CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Pre-U Certificate

MARK SCHEME for the May/June 2015 series

1348 FURTHER MATHEMATICS

1348/01 Paper 1 (Further Pure Mathematics),
maximum raw mark 120

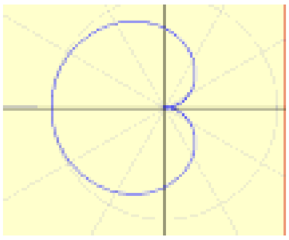
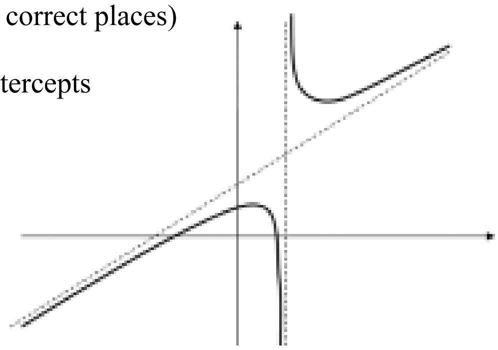
This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

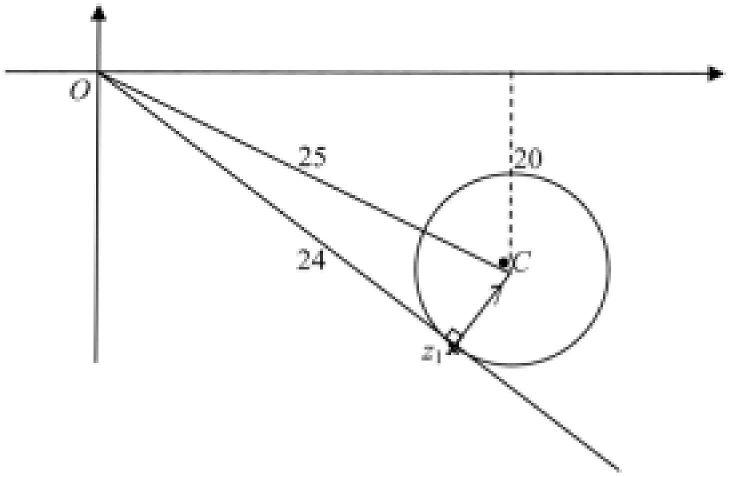
Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2015 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

1	use of formula $V=\frac{1}{6}\left\|\mathbf{a}\bullet\mathbf{b}\times\mathbf{c}\right\|$ or equivalent *NB* $\mathbf{b}\times\mathbf{c}=-4\mathbf{i}+10\mathbf{j}+2\mathbf{k}$ attempt at relevant scalar triple product = $\begin{vmatrix}2&3&-2\\2&0&4\\6&1&7\end{vmatrix}=18$ (or a scalar and a vector product) $V=3$ **cso**	M1 M1 A1 [3]
2	$a_4=\frac{y^{(4)}(1)}{4!}=\frac{(-2)^3}{1.3.5.7}$ from the Taylor series expansion $\Rightarrow\ y^{(4)}(1)=\frac{(-2)^3}{1.3.5.7}\times 4!=-\frac{64}{35}$ **ALTERNATIVE** Diff$^{\text{te}}$. four times to get $\frac{\mathrm{d}^4y}{\mathrm{d}x^4}=\sum_{n=4}^{\infty}\frac{(-2)^{n-1}n(n-1)(n-2)(n-3)(x-1)^{n-4}}{1.3.5....(2n-1)}$ When $x=1$, this is $\frac{(-2)^3\times 4(3)(2)(1)}{1.3.5.7}+0=-\frac{64}{35}$	M1 A1 A1 M1 A1 A1 [3]
3	$n=1$, $\mathbf{M}\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}3\\4\\5\end{pmatrix}=\begin{pmatrix}2(1)+1\\2(1)^2+2(1)\\2(1)^2+2(1)+1\end{pmatrix}$ $\Rightarrow$ result true for $n=1$ (both sides shown) induction hypothesis that $\mathbf{M}^k\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}2k+1\\2k^2+2k\\2k^2+2k+1\end{pmatrix}$ attempt at $\mathbf{M}^{k+1}\begin{pmatrix}1\\0\\1\end{pmatrix}=\mathbf{M}\begin{pmatrix}2k+1\\2k^2+2k\\2k^2+2k+1\end{pmatrix}=\begin{pmatrix}2k+1-4k^2-4k+4k^2+4k+2\\4k+2-2k^2-2k+4k^2+4k+2\\4k+2-4k^2-4k+6k^2+6k+3\end{pmatrix}$ $=\begin{pmatrix}2k+3\\2k^2+6k+4\\2k^2+6k+5\end{pmatrix}$ $=\begin{pmatrix}2(k+1)+1\\2(k+1)^2+2(k+1)\\2(k+1)^2+2(k+1)+1\end{pmatrix}$ This is the result with k replaced by $(k+1)$. Hence IF the result is true for $n=k$ THEN it also true for $n=k+1$. Since the result is true for $n=1$, it follows that it is true for all positive integers n.	B1 M1 M1 A1 A1 E1 [6]

