Cambridge Assessment International Education

Cambridge Pre-U

FURTHER MATHEMATICS **9795/01**

Paper 1 Further Pure Mathematics **May/June 2022**

3 hours

* 9 3 6 4 8 5 2 1 4 1 *

You must answer on the answer booklet/paper.

You will need: Answer booklet/paper
Graph paper
List of formulae (MF20)

INSTRUCTIONS

- Answer **all** questions.
- If you have been given an answer booklet, follow the instructions on the front cover of the answer booklet.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number on all the work you hand in.
- Do **not** use an erasable pen or correction fluid.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- At the end of the examination, fasten all your work together. Do **not** use staples, paper clips or glue.

INFORMATION

- The total mark for this paper is 120.
- The number of marks for each question or part question is shown in brackets [ ].

This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

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[Turn over

1 **(a)** Express $\dfrac{1}{(2n-1)(2n+3)}$ in partial fractions. [2]

(b) Hence evaluate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+3)}$. [3]

2 The curve C has equation $y = \dfrac{x}{1-x+x^2}$.

(a) **(i)** Show algebraically that C exists only for $-\frac{1}{3} \leqslant y \leqslant 1$. [3]

(ii) Hence, or otherwise, find the coordinates of the turning points of C. [3]

(b) Sketch C, showing all significant features. [3]

3 **(a)** **(i)** Determine the possible values of the real numbers a and b for which $(a + \mathrm{i}b)^2 = 28 + 96\mathrm{i}$. [3]

(ii) Deduce the solutions of the equation $z^4 = 28 + 96\mathrm{i}$. [3]

(b) The locus of points in the Argand diagram given by $|z - 28 - 96\mathrm{i}| = d$ passes through the origin. Sketch this locus and state the value of the constant d. [2]

4 A curve has equation $y = \cosh x$. The length of the arc of the curve between the points where $x = 0$ and $x = 1$ is denoted by L.

(a) Determine, in terms of e, the exact value of L. [4]

A rational approximation for L is to be found using the first few terms of the Maclaurin series for $\cosh x$.

(b) **(i)** Calculate the approximation for L found when the first three non-zero terms are used. [3]

(ii) Explain why any approximation for L found by this method will be an under-estimate, no matter how many terms of the series are used. [1]

5 A group G of order 6 consists of functions (of x) under the operation of composition of functions. Two of the elements of G are $\mathrm{p}(x) = \dfrac{1}{x}$ and $\mathrm{q}(x) = 1 - x$.

(a) State the identity element, $\mathrm{i}(x)$, of G. [1]

(b) Determine, as functions of x, the remaining three elements of G. [3]

(c) List all the subgroups of G. [4]

6 Solve the differential equation $x\dfrac{\mathrm{d}y}{\mathrm{d}x} - y = \dfrac{x^2}{\sqrt{1+x^2}}$, given that $y = 3\ln 2$ when $x = \frac{3}{4}$, giving your answer in the form $y = \mathrm{f}(x)$. [8]

7 Let $\mathbf{M} = \begin{pmatrix} 2k-1 & k-1 \\ 1-k & 1-8k \end{pmatrix}$, where k is a non-zero constant.

(a) Determine the value of k for which $\mathbf{M}$ is singular. [3]

(b) **(i)** Find the value of k for which the transformation T given by the matrix $\mathbf{M}$ is a rotation about the origin. [3]

(ii) Describe T fully in this case. [2]

8 The equation $x^3 - px^2 + qx - r = 0$, where p, q and r are constants, has roots α, β and γ. Express each of the following in terms of p, q and r.

(a) $\alpha^2 + \beta^2 + \gamma^2$ [2]

(b) $\alpha^2(\beta + \gamma) + \beta^2(\gamma + \alpha) + \gamma^2(\alpha + \beta)$ [3]

(c) $\alpha^3 + \beta^3 + \gamma^3$ [3]

9 Let $S_n = \sum_{r=1}^{n}(\cos^r\theta \cos r\theta)$. Use mathematical induction to prove that, for all positive integers n,

$$S_n = \frac{\cos^{n+1}\theta \sin n\theta}{\sin\theta}.$$ [7]

10 **(a)** Use the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials to show that

$$\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}.$$ [2]

(b) **(i)** Use the substitution $u = e^{2x}$ to show that $\tanh 2x - \tanh x = 0.3$ can be written as a cubic equation in u. [3]

(ii) Hence solve the equation $\tanh 2x - \tanh x = 0.3$, giving each answer in its simplest exact logarithmic form. [5]

11 The planes Π_1 and Π_2 have equations $\mathbf{r}.(8\mathbf{i} + \mathbf{j} - 3\mathbf{k}) = 20$ and $\mathbf{r}.(-\mathbf{i} + \mathbf{j} + \mathbf{k}) = 3$ respectively. The points V and W have coordinates $(3, -1, 1)$ and $(3, 2, 4)$ respectively.

(a) Show that V is in Π_1 and that W is in Π_2. [1]

The line of intersection of Π_1 and Π_2 is denoted by L.

(b) Find a vector equation for L in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$, where the vectors $\mathbf{a}$ and $\mathbf{d}$ have integer components. [4]

A point U on L has coordinates which are all **positive** integers.

(c) Show that there is only one possible position for U and state its coordinates. [3]

(d) Determine the volume of tetrahedron $OUVW$. [3]

  [Turn over

12 Let $I_n = \int_0^{\frac{1}{2}\pi} \sin^n \theta \, d\theta$, where $n \geqslant 0$.

(a) Prove that $nI_n = (n-1)I_{n-2}$ for $n \geqslant 2$. [4]

The curve B has polar equation $r = 4\sin^2\theta\cos\theta$ for $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$.

(b) Sketch B. [3]

(c) **(i)** Show that the area of the plane enclosed by B can be written in the form $aI_4 + bI_6$ for integers a and b to be determined. [2]

(ii) Deduce the exact value of this area. [3]

(d) Determine a cartesian equation for B. [2]

13 **(a)** Determine the five smallest positive values of θ for which $\cos 5\theta = \frac{1}{2}$. [2]

(b) **(i)** Let $z = \cos\theta + i\sin\theta$. Show that $z^n + z^{-n} = 2\cos n\theta$ for positive integers n. [2]

(ii) Hence express $2\cos 5\theta$ as a polynomial in x, where $x = 2\cos\theta$. [5]

(iii) By considering the result of part **(a)**, find, in an exact trigonometric form, the roots of $x^4 + x^3 - 4x^2 - 4x + 1 = 0$. [3]

(c) Use the result of part **(b)(iii)** to show that $\sin\left(\frac{1}{30}\pi\right)\sin\left(\frac{7}{30}\pi\right)\sin\left(\frac{11}{30}\pi\right)\sin\left(\frac{13}{30}\pi\right) = \frac{1}{16}$. [4]