Cambridge Assessment International Education

Cambridge Pre-U

FURTHER MATHEMATICS **9795/02**

Paper 2 Further Applications of Mathematics **May/June 2022**

MARK SCHEME

Maximum Mark: 120

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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This document consists of **21** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Question	Answer	Marks	Guidance
1(a)	$e^{-\lambda}\dfrac{\lambda^{16}}{16!}=e^{-\lambda}\dfrac{\lambda^{18}}{18!}$	**B1**	Correct formulae
	$\lambda^2 = 17\times18\ (=306)$ or $\dfrac{1}{\lambda^2}=\dfrac{1}{306}$	**M1**	Deal correctly with exponential
		M1	Deal correctly with powers and factorials
	$\lambda = \sqrt{306}\ (= 17.4928\ldots)$	**A1**	Answer, awrt 17.5 or $\sqrt{306}$ or equivalent (e.g. $3\sqrt{34}$)
1(b)	$\text{Po}(2\lambda)$	**M1**	Stated or implied
	$\mathrm{e}^{-2\lambda}\dfrac{(2\lambda)^8}{8!}=4\times\mathrm{e}^{-\lambda}\dfrac{\lambda^8}{8!}$	**M1**	Allow 1 error, including 4 on wrong side with all else correct
		A1	All correct
	$\lambda = 6\ln 2$	**A1**	Mark final answer. 6 ln 2 or exact equivalent, e.g. ln 64, 2ln8, 3ln4, –ln(1/64) etc. but must be cancelled (not e.g. ln(256/4))

Question	Answer	Marks	Guidance
2(a)	$G_U(t) = E(t^U) = \Sigma t^u\, e^{-\lambda}\lambda^u/u!$	**M1**	Correct formula for G(t), any variable letter (but not t^x). Condone missing limits but any limits shown must be correct. If shown as series explicitly, see conditions for final **A1** to be awarded.
	$= e^{-\lambda}\Sigma(t\lambda)^u/u!$	**M1**	Combine t and λ
	$= e^{-\lambda}e^{t\lambda}$	**A1**	State or clearly show sum of series is $e^{t\lambda}$
	$= e^{\lambda(t-1)}$ **AG**	**A1**	If sum shown explicitly it must start at 0, have at least 3 terms and be infinite. If not, or if just small slip, do not award this **A1**.
2(b)(i)	$G(1) = 1$	**B1**	If not specified assume order $G(1)$, $G'(1)$, $G''(1)$
	$G'(1) = 3$	**B1**	
	$6.75 = G''(1) + G'(1) - [G'(1)]^2$	**M1**	Use of given formula or via use of both $\text{var}(X) = E(X^2) - [E(X)]^2$ and $G''(1) = E(X^2) + E(X)$.
	$\Rightarrow \quad G''(1) = 12.75$	**A1**	
2(b)(ii)	mean = $4 \times 3 = 12$	**B1**	If not specified assume order mean, variance.
	variance = $4 \times 6.75 = 27$	**B1**	Could see $159 + 12 - 144$ oe

Question	Answer	Marks	Guidance
3(a)	$\text{var}(kZ) = k^2$…	**M1**	Finding the variance of kZ. Can be implied from correct answer.
	…so $kZ \sim N(0, k^2)$ and hence MGF of kZ is $\exp(k^2t^2/2)$	**A1**	Full argument (but condone just $E(kZ) = 0$ rather than $kZ \sim N(0, k^2)$). Correct answer with no working or justification: **SCB1**.
3(b)	MGF of each Z_i/n is $\exp\left(\frac{1}{2}\left(\frac{1}{n}\right)^2 t^2\right)$	**M1**	Using rule from **(a)** with $k = 1/n$
	$= \exp\left(\frac{t^2}{2n^2}\right)$	**A1**	
	Since $\bar{Z} = \frac{1}{n}\sum Z_i = \sum \frac{Z_i}{n}$ MGF of $\bar{Z}$ is $\left(\exp\left(\frac{t^2}{2n^2}\right)\right)^n$	**M1**	It is sufficient to show understanding that each Z_i/n has the same MGF and that the MGF of a sum of (independent) RVs is the product of their MGFs
	$= \exp\left(\frac{t^2 n}{2n^2}\right) = \exp\left(\frac{t^2}{2n}\right)$ **AG**	**A1**	AG so intermediate step must be shown.