4	(i)	Closed curve containing the pole, O Cusp at the pole Essentially all correct, including at least $(r, \theta) = (1, \pi)$ 	B1 B1 B1 [3]
	(ii)	$A = \left(\frac{1}{2}\right)\int_0^{2\pi}\sin^2\frac{1}{2}\theta\,\mathrm{d}\theta$ Condone missing $\frac{1}{2}$ until final A1 use of double-angle identity $A = \frac{1}{2}\int_0^{2\pi}\left(\frac{1}{2}-\frac{1}{2}\cos\theta\right)\mathrm{d}\theta$ correct integration. $= \frac{1}{4}[\theta - \sin\theta]$ $= \frac{1}{2}\pi$	M1 M1 A1 A1 [4]
5	(i)	VA $x = 3$ $y = \frac{2x^2+5x-25}{x-3} = \frac{2x(x-3)+11(x-3)\ \ [+8]}{x-3}$ $y = 2x + 11$ oblique asymptote	B1 M1 A1 [3]
	(ii)	for $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x-3)(4x+5)-\left(2x^2+5x-25\right)}{(x-3)^2}$ (good attempt at the *Quotient Rule*) correct unsimplified **OR** $\frac{\mathrm{d}}{\mathrm{d}x}\left(2x+11+\frac{8}{x-3}\right) = 2-\frac{8}{(x-3)^2}$ solving quadratic equation $4x^2 - 7x - 15 = 2x^2 + 5x - 25$ i.e. $2x^2 - 12x + 10 = 0$ **OR** $(x-3)^2 = 4$ (1, 9) and (5, 25) (Give one A1 for both x's correct without either y)	M1 A1 M1 A1 A1 [5]
	(iii)	General shape (with asymptotes and turning points in approximately correct places) $2x^2 + 5x - 25 = 0$ solved to find x-intercepts $x = -5$ or $2\frac{1}{2}$ 	B1 M1 A1 [3]

