Cambridge Assessment International Education

Cambridge Pre-U

FURTHER MATHEMATICS **9795/01**

Paper 1 Further Pure Mathematics **October/November 2020**

MARK SCHEME

Maximum Mark: 120

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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This document consists of **19** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Question	Answer	Marks	Guidance
1	Use of $\Sigma r^3 = \frac{1}{4}n^2(n+1)^2$, $\Sigma r^2 = \frac{1}{6}n(n+1)(2n+1)$ and $\Sigma r = \frac{1}{2}n(n+1)$	**M1**	
	$\Rightarrow \sum_{r=1}^{n}\left(4r^3 - 6r^2 + 4r - 1\right)$ $= n^2(n+1)^2 - n(n+1)(2n+1) + 2n(n+1) - n$	**A1**	
	$= n(n^3 + 2n^2 + n - 2n^2 - 3n - 1 + 2n + 2 - 1)$ or $n^4 + 2n^3 + n^2 - 2n^3 - 3n^2 - n + 2n^2 + 2n - n$	**M1**	Multg. out and collecting terms
	$= n^4$ **AG** legitimately obtained	**A1**	

Question	Answer	Marks	Guidance
2(a)(i)	$-1 = p - q + r$ ❶ $53 = 81p + 9q + r$ ❷ $45 = 121p - 11q + r$ ❸	**M1**	3 substns. of x, y values
		A1	$\geqslant 2$ correct
2(a)(ii)	$\begin{pmatrix} 1 & -1 & 1 \\ 81 & 9 & 1 \\ 121 & -11 & 1 \end{pmatrix}\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} -1 \\ 53 \\ 45 \end{pmatrix}$	**B1ft**	

Question	Answer	Marks	Guidance
2(b)	**METHOD I** Eliminate r from (e.g.) ❶, ❷ $54 = 80p + 10q$ and ❷, ❸ $46 = 120p - 10q$	M1A1	
	Solving for one variable (e.g.) $100 = 200p$	M1	
	$p = \frac{1}{2},\ q = \frac{7}{5},\ r = -\frac{1}{10}$	A1	
	METHOD II Row ops. on the augmented matrix:	M1	
	$\left.\begin{array}{ccc\|c} 1 & -1 & 1 & -1 \\ 81 & 9 & 1 & 53 \\ 121 & -11 & 1 & 45 \end{array}\right. \to \begin{array}{ccc\|c} 1 & -1 & 1 & -1 \\ 80 & 10 & 0 & 54 \\ 120 & -10 & 0 & 46 \end{array} \begin{array}{l} \\ R_2' = R_2 - R_1 \\ R_3' = R_3 - R_1 \end{array}$	A1	
	$\to \begin{array}{ccc\|c} 1 & -1 & 1 & -1 \\ 80 & 10 & 0 & 54 \\ 200 & 0 & 0 & 100 \end{array} \begin{array}{l} \\ \\ R_3' = R_3 + R_2 \end{array}$	M1	to some upper/lower echelon form
	$p = \frac{1}{2},\ q = \frac{7}{5},\ r = -\frac{1}{10}$	A1	
	METHOD III $\underline{\mathbf{x}} = \mathbf{C}^{-1}\,\underline{\mathbf{u}}$ attempted	M1	
	$\mathbf{C}^{-1} = \begin{pmatrix} -0.01 & 0.005 & 0.005 \\ -0.02 & 0.06 & -0.04 \\ 0.99 & 0.055 & -0.045 \end{pmatrix}$ or $\frac{1}{200}\begin{pmatrix} -2 & 1 & 1 \\ -4 & 12 & -8 \\ 198 & 11 & -9 \end{pmatrix}$	M1A1	Good attempt at inverse; correct
	$\underline{\mathbf{x}} = \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 0.5 \\ 1.4 \\ -0.1 \end{pmatrix}$	A1	