Question	Answer	Marks	Guidance
3(b)	**Alternative method for question 3(b)**		
	MGF of $Z_1 + \ldots + Z_n$ is $[\exp(t^2/2)]^n$	**M1**	It is sufficient to show understanding that each Z_i has the same MGF and that the MGF of a sum of (independent) RVs is the product of the MGFs
	$= \exp(nt^2/2)$	**A1**	
	MGF of $\bar{Z}$ is $\exp\left(\frac{1}{2}\left(\frac{1}{n}\right)^2 nt^2\right)$	**M1**	AG. Using $k = 1/n$ in in rule derived from **(a)** for MGF. Must see intermediate step or quote both choice of k and use of rule. Condone lack of justification for use of rule from **(a)** in situation where variance is not 1.
	$= \exp\left(\frac{t^2}{2n}\right)$ **AG**	**A1**	AG so it must be clear that rule from **(a)** is being used
	Alternative method for question 3(b)		
	$\mathrm{E}(\bar{Z}) = 0$	**B1**	Quoted is sufficient
	$\mathrm{var}(\bar{Z})$ or $\sigma^2 = \frac{1}{n}$	**M1**	Quoted is sufficient
	$\therefore \bar{Z} \sim N\left(0,\ \frac{1}{n}\right)$	**M1**	or stating sum of normals is normal and multiple of a normal is normal
	MGF of $\bar{Z}$ is $\exp\left(\frac{1}{2}\left(\frac{1}{n}\right)t^2\right) = \exp\left(\frac{t^2}{2n}\right)$ as required **AG**	**A1**	AG so must be clear that $\sigma^2 = \frac{1}{n}$ is being substituted into given rule and that doing so is justified (by **M1M1** but condone **B0**).

Question	Answer	Marks	Guidance
4(a)	B(60, 0.08)	**M1**	Stated or implied
	≈ Po(4.8)	**A1**	Stated or implied
	P(⩾ 6) = 1 – P(⩽ 5)	**M1**	0.209 from 1 – P(⩽ 6): **M1 A1 M1 A0 B1**
	= 0.349	**A1**	If **M1 M0** then **SC1** (oe 2/4) for 0.347(4) from binomial
	n large, *p* small (or *np* < ~5)	**B1**	
4(b)	*M* ~ B(200, 0.45) ≈ N(90, 49.5)	**M1**	N(90, attempt at *npq*)
	O ~ B(200, 0.08) ≈ N(16, 14.72)	**M1**	N(16, attempt at *npq)* or N(16, 16) (or N(96, 96)) from (incorrect) Poisson
	M – 6*O* ~ N(–6, ...)	**M1**	Consider *M* – 6*O* (or 6*O* – *M* or (*O* – *M*/6) etc.), allow variance 137.82 (from 6 rather than 6^2) or 625 (from Poisson)
	N(–6, 579.42)	**A1***	Correct variance
	P(> 0) = ...	**M1dep**	No cc: 0.40158. Wrong cc: 0.4096. Both **M1A0**. Variance must be correct.
	= 0.393(567)	**A1dep**	In range [0.393, 0.394] {(0.290, 0.3046 or 0.320 from N(–6, 137.82)}