6 **(i)**	$z^n = \cos n\theta + \mathrm{i}\sin n\theta$ and $z^{-n} = \cos n\theta - \mathrm{i}\sin n\theta \Rightarrow z^n + \dfrac{1}{z^n} \equiv 2\cos n\theta$	B1 [1]
(ii)	$\left(z+\dfrac{1}{z}\right)^5 \equiv \left(z^5+\dfrac{1}{z^5}\right)+5\left(z^3+\dfrac{1}{z^3}\right)+10\left(z+\dfrac{1}{z}\right)$	M1 A1
	Repeated use of result $\Rightarrow 32\cos^5\theta \equiv 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$	M1
	$\Rightarrow 16\cos^5\theta \equiv \cos 5\theta + 5\cos 3\theta + 10\cos\theta$ **Answer Given**	A1 [4]
(iii)	$16\cos^5\theta = \pm\cos\theta$ Condone sign error here to allow for first A1	M1
	$\cos\theta = 0 \Rightarrow \theta = \frac{1}{2}\pi$ or $\frac{3}{2}\pi$ (both)	A1
	$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{1}{3}\pi$ or $\frac{5}{3}\pi$ (both)	A1
	$\cos\theta = -\frac{1}{2} \Rightarrow \theta = \frac{2}{3}\pi$ or $\frac{4}{3}\pi$ (both)	A1
	but allow one A1 for $\theta = \frac{1}{3}\pi$ and $\theta = \frac{2}{3}\pi$ if both 2nd answers missing	[4]
7 **(i)**	Circle with centre (20, –15)	B1
	Circle of radius = 7 stated or deducible from diagram Allow B1 for a circle entirely in the 4th Quad.	B1
	Interior of circle shaded (Ignore boundary in/out)	B1 [3]
(ii)		
	for z_1 in correct place	B1
	for (sufficient) distances	B1
	for $\arg(z_1) = -\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{7}{24}\right)$ or equivalent, using other inverse trig. functions	M1
	[NB – this is $\left(-\tan^{-1}\frac{4}{3}\right)$ using a result in Q13]	
	for – 0.927 (or 2π – 0.927 = 5.356)	A1 [4]

8 **(i)**

	e	*a*	*b*	*c*	*ab*	*bc*	*ca*	*abc*
e	*e*	*a*	*b*	*c*	*ab*	*bc*	*ca*	*abc*
a	*a*	*e*	*ab*	*ca*	*b*	*abc*	*c*	*bc*
b	*b*	*ab*	*e*	*bc*	*a*	*c*	*abc*	*ca*
c	*c*	*ca*	*bc*	*e*	*abc*	*b*	*a*	*ab*
ab	*ab*	*b*	*a*	*abc*	*e*	*ca*	*bc*	*c*
bc	*bc*	*abc*	*c*	*b*	*ca*	*e*	*ab*	*a*
ca	*ca*	*c*	*abc*	*a*	*bc*	*ab*	*e*	*b*
abc	*abc*	*bc*	*ca*	*ab*	*c*	*a*	*b*	*e*

B4, 3, 2, 1, 0

[4]

(ii) Subgroups of order 2: $\{e, a\}$, $\{e, b\}$,$\{e, c\}$,$\{e, ab\}$,$\{e, bc\}$,$\{e, ca\}$,$\{e, abc\}$

6 distinct given B1

all 7, no repeats B1

Subgroups of order 4: $\{e, a, b, ab\}$, $\{e, b, c, bc\}$, $\{e, c, a, ca\}$
$\{e, a, bc, abc\}$, $\{e, b, ca, abc\}$, $\{e, c, ab, abc\}$
$\{e, ab, bc, ca\}$

Either (all 3 of 1st type) + **B1** (all3 of 2nd type) + **B1** (last one) B2

Or (5 distinct) + **B1** (6th) + **B1** (all 7 and no repeats)

[5]

9 **(i)** Comp. Fn. is $u = A\cos 2x + B\sin 2x$ Allow $Ae^{2\mathrm{i}x} + Be^{-2\mathrm{i}x}$ here M1 A1

For Part. Int. try $u = ax + b$ ($u' = a$ and $u'' = 0$) and substd. into given d.e. M1

$a = 2$, $b = \frac{1}{4}$ A1

Gen. Soln. is $u = A\cos 2x + B\sin 2x + 2x + \frac{1}{4}$ **ft** B1

Must have two arbitrary constants
Condone apparently complex coefficients here
Don't allow $Ae^{2\mathrm{i}x} + Be^{-2\mathrm{i}x}$ here