Question	Answer	Marks	Guidance
3(a)(i)	VA $x = 4$ HA $y = 1$	B1B1	
3(a)(ii)	Standard (positive) reciprocal curve	B1	
	Asymptotes in (**ft**) approx. correct positions	B1	withhold if curve appears to cross either
	$(0, \frac{1}{4})$ and $(1, 0)$ seen or noted on diagram	B1B1	
3(b)	$y = \dfrac{x^2 - 2x + 1}{x - 4} = \dfrac{x(x-4) + 2(x-4) + 9}{x - 4} = x + 2 + \dfrac{k}{x - 4}$	M1	or equivalent method
	$y = x + 2$	A1	condone incorrect 'k'

Question	Answer	Marks	Guidance
4	Area $= \frac{1}{2}\int\left(3 + \sqrt{2}\sin\theta\right)^2 \mathrm{d}\theta$	M1	Attempt to integrate kr^2 (r a fn. of θ)
	$= \frac{1}{2}\int\left(9 + 6\sqrt{2}\sin\theta + 2\sin^2\theta\right)\mathrm{d}\theta$	B1	r^2 correct (ignore limits until the end)
	$= \int\left(5 + 3\sqrt{2}\sin\theta - \frac{1}{2}\cos 2\theta\right)\mathrm{d}\theta$	M1	double-angle formula used
	$= \left[5\theta - 3\sqrt{2}\cos\theta - \frac{1}{4}\sin 2\theta\right]$	A1	correct integration
	$= \left(\frac{15}{4}\pi + 3 + \frac{1}{4}\right) - \left(\frac{5}{4}\pi - 3 - \frac{1}{4}\right)$	M1	clear evidence of use of correct limits
	$= \dfrac{5\pi + 13}{2}$	A1	any correct, exact form

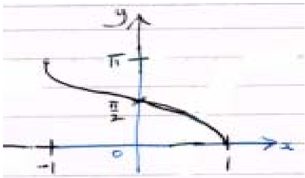
Question	Answer	Marks	Guidance
5(a)	$\alpha\beta\gamma = 6$	**B1**	
5(b)(i)	New roots are $\alpha + \dfrac{12\alpha}{\alpha\beta\gamma}$	**M1**	Attempt to use **(a)**'s result to simplify
	$= \alpha + \dfrac{12\alpha}{6} = 3\alpha$	**A1**	**AG** (including the other 2 roots, possibly 'similarly')
	Alt. $\dfrac{\alpha\beta\gamma + 12}{\beta\gamma} = \dfrac{18}{\beta\gamma} = \dfrac{18\alpha}{\alpha\beta\gamma} = \dfrac{18\alpha}{6} = 3\alpha$	**(M1A1)**	**M1** use of common denomr. and **(a)**'s $\alpha\beta\gamma$ value **A1** correct
5(b)(ii)	$x = \dfrac{y}{3}$ substituted into $2x^3 + 3x^2 - 5x - 12 = 0$	**M1**	
	$\Rightarrow \frac{2}{27}y^3 + \frac{3}{9}y^2 - \frac{5}{3}y - 12 \,(= 0)$	**A1**	unsimplified
	$\Rightarrow 2y^3 + 9y^2 - 45y - 324 = 0$	**A1**	or any (non-zero) integer multiple
	Alt. $\Sigma\alpha' = 3\Sigma\alpha = -\frac{9}{2}$, $\Sigma\alpha'\beta' = 9\Sigma\alpha\beta = -\frac{45}{2}$ and $\alpha'\beta'\gamma' = 27\alpha\beta\gamma = 162$	**(M1)**	Attempt to connect new roots to old
		(A1)	All three correct
	$\Rightarrow 2y^3 + 9y^2 - 45y - 324 = 0$	**(A1)**	or any (non-zero) integer multiple