Question	Answer	Marks	Guidance
5(a)(i)	Quicker/cheaper etc.	**B1**	Accept valid negatives about census rather than positives about sample provided that it is clear why this might just be advantageous to Sheila (i.e. not just 'census has more data than sample'). Ignore irrelevant statements.
5(a)(ii)	Either 'It means that the sample will be a better representation of the population' or 'It means that the sample will not be biased'.	**B1**	Ignore irrelevant statements.
5(b)(i)	$\frac{20}{19}\left(\frac{48360}{20}-47^2\right)$	**B1**	47 (or 2209 (i.e. 47^2)) stated or implied by correct answer
		M1	Correct formula for unbiased variance
	= 220	**A1**	220 only {209: B1M0A0}
5(b)(ii)	$47 \pm 2.093\sqrt{\frac{220}{20}}$	**M1**	Use n or $\sqrt{n}$ divisor
		A1	$47 \pm t\sqrt{(s^2/n)}$, allow z (= 1.96) here
		B1	Correct t value
	= (40.06, 53.94)	**A1**	Answers, end-points correct to 4 sf or better {(40.50, 53.50) from normal: **M1A1B0A0**}
5(b)(iii)	Times need to be normally distributed	**M1**	State 'normal'
	Times may not have a symmetric distribution as there could be some very long times.	**A1**	Any correct or plausible statement to the effect that either: the times may not be normally distributed (e.g. symmetry or non-negativity); or the times may not all come from the same distribution (e.g. people may travel by different modes of transport); or the times may not all be independent (e.g. people may travel on the same train, which is delayed). Note: It is insufficient to observe that "we cannot know if they are normal"

Question	Answer	Marks	Guidance
6(a)	$\lim_{x\to-\infty} \mathrm{F}(x) = \frac{1}{\pi}\left(\frac{\pi}{2} + -\frac{\pi}{2}\right) = 0$	**B1**	**AG** For either **B1**, calculation must be explicit but need not be completely rigorous (e.g. condone 'F(–∞) =')
	$\lim_{x\to\infty} \mathrm{F}(x) = \frac{1}{\pi}\left(\frac{\pi}{2} + \frac{\pi}{2}\right) = 1$	**B1**	If **B0B0** then **SCB1** for stating both limits of $\tan^{-1}x$ as $x \to \pm\infty$ or showing them clearly on a graph of $y = k\tan^{-1}x$ oe. **B0** for ±90° Note: If wrong way round i.e. attempt to show that $x \to \pm\infty$ as F → 0, 1 then **B0B0** but **SCB1** is still available
	$\mathrm{F}'(x) = \frac{1}{\pi(1+x^2)}$	**M1**	Method to show increasing for all x. Argument could be from well drawn sketch of $y = \mathrm{F}(x)$ (as transformation of $y = \tan^{-1}x$) or carefully reasoned from graph of $y = k\tan^{-1}x$. Insufficient to reason from graph of $\tan x$ unless relationship to F(x) clearly explained.
	$\mathrm{F}'(x) > 0$ for all x	**A1**	Correctly shown, allow > or ⩾. Do not condone incorrect statements (e.g. $\mathrm{F}'(x) \geqslant 1$ for all x).
6(b)	1 – F(1)	**M1**	
	= ¼	**A1**	

Question	Answer	Marks	Guidance
6(c)	$\mathrm{F}'(x)=\dfrac{1}{\pi(1+x^2)}$	M1	Differentiate …
		A1	… correctly
	$\int_{-\infty}^{\infty} x^2\mathrm{F}'(x)\,\mathrm{d}x$	M1	Forming $\int x^2\mathrm{F}'(x)\,\mathrm{d}x$, correct limits somewhere
	$=\int_{-\infty}^{\infty}\dfrac{1}{\pi}-\dfrac{1}{\pi(1+x^2)}\,\mathrm{d}x=\dfrac{1}{\pi}\left[x-\tan^{-1}x\right]_{-\infty}^{\infty}$	M1	Method for integral (independent). Must result in compete answer and not just another integral. Might be done via substitution (e.g. $x=\tan\theta$ but need complete integral and correct limits for **M1**)
	$=\dfrac{2}{\pi}\lim\limits_{N\to\infty}\left[x-\tan^{-1}x\right]_0^N=\dfrac{2}{\pi}\lim\limits_{N\to\infty}\left[N-\tan^{-1}N\right]$ $=\dfrac{2}{\pi}\lim\limits_{N\to\infty}\left[N\right]-1$ which does not exist	A1	Correctly show improper integral does not exist. Condone unrigorous approach (e.g. substituting ∞ in) but integral must be complete and correct (can be awarded after **M1A0** if $\mathrm{F}'(x)=k/(1+x^2)$). Alternative method for final **M1A1**. Good sketch of $y=x^2\mathrm{F}'(x)$ and clearly showing area (on one or both sides) is not finite (e.g. by using a rectangle of fixed height < 1 and ever-increasing width and reasoning that the area under the graph must exceed the area of this rectangle (once beyond a certain width)).

