FURTHER MATHEMATICS

Paper 1348/01
Further Pure Mathematics

Key Messages

Candidates are best served by a thoughtful approach to each question that considers – in particular – how the question is structured; many candidates frequently ignore the very careful signposts offered within the question. In a similar vein, candidates must be aware of how appropriate the working they are writing down is to the demand of that question or question-part; producing up to a page-and-a-half of working for a result that has been assigned just one or two marks is clearly inappropriate, and candidates should allow themselves to be guided by the number of marks.

Another key point is that many candidates seem unwilling to deploy their 'single maths' skills in the further maths setting and they should be aware that understanding and fluency in GCSE-level and Maths-9794-level techniques are taken as background/assumed knowledge within the further maths papers.

General Comments

Overall, the quality of the candidates' work was very impressive, with a good proportion gaining 100 or more marks. These candidates are to be commended, especially considering the number of places in the paper which required considerable care and due diligence in order to avoid oversights of the small details that many candidates found all too easy to miss.

In **Questions 6** and **12**, in particular, the clear guidance as to how best to proceed was overlooked by many candidates, who seemed to prefer lengthy and unhelpful approaches when a moment's consideration of the questions' wording and structure would have led to a more concise or useful approach being adopted. The importance of remembering 'single maths' skills when taking further maths papers was particularly true in **Question 4** with the integration of $\sin^2 x$, that first required the very standard 'single maths' use of a double-angle identity; the straightforward trigonometric equations that arose at the end of **Question 5**; and the use of either or both of *Pythagoras' Theorem* and some elementary trigonometry work within suitably constructed right-angled triangles in **Question 7** in order to find the argument of a specific point in the complex plane.

Those questions that candidates found most difficult were **Questions 2**, **7** and **12**, while **Questions 1**, **5**, **8** and **9** were handled most comfortably and confidently.

Centres are reminded that there are some changes to the syllabus for 2016 and there are also revised specimen papers on the website: www.cie.org.uk .

Comments on Specific Questions

Question 1

This was a simple starter question, requiring the quoting of a formula and then its use. Almost all candidates managed to attend to these matters both quickly and accurately.

Answer: 3

Question 2

The sigma-notation did not appear to be understood by a number of candidates. In most cases, candidates seemed unsure of what to do with the power of (x – 1) in any chosen term, so they ended up with an answer of zero upon substituting $x = 1$. It had been expected that candidates would do this question by little more than quoting the result and then dealing with the detail.

Answer: $-\frac{64}{35}$

Question 3

This was a relatively straightforward induction question. However, it was often undertaken in a poor or incomplete way. Amongst the better candidates, the most frequently lost mark was the one at the end for explaining why what they have done up to this point actually means that the result has indeed been proven. Candidates should be reminded of the need for this 'induction round-up'.

Question 4

Candidates should be aware that a sketch does not require the use of graph paper and plotting points. A clear sketch in their answer booklet is all that is required. On this occasion, a closed curve through the pole (O), with a cusp-like shape and some indication of scale was all that was required. The area required the use of a standard, quote-able formula and some very routine integration; however, many candidates made very heavy going of what had been intended to be a quick and easy question.

Answer: **(ii)** $\frac{1}{2}\pi$

Question 5

Once again, the curve-sketching question, on the topic of *Rational Functions*, was by far the most popular and highest-scoring question on the paper, yielding very high marks for almost all candidates. Once again, for the 'sketch', it is important to stress that candidates should not use graph paper, as this has the unfortunate effect of forcing candidates into a particular scale, and also then into the ensuing 'wobbly' curves that occur when trying to accommodate it unreasonably. It is also important to note the demand, stated clearly within the syllabus, that key points – especially those points where the curve meets or crosses the coordinate axes – should be noted somewhere (on or alongside the sketch). On this occasion, the y-intercept was not required because it had little bearing on the validity of the sketch; but the x-intercepts were, and several candidates lost marks for not supplying them.