Question	Answer	Marks	Guidance
6(a)	$\mathbf{X}^2=\begin{pmatrix}4&0\\3&1\end{pmatrix}$, $\mathbf{X}^3=\begin{pmatrix}8&0\\7&1\end{pmatrix}$, $\mathbf{X}^4=\begin{pmatrix}16&0\\15&1\end{pmatrix}$	**B1B1B1**	
6(b)	$\mathbf{X}^n=\begin{pmatrix}2^n&0\\2^n-1&1\end{pmatrix}$	**B1**	(i.e. assuming this is true for some n)
	Then $\mathbf{X}^{n+1}=\begin{pmatrix}2^n&0\\2^n-1&1\end{pmatrix}\begin{pmatrix}2&0\\1&1\end{pmatrix}=\begin{pmatrix}2^{n+1}&0\\2^{n+1}-1&1\end{pmatrix}$	**M1A1**	(shown, with sufficient justification)
	True for $n = 1$ (2, 3 & 4) and true $n \Rightarrow$ true $n + 1$ …	**E1**	for induction "round up"
6(c)	$\mathbf{X}^{-1}=\dfrac{1}{2-0}\begin{pmatrix}1&0\\-1&2\end{pmatrix}=\begin{pmatrix}\frac{1}{2}&0\\-\frac{1}{2}&1\end{pmatrix}$	**M1**	$\mathbf{X}^{-1}$ form from either formula or for real
		A1	'Real' inverse correctly found
	$=\begin{pmatrix}2^{-1}&0\\2^{-1}-1&1\end{pmatrix}$ … so *Yes*	**A1**	'Formula' inverse correctly found

Question	Answer	Marks	Guidance
7(a)(i)	$\omega=1+\mathrm{i}\sqrt{3}=r\,\mathrm{e}^{\mathrm{i}\theta}$ with $r=2$ and $\theta=\frac{\pi}{3}$	**B1B1**	
7(a)(ii)	Then $\omega^7=r^7\,\mathrm{e}^{\mathrm{i}(7\theta)}=128\,\mathrm{e}^{\mathrm{i}\pi/3}=64\omega$	**M1M1A1**	**M1** (mod) **M1** (arg) **A1**

Question	Answer	Marks	Guidance
7(b)	$z^7 = (128, -\frac{\pi}{3})$	B1	
	$= (128, 2n\pi - \frac{\pi}{3})$ for $n = 1, 2, \ldots, 7$	M1	seven candidate args attempted at 2π intervals
	$\Rightarrow z = (2, 2n\frac{\pi}{7} - \frac{\pi}{21})$ for $n = 1, 2, \ldots, 7$	M1M1	**M1** (mods) **M1** (args)
	$= 2\text{cis}\theta$ for $\theta = (6n-1)\frac{\pi}{21}$; $n = 1, 2, \ldots, 7$	A1	allow $n = 0, 1, \ldots, 6$ or $n = 0, \pm 1, \pm 2, \pm 3$ etc

Question	Answer	Marks	Guidance
8(a)	NO, since non-abelian $\Rightarrow$ non-cyclic	B1	with justification
8(b)	$a^3b = ba \Rightarrow a^4b = aba$	M1	Method using valid results
	$\Rightarrow eb = aba \Rightarrow b = aba$	A1	since $a^4 = e$ and $eb = b$ **AG**
	$b = aba \Rightarrow ab = a^2ba$	M1	Method using valid results; one main step
	$\Rightarrow aba = a^2ba^2$	M1	Method using valid results; concluding step(s)
	$\Rightarrow b = a^2ba^2$	A1	**AG**
	$b = aba \Rightarrow ba^3 = aba^4$	M1	Method using valid results
	$\Rightarrow ba^3 = abe = ab$	A1	since $a^4 = e$ and $xe = e$ **AG**
	Alt. $a^3b = ba \Rightarrow a^3b\,a^3 = ba^4 \Rightarrow a^3b\,a^3 = b \Rightarrow a^4b\,a^3 = ab \Rightarrow ba^3 = ab$		