Question	Answer	Marks	Guidance
6(d)	$F(y) = P(Y \leqslant y) = P(1 - X \leqslant y) = P(X \geqslant 1 - y)$	**M1**	Correctly express $F(y)$ as an inequality in X and y.
	$= 1 - F(1 - y)$	**M1**	Correctly deal with changed inequality $(1 - \ldots)$
	$= \frac{1}{\pi}\left(\frac{\pi}{2} - \tan^{-1}(1-y)\right)$ (for all values of y)	**A1**	aef e.g. $1 - \frac{1}{\pi}\left(\frac{\pi}{2} + \tan^{-1}(1-y)\right)$, $\frac{1}{2} - \frac{1}{\pi}\tan^{-1}(1-y)$, $\frac{1}{2} + \frac{1}{\pi}\tan^{-1}(y-1)$, $\frac{1}{\pi}\left(\frac{\pi}{2} + \tan^{-1}(y-1)\right)$, etc ISW Ignore extraneous piecewise definitions. Note: $\frac{1}{\pi}\left(\frac{\pi}{2} + \tan^{-1}(1-y)\right)$ from $x = 1 - y$ (or no working) scores 0/3 Alternative: **M1** for correctly deducing $f(y) = \frac{1}{\pi(1+(1-y)^2)}$ **M1** for $F(y) = \int_{-\infty}^{y} \frac{1}{\pi(1+(1-t)^2)}\,dt$ (limits must be correct but condone 'double' use of y) and **A1** for correct final answer.

Question	Answer	Marks	Guidance
7(a)	$96000/40 - k\times40^2 = 0$	M1	Using NII (or balancing forces) with correct expressions for
	$k = 1.5$	A1	forwards force and resistance.
7(b)	$\frac{96000}{20} - 1.5\times20^2 - 800g\sin\theta = 800a$	M1	Using NII with expressions for all relevant forces
		A1	All correct
	$a = 4.25\text{ms}^2$ up the slope	A1	Condone missing direction
	Alternative method for question 7(b)		
	Work done against resistance and gravity $= Rv + mg\frac{dh}{dt} = kv^3 + 800\times10\times v\sin\theta$ $= 1.5\times20^3 + 8000\times20\times0.1$ $= 28000$	M1	Understanding that the power is also being used to overcome friction and increase PE and finding the power associated with the change of KE
	so acceleration is using 96 kW – 28 kW = 68 kW $68000 = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = mva = 800\times20a = 16000a$	A1	
	$a = 4.25\text{ms}^2$ up the slope	A1	Condone missing direction
7(c)(i)	$\frac{96000}{v} - 1.5v^2 = 800\times0.5$	M1	Using NII with expressions for all relevant forces
		A1	(or using $P = kv^3 + mva$ from alternative in **(b)** with no PE)
	$v^3 + \frac{800}{3}v = 64000$	A1	Must be correct form (i.e. 'v^3 +'). [$a = 800/3$, $b = 64000$]

Question	Answer	Marks	Guidance
7(c)(ii)	Evaluate relevant expression at 37.75 and 37.85	**M1**	e.g. –137 & 318 (or –205 & 477) or 63862 (or 63863) & 64318 or 95794 & 96477
	State "sign change, therefore solution between" if there is or state relevant inequalities and draw conclusion oe	**A1**	Or: explicitly solve (e.g. by fixed point iteration) to get solution or clear convergence to 37.8 (3 sf) or better **B2** Just e.g. 64090 or 90 from 37.8 is insufficient for **M1A1**

