all positive integers n. [6]

2 The points $P(a, b, 0)$, $Q(c, d, 0)$ and $R(e, f, 0)$ all lie in the x-y plane.

(a) Using the vector product, or otherwise, find the area of triangle PQR. [4]

(b) Show that this area is also given by $\frac{1}{2}|\det(\mathbf{M})|$, where $\mathbf{M}$ is the matrix $\begin{pmatrix} 1 & a & b \\ 1 & c & d \\ 1 & e & f \end{pmatrix}$. [2]

3 **(a)** Find a vector equation for the line of intersection of the planes with equations

$$\mathbf{r}.\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = 11 \quad \text{and} \quad \mathbf{r}.\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = 16.$$ [5]

(b) Given that the system of equations

$$\begin{aligned} 2x + 3y + z &= 11 \\ x - y + 2z &= 16 \\ 4x + 11y - z &= k \end{aligned}$$

does **not** have a unique solution, determine the value of the constant k for which the system is consistent. [2]

4 Consider the set $S = \{3, 6, 9, 12\}$ together with $\times_{15}$, the operation of multiplication modulo 15.

(a) Construct the multiplication table for $(S, \times_{15})$ and show that it is a group, G.

[You may assume that $\times_{15}$ is an associative operation.] [5]

The group H consists of the set $T = \{1, 7, 9, 15\}$ together with $\times_{16}$, the operation of multiplication modulo 16. The multiplication table for H is shown below.

	1	7	9	15
1	1	7	9	15
7	7	1	15	9
9	9	15	1	7
15	15	9	7	1

(b) State, giving a reason, whether H is

(i) abelian, [1]

(ii) cyclic. [1]

(c) State also, with justification, whether G and H are isomorphic. [1]

5 Solve the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 4y = 1 - 8x^2$, given that $y = 8$ and $\frac{dy}{dx} = 3$ when $x = 0$. [10]

6 The point P in the complex plane represents the complex number $z = x + iy$.

(a) On a single Argand diagram, sketch the locus of P in each of the following cases:

(i) $|z - 2i| = 3$; [2]

(ii) $|z - i| = |z - 2 + i|$. [2]

(b) Write down the cartesian equation of the locus of part **(a)(i)** and determine the cartesian equation of the locus of part **(a)(ii)**. [3]

(c) The region of the complex plane for which

$$|z - 2i| \leqslant 3 \quad \text{and} \quad |z - i| \geqslant |z - 2 + i|$$

is denoted by R. Calculate, in an exact form, the area of R. [3]

7 A curve has equation $y = \frac{x^2 - x + 1}{x^2 + x + 1}$.

(a) Show that the x-coordinates of any points of intersection of the curve with the line $y = k$ are given by

$$(k - 1)x^2 + (k + 1)x + (k - 1) = 0,$$

and deduce the coordinates of the turning points of the curve. [6]

(b) Sketch the curve. [4]

8 The curve C has polar equation $r = 5 - 4\cos\theta$ for $0 \leqslant \theta < 2\pi$.

(a) Sketch C. [3]

(b) The points P, Q, R and S lie on C such that PR and QS are straight lines through the pole, O, and PR is perpendicular to QS.

(i) Given that P is the point with polar coordinates $(5 - 4\cos\theta, \theta)$, where $0 < \theta < \frac{1}{2}\pi$, write down the polar coordinates of Q, R and S, in terms of θ, $\cos\theta$ and $\sin\theta$. [4]

(ii) Show that $(OP)(OR) + (OQ)(OS)$ is constant for all values of θ, and determine its value. [2]

9 The equation $x^3 + px^2 + qx + r = 0$, where $r \neq 0$, has roots α, β and γ.

(a) By considering the expression $(\alpha\beta - 1)(\beta\gamma - 1)(\gamma\alpha - 1)$, show that one root of this equation is the reciprocal of another root if and only if $r(r - p) = 1 - q$. [4]

(b) Hence or otherwise show that, in this case, $-r$ is a root of the equation. [2]

(c) Solve the equation $12x^3 + 11x^2 - 63x + 36 = 0$. [4]

10 **(a)** Use de Moivre's theorem to prove that $\sin 3x \equiv 3\sin x - 4\sin^3 x$ and deduce the identity

$$\frac{1}{2}\left(\frac{1}{\sin x} - \frac{1}{\sin 3x}\right) \equiv \frac{\cos 2x}{\sin 3x}.$$ [6]

(b) Use the method of differences to find $\displaystyle\sum_{r=0}^{n} \frac{\cos(2 \times 3^r x)}{\sin(3^{r+1}x)}$. [4]

11 **(a)** Let $I_n = \displaystyle\int \cos^n \theta \, d\theta$, where n is a positive integer.

Show that $nI_n = \sin\theta\cos^{n-1}\theta + (n-1)I_{n-2}$ for $n \geqslant 3$. [5]

(b) A curve is defined parametrically for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$ by

$$x = 2\theta + \sin 2\theta, \quad y = 1 + \cos 2\theta.$$

(i) Show that the length of the curve is 4. [6]

(ii) When the curve is rotated through 2π radians about the x-axis, a surface of revolution is formed having area S. Determine the exact value of S. [7]

12 **(a)** Without using a calculator, show that if $\sinh x = 1$ then $\tanh \frac{1}{2}x = \sqrt{2} - 1$. [5]

(b) Show that $\dfrac{d}{dx}\left(2\tan^{-1}\left(\tanh \frac{1}{2}x\right)\right) = \dfrac{1}{\cosh x}$. [4]

(c) Use the substitution $\tan\theta = \sinh^2 x$ to find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{\sqrt{\tan\theta}\sec^2\theta}{1 + \tan\theta} \, d\theta.$$

[You may use, without proof, the result that $\tan\frac{1}{8}\pi = \sqrt{2} - 1$.] [7]