[5]

(ii) $xy = u \Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x} + y = \frac{\mathrm{d}u}{\mathrm{d}x}$ and $x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}^2u}{\mathrm{d}x^2}$ M1 A1

$\Rightarrow$ (*) becomes $x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 4xy = 8x + 1$ A1

$\Rightarrow y = \frac{A\cos 2x}{x} + \frac{B\sin 2x}{x} + 2 + \frac{1}{4x}$ **ft** B1

Accept $xy = \ldots$
Must have two real arbitrary constants

[4]

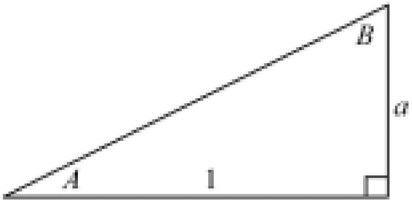
10	**(i)**	d.v. = $\begin{pmatrix}1\\7\\-6\end{pmatrix}\times\begin{pmatrix}3\\-5\\8\end{pmatrix}=\begin{pmatrix}26\\-26\\-26\end{pmatrix}\equiv\begin{pmatrix}-1\\1\\1\end{pmatrix}$	M1 A1
		finding one point on line: e.g. (0, 8, 11), (8, 0, 3), (11, –3, 0)	M1 A1
		ft Line equation in vector form $\mathbf{r}=\begin{pmatrix}8\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}$ (must have **r** = …, ***l*** = …)	B1
			[5]
		ALTERNATIVE 1	
		finding two points on line: e.g. (0, 8, 11), (1, 7, 10), (8, 0, 3), (11, –3, 0) , …	M1 A1 A1
		ft d.v. $\mathbf{r}=\begin{pmatrix}-1\\1\\1\end{pmatrix}$ (e.g.)	B1
		ft Line equation in vector form $\mathbf{r}=\begin{pmatrix}8\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}$ (must have **r** = …)	B1
			[5]
		ALTERNATIVE 2	
		Eliminating x (say): $z = y + 3$	M1 A1
		Setting $y = \lambda$ (or equivalent) and finding z and x in terms of the parameter	M1
		$z = \lambda + 3$ and $x = 8 - \lambda$	A1
		ft Line equation in vector form $\mathbf{r}=\begin{pmatrix}8\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}$ (must have **r** = …)	B1
			[5]

(ii)	$\begin{vmatrix} 1 & 7 & -6 \\ 3 & -5 & 8 \\ k & 2 & 3 \end{vmatrix} = 26k - 130$	M1 A1
	$= 0$ when $k = 5$ **ft** from a linear eqn.	A1
	e.g. $\begin{matrix} x+7y-6z=-10 \\ 3x-5y+8z=48 \\ 5x+2y+3z=16 \end{matrix} \to \begin{matrix} -33y+33z=66 \\ -26y+26z=78 \end{matrix} \to \begin{matrix} -y+z=2 \\ -y+z=3 \end{matrix}$	
	eliminating one variable (twice); correctly	M1 A1
	Inconsistency noted/explained (allow valid **ft**)	A1 [5]
	ALTERNATIVE	
	Substituting $x = 8 - \lambda$, $y = \lambda$ and $z = 3 + \lambda$ into $kx + 2y + 3z = 16$	M1
	$\Rightarrow \lambda(5 - k) + (11k - 22) = 0$ λ terms; other terms	A1 A1
	$k = 5$ gives non-unique soln.	A1
	Then 33 = 0 **ft** and system inconsistent (allow valid **ft**)	A1 B1 [5]
11 (a)	Roots $\{qr, rp, pq\} = \left\{\frac{pqr}{p}, \frac{pqr}{q}, \frac{pqr}{r}\right\}$	M1
	$= \left\{\frac{4}{p}, \frac{4}{q}, \frac{4}{r}\right\}$ from $pqr = 4$, so subst. $y = \frac{4}{x}$	A1
	Setting $x = \frac{4}{y} \Rightarrow \frac{64}{y^3} + 2.\frac{16}{y^2} + 3.\frac{4}{y} - 4 = 0$	M1
	$\Rightarrow 64 + 32y + 12y^2 - 4y^3 = 0$	M1 A1
	$\Rightarrow y^3 - 3y^2 - 8y - 16 = 0$	A1 [6]
	ALTERNATIVE (for last four-mark section)	
	$\sum \alpha' = qr + rp + pq = 3$	B1
	$\sum \alpha'\beta' = pqr(p + q + r) = 4 \times (-2) = -8$	B1
	$\sum \alpha'\beta'\gamma' = (pqr)^2 = 16$	B1
	New eqn. is $x^3 - 3x^2 - 8x - 16 = 0$ (must have "= 0")	B1 [6]
(b) (i)	β and γ are complex conjugates (since the coefficients are real)	B1 [1]
(ii)	$\alpha\beta\gamma = 4 \Rightarrow \beta\gamma = \frac{4}{\alpha}$	
	and $\lvert \beta \rvert = \lvert \gamma \rvert$ since conjugates $\Rightarrow \lvert\beta\gamma\rvert = \lvert\beta\rvert.\lvert\gamma\rvert = \lvert\beta\rvert^2 = \frac{4}{\alpha}$	M1