Question	Answer	Marks	Guidance
8(a)	Vertical speed before 1st collision is $\sqrt{2gh}$ = 7 m s^{-1}	**B1**	Obtain or imply 7
	$\parallel$ component after is 7 sin 30° = 3.5 (m s^{-1})	**B1FT**	FT their 7 (so 0.5×'7')
	$\perp$ component afterwards is 0.8×7 cos 30°	**M1**	Multiplying their $\perp$ component by e
	= 2.8√3 or awrt 4.85 (m s^{-1})	**A1FT**	FT their 7 so (so 0.4√3×'7')
8(b)	$y = 2.8\sqrt{3}t - 2.5\sqrt{3}t^2$	**M1**	
	= 0 at t = 1.12	**A1FT**	FT their 7 (so 0.16×'7')
	$OA = x = 3.5t + 2.5t^2$	**M1**	**M0** if – instead of +
	= 7.056 (m)	**A1FT**	Answer, in range [7.05, 7.06]. FT their 7 (so 0.144×'7'^{2}) **SCB1** for $t = (7\sqrt{3})/15$ (= 0.808...) possibly followed by **SCB2** for distance =5.55(33...) (= 833/150) m

Question	Answer	Marks	Guidance
9	Initial energy = $mgl(1 - \cos 60°) = 5ml$	B1	Could be left as $\frac{1}{2}mgl$
	$\frac{1}{2}mv^2 + mgl(1 - \cos\theta) = 5ml$	M1	Conservation of energy; equating KE + PE to their initial energy...
	$v^2 = 10l(2\cos\theta - 1)$ oe	A1	...to obtain (correct) expression for v^2 in terms of θ and l (and g)
	NII Radial: $T - mg\cos\theta = ma_r$	M1	Using NII in the radial direction correctly
	$a_r = \dfrac{v^2}{l}$	B1	Using formula for radial acceleration
	$T\cos\theta = mg$ (or $10m$)	M1	Using vertical acceleration = 0
	$\therefore \dfrac{mg}{\cos\theta} - mg\cos\theta = \dfrac{m \times 10l(2\cos\theta - 1)}{l}$ $\therefore \dfrac{1}{\cos\theta} - \cos\theta = 2\cos\theta - 1$ $\therefore 3\cos^2\theta - \cos\theta - 1 = 0$	A1	Substituting T, v^2 and a_r into NII radial and reducing to quadratic equation in $\cos\theta$
	$\cos\theta = (1 + \sqrt{13})/6 \Rightarrow \theta =$ awrt 39.9° or 0.696 rads	A1	Correct final answer and no other

Question	Answer	Marks	Guidance
9	**Alternative method for question 9**		
	Initial energy = $mgl(1 - \cos 60°) = 5ml$	**B1**	Could be left as ½mgl
	$\tfrac{1}{2}mv^2 + mgl(1 - \cos\theta) = 5ml$	**M1**	Conservation of energy; equating KE + PE to their initial energy...
	$v^2 = 10l(2\cos\theta - 1)$ oe	**A1**	...to obtain (correct) expression for v^2 in terms of θ and l (and g)
	NII tangential: $-mg\sin\theta = ma_t$	**M1**	Using NII in the tangential direction correctly
	$a_r = \dfrac{v^2}{l}$	**B1**	Using formula for radial acceleration
	$a_y = 0 \Rightarrow a_r\cos\theta + a_t\sin\theta = 0$	**M1**	Using vertical acceleration is 0 (could be in the form $\tan\theta = -\dfrac{a_r}{a_t}$)
	$\therefore \dfrac{v^2}{l}\cos\theta + (-g\sin\theta)\sin\theta = 0$ $\therefore \dfrac{10l(2\cos\theta - 1)}{l}\cos\theta - 10\sin^2\theta = 0$ $\therefore (2\cos\theta - 1)\cos\theta - (1 - \cos^2\theta) = 0$ $\therefore 3\cos^2\theta - \cos\theta - 1 = 0$	**A1**	Substituting a_t, v^2 and a_r into $a_y = 0$ and reducing to quadratic equation in $\cos\theta$
	$\cos\theta = (1 + \sqrt{13})/6 \Rightarrow \theta =$ awrt 39.9° or 0.696 rads	**A1**	Correct final answer and no other

