Cambridge Pre-U

Cambridge International Examinations
Cambridge Pre-U Certificate

FURTHER MATHEMATICS (SHORT COURSE) **1348/01**

Paper 1 Further Pure Mathematics **May/June 2015**

3 hours

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Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

The syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document consists of **4** printed pages.

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CAMBRIDGE International Examinations

1 Determine the volume of tetrahedron $OABC$, where O is the origin and A, B and C are, respectively, the points $(2, 3, -2)$, $(2, 0, 4)$ and $(6, 1, 7)$. [3]

2 The Taylor series expansion, about $x = 1$, of the function y is

$$y = 1 + \sum_{n=1}^{\infty} \frac{(-2)^{n-1}(x-1)^n}{1 \times 3 \times 5 \times \ldots \times (2n-1)}.$$

Find the value of $\dfrac{\mathrm{d}^4 y}{\mathrm{d}x^4}$ when $x = 1$. [3]

3 $\mathbf{M}$ is the matrix $\begin{pmatrix} 1 & -2 & 2 \\ 2 & -1 & 2 \\ 2 & -2 & 3 \end{pmatrix}$. Use induction to prove that, for all positive integers n,

$$\mathbf{M}^n \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2n+1 \\ 2n^2+2n \\ 2n^2+2n+1 \end{pmatrix}.$$ [6]

4 A curve has polar equation $r = \sin \frac{1}{2}\theta$ for $0 \leqslant \theta < 2\pi$.

(i) Sketch the curve. [3]

(ii) Determine the area of the region enclosed by the curve. [4]

5 A curve has equation $y = \dfrac{2x^2 + 5x - 25}{x - 3}$.

(i) Determine the equations of the asymptotes. [3]

(ii) Find the coordinates of the turning points. [5]

(iii) Sketch the curve. [3]

6 **(i)** Given the complex number $z = \cos\theta + \mathrm{i}\sin\theta$, show that $z^n + \dfrac{1}{z^n} = 2\cos n\theta$. [1]

(ii) Deduce the identity $16\cos^5\theta \equiv \cos 5\theta + 5\cos 3\theta + 10\cos\theta$. [4]

(iii) For $0 < \theta < 2\pi$, solve the equation $\cos 5\theta + 5\cos 3\theta + 9\cos\theta = 0$. [4]

7 **(i)** On an Argand diagram, shade the region whose points satisfy

$$|z - 20 + 15\mathrm{i}| \leqslant 7.$$ [3]

(ii) The complex number z_1 represents that value of z in the region described in part **(i)** for which $\arg(z)$ is least. Mark z_1 on your Argand diagram and determine $\arg(z_1)$ correct to 3 decimal places. [4]

8 The group G, of order 8, consists of the elements $\{e, a, b, c, ab, bc, ca, abc\}$, together with a multiplicative binary operation, where e is the identity and

$$a^2 = b^2 = c^2 = e, \quad ab = ba, \quad bc = cb \quad \text{and} \quad ca = ac.$$

(i) Construct the group table of G. [You are not required to show how individual elements of the table are determined.] [4]

(ii) List all the proper subgroups of G. [5]

9 The differential equation $(*)$ is

$$\frac{\mathrm{d}^2u}{\mathrm{d}x^2} + 4u = 8x + 1.$$

(i) Find the general solution of $(*)$. [5]

(ii) The differential equation $(**)$ is

$$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 4xy = 8x + 1.$$

By using the substitution $u = xy$, show that $(*)$ becomes $(**)$ and deduce the general solution of $(**)$. [4]

10 **(i)** Find a vector equation for the line of intersection of the planes with cartesian equations

$$x + 7y - 6z = -10 \quad \text{and} \quad 3x - 5y + 8z = 48.$$ [5]

(ii) Determine the value of k for which the system of equations

$$\begin{aligned} x + 7y - 6z &= -10 \\ 3x - 5y + 8z &= 48 \\ kx + 2y + 3z &= 16 \end{aligned}$$

does not have a unique solution and show that, for this value of k, the system of equations is inconsistent. [6]

11 **(a)** The cubic equation $x^3 + 2x^2 + 3x - 4 = 0$ has roots p, q and r. A second cubic equation has roots qr, rp and pq. Show how the substitution $y = \frac{4}{x}$ can be used to determine this second equation. Hence, or otherwise, find this equation in the form $y^3 + ay^2 + by + c = 0$. [6]

(b) The cubic equation $x^3 - 4x^2 + 5x - 4 = 0$ has roots α, β and γ. You are given that α is real and positive, and that β and γ are complex.

(i) Describe the relationship between β and γ. [1]

(ii) Explain why $|\beta| = \frac{2}{\sqrt{\alpha}}$. [2]

(iii) Verify that $\alpha = 2.70$ correct to 3 significant figures, and deduce that $\mathrm{Re}(\beta) = 0.65$ correct to 2 significant figures. [4]

12 Let $I_n = \int_0^2 x^n \sqrt{1 + 2x^2}\, dx$ for $n = 0, 1, 2, 3, \ldots$.

(i) **(a)** Evaluate I_1. [3]

(b) Prove that, for $n \geqslant 2$,

$$(2n + 4)I_n = 27 \times 2^{n-1} - (n - 1)I_{n-2}.$$ [6]

(c) Using a suitable substitution, or otherwise, show that

$$I_0 = 3 + \frac{1}{\sqrt{2}} \ln\left(1 + \sqrt{2}\right).$$ [8]

(ii) The curve $y = \frac{1}{\sqrt{2}}x^2$, between $x = 0$ and $x = 2$, is rotated through 2π radians about the x-axis to form a surface with area S. Find the exact value of S. [5]

13 **(i)** By sketching a suitable triangle, show that $\tan^{-1} a + \tan^{-1}\left(\frac{1}{a}\right) = \frac{1}{2}\pi$, for $a > 0$. [1]

(ii) Given that a and b are positive and less than 1, express $\tan(\tan^{-1} a \pm \tan^{-1} b)$ in terms of a and b. [2]

(iii) By letting $a = \frac{1}{n-1}$ and $b = \frac{1}{n+1}$, use the method of differences to prove that

$$\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right) = \frac{3}{4}\pi.$$ [7]