Answers: **(i)** $x = 3$, $y = 2x + 11$ **(ii)** (1, 9), (5, 25)

Question 6

This was a routine complex numbers question, but it led to several unnecessarily lengthy sets of working from candidates. The allocation of just the one mark to part **(i)** should have given a clear indication that very little was being required of candidates – certainly, those who produced many lines of working should have realised they were doing more than was required. In part **(ii)**, it was often the case that candidates struggled in the use of *de Moivre's theorem* this way round, preferring instead to express $\cos(n\theta)$ as the real part of $(\cos\theta + \text{i}\sin\theta)^n$ in each of three cases. This is a much lengthier approach and most who adopted it made little progress. Part **(iii)** was a relatively straightforward piece of single maths trigonometry. Nonetheless, there were many candidates who were unsuccessful. To begin with, a number of candidates did not notice the difference between the identity in part **(ii)** and the equation in part **(iii)**, thereby attempting to solve $16\cos^5\theta = 0$ (or 1). Many candidates also missed either or both of the $\cos\theta = 0$ and $\cos\theta = -\frac{1}{2}$ cases.

Answers: **(iii)** $\frac{1}{2}\pi$, $\frac{3}{2}\pi$; $\frac{1}{3}\pi$, $\frac{5}{3}\pi$; $\frac{2}{3}\pi$, $\frac{4}{3}\pi$

Question 7

In part **(i)**, the required locus is a circle and its interior. Whilst most candidates identified these details correctly, their circle was often to be found in either the first or the second quadrants. Even for those candidates whose circle was in approximately the right place, the position of z_1 (the point with least argument within this region) was almost never identified correctly. The candidates' work showed, in many cases, a lack of suitable basic trigonometry and *Pythagoras* being used to find the appropriate angle(s).

Answers: **(i)** circle, centre 20 – 15i, radius 7 **(ii)** –0.927 (or 5.356)

Question 8

This was a relatively simple question on this topic. Nevertheless, it was very easy to overlook some of the subgroups – there are seven of order 2 and seven of order 4 – and thus very few candidates gained full marks on the question.

Answers: **(ii)** {*e*, *a*}, {*e*, *b*}, {*e*, *c*}, {*e*, *ab*}, {*e*, *bc*}, {*e*, *ca*}, {*e*, *abc*}, {*e*, *a*, *b*, *ab*}, {*e*, *b*, *c*, *bc*}, {*e*, *c*, *a*, *ca*}, {*e*, *a*, *bc*, *abc*}, {*e*, *b*, *ca*, *abc*}, {*e*, *c*, *ab*, *abc*}, {*e*, *ab*, *bc*, *ca*}

Question 9

This was a relatively gentle question on the *Differential Equations* topic, and proved very popular amongst candidates. A few candidates preferred to differentiate $y = \frac{u}{x}$ in part **(ii)**, rather than the easier $u = xy$ given, frequently confusing themselves as the working is more complicated this way round. Also, despite its relatively straightforward nature, many candidates felt obliged to demonstrate how the auxiliary equation and complementary function arose, rather than quoting and then solving a quadratic, followed by a statement of the relevant CF; this was entirely unnecessary and wasted time, and often led to complex coefficients and complicated expressions.

Answers: **(i)** $u = A\cos 2x + B\sin 2x + 2x + \frac{1}{4}$ **(ii)** $y = \frac{A\cos 2x}{x} + \frac{B\sin 2x}{x} + 2 + \frac{1}{4x}$

Question 10

This was a popular and usually well-attempted question. Algebraic or vector methods apply equally well in part **(i)**, although the simplest method must be that of finding two points on the line and proceeding from there. Once obtained, this line equation can be substituted straight into the third equation in part **(ii)**, to yield both the value of *k* and the evidence for the inconsistency. However, the value of *k* was usually found by evaluating the determinant of the coefficient matrix and then an algebraic elimination process was used to give the required inconsistency, often even from incorrect arithmetic. Many candidates lost one mark unnecessarily by omitting to have an '**r** = ...' (or equivalent) when writing the vector equation of a line.