Question	Answer	Marks	Guidance
10(a)	$F = ma \Rightarrow 0.0024(0.12 - 3x) = 5\times10^{-5}\ddot{x}$	**M1**	Attempting NII for the particle (allow a for acceleration here). Could already be in terms of the substituted variable
	$\ddot{x} + 144x = 5.76$ or $\ddot{x} + 144(x - 0.04) = 0$	**A1**	All correct and simplified to a useful form (could be done after substitution).
	Equilibrium position must be $x = 0.04$	**B1**	Can be implied by correct substitution
	$y = x - 0.04 \Rightarrow \ddot{y} + 144y = 0$ or $\ddot{y} = -144y$	**A1**	Correctly using a substitution (not $x = x - 0.04$) in the DE and arranging to either of these forms and from cwo. Argument must be complete. **A0** for $\ddot{x} = -144x$ oe.
	... which is SHM with $\omega^2 = 144$ (or $\omega = 12$) about the point $x =$ 0.04...	**A1**	SHM explicitly stated from equation of correct form *only*. About 0.04 oe explicitly stated but might have been earlier. $\omega = 12$ can be implied from correct period.
	with period $\frac{2\pi}{\omega} = \frac{1}{6}\pi$	**B1FT**	Aeef, or in interval [0.523, 0.524]. This mark could be awarded even if the substitution is incorrect. FT their ω from $\ddot{x} = -144x$ or $\ddot{y} = -144y$
10(b)	$x = 0.02\cos 12t + 0.04$	**M1**	Using form of SHM solution, for x, which has 0 velocity when $t =$ 0 (constants could be wrong but must be present). Could use sin with a phase shift of ½π.
	$-0.005 = 0.02\cos 12t$	**M1***	0.02 correct and $x = 0.035$ to set up equation (allow wrong sign)
	$\cos^{-1}(-0.005/0.02)$ and value for t found	**M1dep**	Solving to find a value for t
	0.152 (seconds)	**A1**	Answer, awrt 0.152

Question	Answer	Marks	Guidance
11(a)	Max speed at equilibrium position	**B1**	Stated or implied e.g. by $\frac{0.15x_A}{0.4} = \frac{0.15x_B}{0.6} + 0.1$ or $T_A = T_B + 0.1$ or $\frac{\mathrm{d}E}{\mathrm{d}x} = 0$ (which could be implied by completing the square) oe
	x = extension of AP: $\frac{0.15x}{0.4} - \frac{0.15(0.6-x)}{0.6} - 0.1 = 0$	**M1**	Equation in unknown distance (e.g. length of AP or distance moved from initial position or extension of AP). e.g. $\frac{0.15(y-0.4)}{0.4} - \frac{0.15(1-y)}{0.6} - 0.1 = 0$ where y is AP. Condone sign errors and $m = 0.1$.
	$x = 0.4$	**A1**	Correct value of unknown distance (e.g. 0.1 below initial posn)
	Distance below A is 0.8 m	**A1FT**	Distance correctly stated and clear where it is from