	and $\lvert \beta \rvert = \dfrac{2}{\sqrt{\alpha}}$ (since $\alpha > 0$ given) **Answer Given**	A1	[2]
(iii)	p(2.695) = –0.003… < 0 and p(2.705) = 0.049… > 0 so $2.695 < \alpha < 2.705$ (by the "*Change-of-Sign Rule*") $\Rightarrow \alpha = 2.70$ to 3s.f. (Allow numerical methods, e.g. *Newton-Raphson* – to find $\alpha = 2.70$ to 3s.f.)	B1	
	$\alpha + \beta + \gamma = 4 \Rightarrow \beta + \gamma = 4 - \alpha$ Then $2.695 < \alpha < 2.705 \Rightarrow 1.295 < 4 - \alpha < 1.305$	M1	
	Now, if $\beta = u + \mathrm{i}v$, then $\gamma = u - \mathrm{i}v$ and $\beta + \gamma = 2u$, i.e. 2 Re(β) Thus $1.295 < 2\,\mathrm{Re}(\beta) < 1.305 \Rightarrow 0.6475 < \mathrm{Re}(\beta) < 0.6525$	M1	
	and Re(β) = 0.65 to 2 s.f. (A0 for failure to justify 2s.f. accuracy properly)	A1	[4]

12	(i)	**(a)** $I_1 = \int_0^2 x\sqrt{1+2x^2}\,\mathrm{d}x = \left[\tfrac{1}{6}(1+2x^2)^{\frac{3}{2}}\right]_0^2 = \tfrac{13}{3}$	M1 A1 A1 [3]
		(b) $I_n = \int_0^2 x^{n-1}.x\sqrt{1+2x^2}\,\mathrm{d}x$ Correct splitting and use of parts	M1
		$= \left[x^{n-1}.\frac{1}{6}(1+2x^2)^{\frac{3}{2}}\right]_0^2 - \int_0^2 (n-1)x^{n-2}.\frac{1}{6}(1+2x^2)^{\frac{3}{2}}\,\mathrm{d}x$	A1 A1
		$= 2^{n-1}.\frac{27}{6} - 0 \qquad -\frac{1}{6}(n-1)\int_0^2 x^{n-2}.(1+2x^2)\sqrt{1+2x^2}\,\mathrm{d}x$	M1
		$= 2^{n-1}.\frac{27}{6} - \frac{1}{6}(n-1)\{2I_n + I_{n-2}\}$	A1
		$\Rightarrow 6\,I_n = 27\times 2^{n-1} - 2(n-1)I_n - (n-1)I_{n-2}$	
		$\Rightarrow (2n+4)\,I_n = 27\times 2^{n-1} - (n-1)I_{n-2}$ **Answer Given**	A1 [6]
		(c) $I_0 = \int_0^2 \sqrt{1+2x^2}\,\mathrm{d}x$	
		Let $x\sqrt{2} = \sinh\theta$ ($\sqrt{2}\,\mathrm{d}x = \cosh\theta\,\mathrm{d}\theta$, $\sqrt{1+2x^2} = \cosh\theta$)	M1
		full substitution $I_0 = \frac{1}{\sqrt{2}}\int\cosh^2\theta\,\mathrm{d}\theta$ (Ignore limits for now)	M1 A1
		trig. identity $= \frac{1}{2\sqrt{2}}\int(1+\cosh 2\theta)\,\mathrm{d}\theta$	M1
		$= \frac{1}{2\sqrt{2}}\left[\theta + \frac{1}{2}\sinh 2\theta\right]$ **ft** ∫n. of $a + b\cosh 2\theta$ only	A1
		(0, 2) → (0, $\sinh^{-1} 2\sqrt{2}$) Limits properly dealt with	M1