Answers: **(i)** $\mathbf{r} = \begin{pmatrix} 8 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ **(ii)** $k = 5$

Question 11

In general, candidates did not answer this question well. Part of the problem lay in the candidates' misinterpretations of what was being asked of them. In part **(a)**, candidates were asked – in addition to finding the new equation with roots related to those of the first – to show **why** the substitution $y = \frac{4}{x}$ yielded the new equation and not just to verify that it did. In part **(b)**, parts **(i)** and **(ii)** were found to be very routine. Thereafter, in part **(iii)**, the problem lay principally in the justification of the degree of accuracy involved in the given numerical answers. It had been expected that candidates would use the *Change-of-Sign Rule*, but very few did so and thus lost at least two of the four marks available here. Most attempts merely substituted 2.70 into the left hand side of the equation and showed that the outcome was a number close to zero.

Answers: **(a)** $y^3 - 3y^2 - 8y - 16 = 0$ **(b)(i)** complex conjugates

Question 12

There is no doubt that the last question or two on the 9795 paper 1 are intended to differentiate between the most able candidates; it has often been the case – as it was here – that the penultimate question is long and technically demanding, while the last question is relatively short but requires an input of insight in order to untangle what is required. Nonetheless, candidates often seem to ignore the structure supplied. It is also the case that a few moments of careful consideration would show that, despite its appearance, all the parts of this question are essentially free-standing. Thus, an inability to complete one part does not prevent a candidate from continuing with the question. Indeed, part **(i)(a)** is actually a single-maths question, as it involves either integration by 'recognition' or by the most obvious of substitutions. The principal reason for its inclusion becomes abundantly clear when one views the reduction formula in part **(i)(b)**, a process that almost invariably requires the use of *integration by parts* and one of these parts has now been clearly flagged. Candidates who realised this had relatively easy access to the first 9 marks of the question. Those who didn't just seemed to abandon the question, often almost in its entirety, at this point.

For part **(i)(c)**, many unsuccessful candidates often made three or four attempts using unlikely substitutions and this wasted time. A few candidates chose to use $x\sqrt{2} = \tan\theta$ instead of the intended hyperbolic function substitution; this was sufficiently robust to allow them 6 of the 8 marks available, although they did run into difficulties when changing limits. It should then have been clear that the purpose in giving the answer here in part **(i)(c)** was to enable access to part **(ii)**, which now required only the 'surface area of revolution' formula and the preceding results. However, this formula was often incorrectly cited, with the '*y*' part missing.

Answers: **(i) (a)** $\frac{13}{3}$ **(ii)** $\frac{1}{8}\left(51\sqrt{2} - \ln\left(1+\sqrt{2}\right)\right)\pi$

Question 13

Once again, this question was placed last in order to avoid the scenario in which candidates wasted valuable time on something quite tough unless they had attempted all previous questions to the best of their ability. In fact, again, the structure was intended to be helpful and, on this occasion, very specific instructions were given to help guide candidates through the process. It had been expected that candidates would justify in part **(ii)** that the final possible compound angle would actually lie in the interval $(-\frac{1}{2}\pi, \frac{1}{2}\pi)$. However, very few candidates thought to be that thorough. In part **(iii)**, it had been expected that candidates would realise that using the substituted form in the writing of the first term of the given series was invalid, deploy the result of part **(i)** for the sum of the *n* = 1 and *n* = 2 cases to obtain $\frac{1}{2}\pi$, and *then* use the 'method of differences' for the cancelling of terms in the remaining infinite series from *n* = 3 onwards. Again, almost no-one did this, as the option of noting that '$\tan^{-1}\left(\frac{1}{0}\right) = \frac{1}{2}\pi$' was just too inviting. Candidates were not penalised for taking this shortcut, even though this meant that one or other of the earlier parts of the question became redundant as a result.

Answer: **(ii)** $\frac{a \pm b}{1 \mp ab}$.