Question	Answer	Marks	Guidance
9(a)(i)		**B1**	Correct shape with sufficient detail Must be sinusoidal Domain $-1 \leqslant x \leqslant 1$ may be taken as given Range $[0, \pi]$ requires some indication with y-intercept approx. halfway up
9(a)(ii)	$y = \cos^{-1}x \rightarrow \cos y = x$ differentiated	**M1**	
	$\Rightarrow -\sin y \dfrac{\mathrm{d}y}{\mathrm{d}x} = 1$	**A1**	
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-1}{\sin y} = \dfrac{-1}{\sqrt{1-x^2}}$ using $s^2 = 1 - c^2$	**A1**	**AG** legitimately obtained
	Justification of +ve $\sqrt{\ }$ (i.e. –ve answer)	**E1**	e.g. from negative gradient of curve
9(b)	$\int \left(\cos^{-1} x \times 1\right) \mathrm{d}x$ and attempt at integration by parts	**M1**	
	$= x\cos^{-1}x + \int \dfrac{x}{\sqrt{1-x^2}}\,\mathrm{d}x$	**A1A1**	
	2nd term $\int$d. using 'recognition' (*reverse Chain Rule*) or substitution: $u = (1 - x^2)^{1/2} \Rightarrow \mathrm{d}u = \dfrac{-x}{\sqrt{1-x^2}}\,\mathrm{d}x$	**M1**	
	giving $\int \cos^{-1} x \,\mathrm{d}x = x\cos^{-1}x - \sqrt{1-x^2}$ $(+ C)$	**A1**	

Question	Answer	Marks	Guidance
10(a)	$\mathbf{b}-\mathbf{a}=\begin{pmatrix}4\\-1\\-6\end{pmatrix}$, $\mathbf{c}-\mathbf{a}=\begin{pmatrix}1\\1\\-4\end{pmatrix}$, $\mathbf{b}-\mathbf{c}=\begin{pmatrix}3\\-2\\-2\end{pmatrix}$	**B1**	Any two (+/–) of these correct
	Area $\Delta ABC = \frac{1}{2}\lvert(\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{a})\rvert$	**M1**	Use of formula for area
	$= \frac{1}{2}\left\lvert\begin{pmatrix}10\\10\\5\end{pmatrix}\right\rvert = \frac{15}{2}$	**M1**	Good attempt at a relevant vector product
		A1	**CAO**
10(b)(i)	Volume $OABC = \frac{1}{6}\lvert\mathbf{a}\,.\,\mathbf{b}\times\mathbf{c}\rvert$	**M1**	Quoted or used
	$= \frac{1}{6}\begin{vmatrix}1 & 5 & 2\\2 & 1 & 3\\3 & -3 & -1\end{vmatrix}$	**M1**	method for calculating a STP
	$= \frac{1}{6}\times 45 = \frac{15}{2}$	**A1**	
	Alt. Eqn. plane is $\mathbf{r}\,.\begin{pmatrix}2\\2\\1\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}.\begin{pmatrix}2\\2\\1\end{pmatrix}=9$ using **(a)**'s VP and A's **p**.**v**.	**(M1)**	
	SD O to plane is $\dfrac{9}{\sqrt{2^2+2^2+1^2}}=3$.	**(M1)**	
	Then $V=\frac{1}{3}\times(\text{area }\Delta ABC)\times\text{SD}=\frac{1}{3}\times\frac{15}{2}\times 3=\frac{15}{2}$	**(A1)**	