Question	Answer	Marks	Guidance
11(b)	Any correct numerical calculation of EPE	B1	(In this solution x is the max extension of AP but other definitions are possible) Could be seen in **(a)**
	$\frac{0.15(0.3)^2}{2\times0.4}+\frac{0.15(0.3)^2}{2\times0.6}+0.02375+0.09$	B1	Expression for total initial energy (0.141875 J) (here, zero PE level is at B but it could be defined at a different level). Could be embedded.
	$0.141875=\frac{0.15x^2}{2\times0.4}+(1.6-(x+0.4))\times0.1$	M1*	Conservation of energy. EPE + GPE = initial energy (KE = 0)
		A1	Correct formula for EPE and signs all correct
		A1	All correct
	$60x^2 - 32x - 7 = 0$	M1dep	Simplify to 3-term quadratic {from A: $300x^2 - 400x + 77 = 0$}
	$x = 0.7$ so distance below A is $x + 0.4$ m = 1.1 m	A1	Solve to get 0.7and correctly deal with distance (if not dealt with by definition of x)
			Alternative method for last 5 marks using Y as point where lower string becomes slack and y for displacement below this point: **M1** for using conservation of energy to find v^2 at Y (23/8) **M1** for setting up SHM for P for $y > 0$ **A1** for $\omega^2 = 75/2$ and equilibrium position 2/3 below A **M1** for setting up $v^2 = \omega^2(A^2 - x^2)$ with consideration at Y leading to $A = 13/30$ **A1** for $v = 0$ leading to 0.4 + 4/15 + 13/30 = 1.1 m below A

Question	Answer	Marks	Guidance
12(a)	$0.4R_A\times0.6 = 0.4R_B\times0.6 + 0.4F_A\times0.8 + 0.4\times F_B\times0.8$	**M1** **A1**	Need consistent components of all four unknown forces and no weight. Condone sign errors for **M1**. Condone same incorrect (or cancelled) distance for all forces for **M1**. Condone consistent use of angle (e.g. α or φ) or consistent incorrect use of θ for **M1** so e.g. **M1** for $R_A \sin\theta\times d = F_A \cos\theta\times d + R_B \sin\theta\times d + F_B \cos\theta\times d$ If all correct but in terms of e.g. $\cos\alpha$, $\sin\alpha$ and correct values of cos/sin (or α) shown then **A1** can be awarded.
	$F_A = 0.4R_A$; $F_B = 0.4R_B$	**B1**	But must see this applied to both F_A and F_B, not just general (could be embedded and could use μ for 0.4).
	$\Rightarrow \quad 0.6(R_A - R_B) = 0.4\times0.8(R_A + R_B)$	**M1**	Reduction to linear equation in R_A and R_B (or F_A and F_B). Must sub values for sin/cos
	$\Rightarrow \quad 0.28R_A = 0.92R_B$ $\Rightarrow \quad R_A : R_B = 23 : 7$ **AG**	**A1**	Condone final answer presented as fractions. If rounded values used for trig ratios then **A0**.

Question	Answer	Marks	Guidance
12(b)	Para. to rod: $0.8R_A - 0.8R_B + 0.6F_A + 0.6F_B = 10\cos\theta$	**M1** **A1**	First two **M1** marks require components of each of the five forces to be balanced. Condone sin/cos errors and sign errors and/or $10g$. Could be in terms of $\sin\alpha$ and $\cos\alpha$ if values given here or earlier but not θ here unless recovered.
	Perp. to rod: $0.6R_A + 0.6R_B - 0.8F_A + 0.8F_B = 10\sin\theta$	**M1** **A1**	If **M0M0** then **SCB1** for both equations correct but with weight missing
	Substitute $F_A = 0.4R_A$; $F_B = 0.4R_B$ to obtain $0.8(R_A - R_B) + 0.24(R_A + R_B) = 10\cos\theta$ $0.6(R_A + R_B) + 0.32(R_B - R_A) = 10\sin\theta$	**M1**	Substitute $F_A = 0.4R_A$; $F_B = 0.4R_B$ into the force equations to obtain two equations in two unknown forces and θ only
	$\tan\theta = \dfrac{0.28R_A + 0.92R_B}{1.04R_A - 0.56R_B}$	**M1**	Use $R_A : R_B = 23 : 7$ to eliminate R_A or R_B from both equations and divide to find a value for $\tan\theta$ or square and add to eliminate θ and find a value for R_A or R_B
	$= 0.644$	**A1**	or $R_A = 9.66852...$ or $R_B = 2.94259...$
	$\Rightarrow \theta = 32.8°$	**A1**	awrt 32.8° (or awrt 0.572 rads) cwo (could be from e.g. $\sin\theta = (7/125)R_A$)