		$= \frac{1}{2\sqrt{2}}\left[\sinh^{-1} 2\sqrt{2} + 6\sqrt{2}\right]$	
		$\sinh^{-1} 2\sqrt{2} = \ln\left(3+2\sqrt{2}\right)$ to at least here $= \ln\left(1+\sqrt{2}\right)^2 = 2\ln\left(1+\sqrt{2}\right)$	M1
		$I_0 = 3 + \frac{1}{\sqrt{2}}\ln\left(1+\sqrt{2}\right)$ legitimately **Answer Given**	A1 [8]

	ALTERNATIVE (partial)	
	Let $x\sqrt{2}=\tan\theta$ $\left(\sqrt{2}\,\mathrm{d}x=\sec^2\theta\,\mathrm{d}\theta\right)$	M1
	full substitution $I_0=\frac{1}{\sqrt{2}}\int\sec^3\theta\,\mathrm{d}\theta$ (Ignore limits for now)	M1 A1
	use of ∫n. by parts on $\int\sec\theta\sec^2\theta\,\mathrm{d}\theta$ and use of $\tan^2\theta=\sec^2\theta-1$	M1
	$2\sqrt{2}I_0=\sec\theta\tan\theta+\ln\lvert\sec\theta+\tan\theta\rvert$	A1
	Further progress (limits, etc.) essentially impossible	
	ALTERNATIVE	
	$I_0=\int_0^2\sqrt{1+2x^2}\times 1\,\mathrm{d}x=x\sqrt{1+2x^2}-\int\frac{1}{2}\left(1+2x^2\right)^{-\frac{1}{2}}\,.\,4x\,.\,x\,\mathrm{d}x$ ∫n. by parts	M1 A1
	$=x\sqrt{1+2x^2}-\int\frac{\left(2x^2+1\right)-1}{\sqrt{1+2x^2}}\mathrm{d}x$	M1
	$=x\sqrt{1+2x^2}-I_0+\int\frac{1}{\sqrt{1+2x^2}}\mathrm{d}x$	M1
	$\Rightarrow 2I_0=x\sqrt{1+2x^2}+\int\frac{1}{\sqrt{1+2x^2}}\mathrm{d}x$	
	$\Rightarrow I_0=\frac{1}{2}\left(x\sqrt{1+2x^2}+\frac{1}{\sqrt{2}}\sinh^{-1}\left(x\sqrt{2}\right)\right)$ (from MF20)	M1 A1
	Use of limits (0, 2) $\Rightarrow I_0=3+\frac{1}{2\sqrt{2}}\sinh^{-1}(2\sqrt{2})$	
	$\sinh^{-1}2\sqrt{2}=\ln\left(3+2\sqrt{2}\right)=\ln\left(1+\sqrt{2}\right)^2=2\ln\left(1+\sqrt{2}\right)$	M1
	$I_0=3+\frac{1}{\sqrt{2}}\ln\left(1+\sqrt{2}\right)$ legitimately **Answer Given**	A1 [8]
(ii)	$y=\frac{1}{\sqrt{2}}x^2\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=x\sqrt{2}$ and $S=2\pi\int_0^2\frac{x^2}{\sqrt{2}}.\sqrt{1+2x^2}\,\mathrm{d}x$	M1
	$=\pi\sqrt{2}\left(I_2\right)$	A1
	use of R.F. for $n=2$: $8I_2=54-I_0$	M1
	use of *their* **(i)** result $=51-\frac{1}{\sqrt{2}}\ln\left(1+\sqrt{2}\right)$	M1
	$\Rightarrow S=\frac{\pi\sqrt{2}}{8}\left(51-\frac{1}{\sqrt{2}}\ln\left(1+\sqrt{2}\right)\right)$ or $\frac{\pi}{8}\left(51\sqrt{2}-\ln\left(1+\sqrt{2}\right)\right)$	A1 [5]