Question	Answer	Marks	Guidance
10(b)(ii)	Vol. tetrahedron = $\frac{1}{3} \times$ (area ΔABC) $\times$ SD	M1	attempt to use
	$\Rightarrow \frac{15}{2} = \frac{1}{3} \times \frac{15}{2} \times \text{SD} \Rightarrow \text{SD} = 3$	A1	
	Special Case B1 for correct answer obtained (not "deduced" from scratch)	(B1)	
10(c)	OA is $\mathbf{r} = \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ BC is $\mathbf{r} = \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix}$	B1B1	Dirn. Vectors only needed; vector between the two lines may be introduced later on
	Common perpendicular , $\mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 11 \\ -8 \end{pmatrix}$	M1	attempted
	SD = $\lvert (\mathbf{b} - \mathbf{0}) \cdot \hat{\mathbf{n}} \rvert$	M1	$\lvert \dots \rvert = 45$, as in **(b)(i)** by the STP
	$= \frac{1}{\sqrt{189}} \left\lvert \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 11 \\ -8 \end{pmatrix} \right\rvert$ $= \frac{45}{\sqrt{189}}$ or $\frac{45}{3\sqrt{21}}$ or $\frac{15}{\sqrt{21}}$ or $\frac{5}{7}\sqrt{21}$	A1	any exact surd form

Question	Answer	Marks	Guidance
10(c)	**Alt.** A correct general vector from one line to the other: $\begin{pmatrix} 5+3\mu-\lambda \\ 1-2\mu-2\lambda \\ -3-2\mu-3\lambda \end{pmatrix}$	**(B1)**	
	This equated to multiple of the common normal: $\begin{pmatrix} 5+3\mu-\lambda \\ 1-2\mu-2\lambda \\ -3-2\mu-3\lambda \end{pmatrix} = m\begin{pmatrix} 2 \\ 11 \\ -8 \end{pmatrix}$	**(B1)**	
	Solving 3 equations in 3 unknowns	**(M1)**	
	$\mu=-\frac{4}{3}$, $\lambda=\frac{11}{21}$, $m=-\frac{5}{21}$ (not all three needed)	**(A1)**	
	Correct SD (as above) from magnitude of either side	**(A1)**	
11(a)	$y=\frac{2}{3}x^{\frac{3}{2}} \Rightarrow 1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 1+x$	**B1**	
	$L=\int_0^{15}\sqrt{1+x}\,\mathrm{d}x$	**M1**	correct arc-length formula used
	$=\left[\frac{2}{3}(1+x)^{\frac{3}{2}}\right]$	**A1**	correct integration
	$=\frac{2}{3}(64-1)=42$	**A1**	from correct working

Question	Answer	Marks	Guidance
11(b)(i)	$A = 2\pi \int_0^{15} \frac{2}{3} x^{\frac{3}{2}} \sqrt{1+x}\, dx$	B1	correct statement of SA integral
	$= \frac{4}{3}\pi \int_0^{15} x\sqrt{x^2+x}\, dx$	M1	clear evidence of moving to required form
	$= \frac{4}{3}\pi \int_0^{15} x\sqrt{(x+\frac{1}{2})^2 - \frac{1}{4}}\, dx$	A1	

Question	Answer	Marks	Guidance
11(b)(ii)	Substitution $x + \frac{1}{2} = \frac{1}{2}\cosh\theta$ $dx = \frac{1}{2}\sinh\theta \, d\theta$	M1	Use of a suitable hyp. fn. or trig. fn. substn. This incorporated into the first A mark
	Limits: $x = 0,\ \theta = 0$ and $x = 15,\ \theta = \ln\left(31 + 8\sqrt{15}\right) = \alpha$ and/or $\cosh\theta = 31,\ \sinh\theta = 8\sqrt{15}$	B1	**SOI** at any stage (ignore limits until end)
	$A = \frac{4}{3}\pi\int_0^{\alpha}\frac{1}{2}(\cosh\theta - 1).\frac{1}{2}\sinh\theta.\frac{1}{2}\sinh\theta\,d\theta$	A1	Simplified to at least this state
	$= \frac{1}{6}\pi\int_0^{\alpha}\left(\cosh\theta\sinh^2\theta - \frac{1}{2}[\cosh 2\theta - 1]\right)d\theta$	M1M1	splitting into integrable terms
	$= \frac{1}{6}\pi\left[\frac{1}{3}\sinh^3\theta - \frac{1}{4}\sinh 2\theta + \frac{1}{2}\theta\right]$ $= \frac{1}{6}\pi\left(\frac{1}{3}\ .\ 8^3\ .\ 15\sqrt{15} - \frac{1}{2}.\ 8\sqrt{15}\ .\ 31 + \frac{1}{2}\ln[31 + 8\sqrt{15}]\right)$ $= \frac{1}{6}\pi\left(2560\sqrt{15} - 124\sqrt{15} + \frac{1}{2}\ln[31 + 8\sqrt{15}]\right)$	A1A1	for the first two terms
	$= 406\pi\sqrt{15} + \frac{1}{12}\pi\ln\left(31 + 8\sqrt{15}\right)$	A1	**AG** legitimately obtained
	Alt. Substitution $x = \frac{1}{2}\sec\theta - \frac{1}{2}$ $dx = \frac{1}{2}\sec\theta\tan\theta\ d\theta$	(M1)	Use of a suitable hyp. fn. or trig. fn. substn. This incorporated into the first A mark
	Limits: when $x = 0,\ \theta = 0$ when $x = 15,\ \sec\theta = 31,\ \tan\theta = 8\sqrt{15}$	(B1)	**SOI** at any stage (ignore limits until end)

Question	Answer	Marks	Guidance
11(b)(ii)	$A = \frac{4}{3}\pi\int_0^{\alpha} \frac{1}{2}(\sec\theta - 1).\frac{1}{2}\tan\theta.\frac{1}{2}\sec\theta\tan\theta\,\mathrm{d}\theta$	(A1)	Simplified to at least this state
	$= \frac{1}{6}\pi\int_0^{\alpha}\left(\sec^2\theta\tan^2\theta - \sec\theta\tan^2\theta\right)\mathrm{d}\theta$	(M1)	Splitting into integrable terms
	Now $\int_0^{\alpha}\sec^2\theta\tan^2\theta\,\mathrm{d}\theta = \frac{1}{3}\tan^3\theta$	(A1)	
	and $\int_0^{\alpha}\sec\theta\tan^2\theta\,\mathrm{d}\theta = \int_0^{\alpha}\tan\theta\times\sec\theta\tan\theta\,\mathrm{d}\theta$	(M1)	for use of parts or equivalent
	$= \tan\theta\times\sec\theta - \int_0^{\alpha}\sec^3\theta\,\mathrm{d}\theta$ $= \sec\theta\tan\theta - \int_0^{\alpha}\left(\sec\theta + \sec\theta\tan^2\theta\right)\mathrm{d}\theta$ $\Rightarrow I = \sec\theta\tan\theta - \ln\left(\sec\theta + \tan\theta\right) - I$ giving $I = \frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\ln\left(\sec\theta + \tan\theta\right)$	(A1)	
	Thus $A = \frac{1}{6}\pi\left[\frac{1}{3}\tan^3\theta - \frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\ln(\sec\theta + \tan\theta\right]$ $= \frac{1}{6}\pi\left(\frac{1}{3}\,.\,\left(8\sqrt{15}\right)^3 - \frac{1}{2}.\,31.8\sqrt{15}\;+\frac{1}{2}\ln[31+8\sqrt{15}]\right)$ $= \frac{1}{6}\pi\left(2560\sqrt{15} - 124\sqrt{15} + \frac{1}{2}\ln[31+8\sqrt{15}]\right)$ $= 406\pi\sqrt{15}\; + \;\frac{1}{12}\pi\ln\left(31+8\sqrt{15}\right)$	(A1)	**AG** legitimately obtained