13 **(i)**	 $\tan A = \frac{a}{1}$, $\tan B = \frac{1}{a}$ $\Rightarrow \tan^{-1} a + \tan^{-1} \frac{1}{a} = A + B = \frac{\pi}{2}$	B1 [1]
(ii)	$\tan\left(\tan^{-1} a \pm \tan^{-1} b\right) = \frac{\tan\left(\tan^{-1} a\right) \pm \tan\left(\tan^{-1} b\right)}{1 \mp \tan\left(\tan^{-1} a\right) \tan\left(\tan^{-1} b\right)} = \frac{a \pm b}{1 \mp ab}$	M1 A1 [2]
(iii)	$\tan^{-1}\left(\frac{1}{n-1}\right) - \tan^{-1}\left(\frac{1}{n+1}\right) = \tan^{-1}\left(\frac{2}{n^2}\right)$ noted at any stage	M1 A1
	$\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right) = \tan^{-1}(2) + \tan^{-1}\left(\frac{1}{2}\right) + \sum_{n=3}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right)$ Splitting off 1st 2 terms $= \frac{\pi}{2} + \sum_{n=3}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right)$ using (i)'s result	B1
	use of difference method (finite or infinite series)	M1
	$\Rightarrow \sum_{n=1}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right) = \frac{\pi}{2} +$ $\left(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{6} + \tan^{-1}\frac{1}{7} \ldots\right)$ $-\left(\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{6} + \tan^{-1}\frac{1}{7} + \ldots\right)$ Cancelling of terms made clear	M1
	$= \frac{\pi}{2} + \left(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}\right)$	A1
	and $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\cdot\frac{1}{3}}\right) = \tan^{-1} 1 = \frac{\pi}{4}$ using (ii)'s result	B1
	leading to … **Answer Given**	[7]

	ALTERNATIVE	
	$\tan^{-1}\left(\frac{1}{n-1}\right)-\tan^{-1}\left(\frac{1}{n+1}\right)=\tan^{-1}\left(\frac{2}{n^2}\right)$ noted at any stage	M1 A1
	Use of difference method (finite or infinite series)	M1
	$\sum_{n=1}^{N}\tan^{-1}\left(\frac{2}{n^2}\right)=\left(\tan^{-1}\frac{1}{0}+\tan^{-1}1+\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}+...\right)$ $-\left(\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}+...\right)$	A1
	Cancelling of remaining terms made clear	M1
	$\tan^{-1}\infty+\tan^{-1}1=\frac{\pi}{2}+\frac{\pi}{4}$	A1
	$=\frac{3\pi}{4}$ **Given Answer**	A1 [7]