Question	Answer	Marks	Guidance
12	$\frac{d^2y}{dx^2}+\frac{dy}{dx}\sinh x+4y\cosh x=8e^x$ **(*)**		
12(a)(i)	$e^x=2$, $\sinh x=\frac{3}{4}$, $\cosh x=\frac{5}{4}$	**B1**	seen or implied
	$\frac{d^2y}{dx^2}=16-4\times\frac{3}{4}-4\times3\times\frac{5}{4}$	**M1**	substitution of values ($x=\ln 2$, $y=3$, $\frac{dy}{dx}=4$)
	$=-2$	**A1**	
	$y=3+4(x-\ln 2)-\frac{1}{2}\times2\,(x-\ln 2)^2+\ldots$	**M1A1**	use of Taylor series form; correct
12(a)(ii)	When $x=0.75$, $y=3.224$ (to 3 d.p.)	**B1**	exactly this
12(b)(i)	Using $\sinh x=x$, $\cosh x=1+\frac{1}{2}x^2$, $e^x=1+x+\frac{1}{2}x^2$	**M1**	Attempt to use at least 1 truncated series *in situ*
	(*) becomes $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(4+2x^2)y=8+8x+4x^2$	**A1**	
	Using $\sinh x=x$, $\cosh x=1$, $e^x=1+x$ **(*)** becomes $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+4y=8+8x$	**A1**	
	Using $\sinh x=0$, $\cosh x=1$, $e^x=1$ **(*)** becomes $\frac{d^2y}{dx^2}+4y=8$	**A1**	

Question	Answer	Marks	Guidance
12(b)(ii)	$\frac{d^2y}{dx^2}+x\frac{dy}{dx}+4y=8+8x$ For a PI, try $y = a + bx$	M1	
	$\Rightarrow\ 0 + bx + 4a + 4bx = 8 + 8x$	M1	substitution and comparing terms
	$\Rightarrow\ a=2$ and $b=\frac{8}{5}$ i.e. PI is $y=2+\frac{8}{5}x$ **Note:** $y = a + bx + cx^2$ (or higher-order polynomial) works also	A1	
12(b)(iii)	$\frac{d^2y}{dx^2}+4y=8$ has PI $y=2$	B1	
	and CF $y = A\cos 2x + B\sin 2x$	M1A1	
	giving Gen. Soln. $y = A\cos 2x + B\sin 2x + 2$	B1ft	
	$\frac{dy}{dx}=-2A\sin 2x+2B\cos 2x$	B1	
	Exactly: $x = \ln 2, y = 3$: $1 = A\cos(\ln 4) + B\sin(\ln 4)$ $x = \ln 2$, $\frac{dy}{dx}=4$: $2 = -A\sin(\ln 4) + B\cos(\ln 4)$ **Numerically:** $x = \ln 2, y = 3$: $1 = 0.183\,457A + 0.983\,028B$ $x = \ln 2$, $\frac{dy}{dx}=4$: $2 = -0.983\,028A + 0.183\,457B$	M1	use of initial conditions in both cases

Question	Answer	Marks	Guidance
12(b)(iii)	Solving simultaneous eqns. (exactly or numerically) **Exactly:** $\sin(\ln 4) = A\sin(\ln 4)\cos(\ln 4) + B\sin^2(\ln 4)$ $2\cos(\ln 4) = -A\sin(\ln 4)\cos(\ln 4) + B\cos^2(\ln 4)$ **Adding** $\Rightarrow$ $B = 2\cos(\ln 4) + \sin(\ln 4)$ and then $A = \cos(\ln 4) - 2\sin(\ln 4)$ **Numerically:** $A = -1.7826$, $B = 1.3499$ (Note: Working to 4dp gives $A = -1.7826$, $B = 1.3500$ Working to 3dp gives $A = -1.7834$, $B = 1.3493$ Working to 2dp gives $A = -1.7929$, $B = 1.3497$)	**M1A1**	for both A, B
	Then $y(0.75) = 3.220$ or 3.221 (must be one of these)	**A1**	(from all above accuracies)