Cambridge International Examinations
Cambridge Pre-U Certificate

FURTHER MATHEMATICS **1348/01**

Paper 1 Further Pure Mathematics **May/June 2017**

MARK SCHEME

Maximum Mark: 120

Published

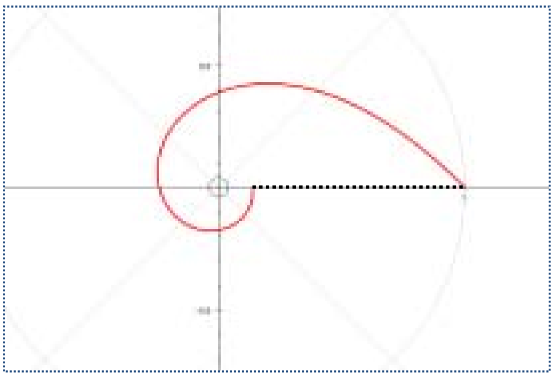
This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

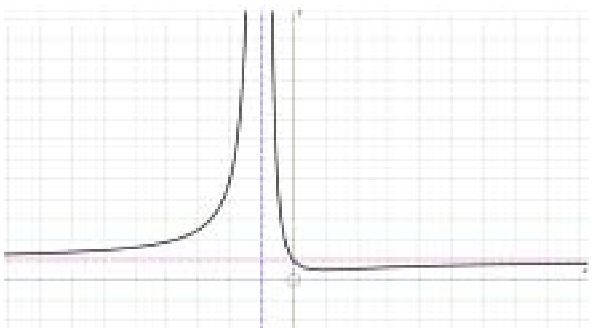
Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

Question	Answer	Marks	Part Marks
1	$(a + \mathrm{i}b)^2 = (a^2 - b^2) + \mathrm{i}.2ab$	**B1**	
	$(a^2 - b^2) = 21$ and $ab = -10$	**M1**	Comparing real and imaginary parts
	e.g. eliminating one variable and solving for the other	**M1**	Allow implied by e.g. $a = 5$, $b = 2$ (or v.v.)
	$a = \pm 5,\ b = \mp 2$	**A1**	Ignore any complex answers
2	$\Sigma\alpha = -2$ and $\Sigma\alpha\beta = 3$	**B1**	Both ($\alpha\beta\gamma = -7$ not required)
	$\alpha^2 + \beta^2 + \gamma^2 = (\Sigma\alpha)^2 - 2\Sigma\alpha\beta = -2$	**M1A1**	**FT**
	1 real and 2 complex (conjugate) roots	**B1**	Accept any comment that "not all roots are real
	Alternative Form an equation with roots $\alpha^2, \beta^2, \gamma^2$; $y^3 + 2y^2 - 19y - 49 = 0$	**M1A1**	
	$\Sigma\alpha^2 = -\dfrac{b}{a} = -2$	**B1**	**FT**
	1 real and 2 complex (conjugate) roots	**B1**	Accept any comment that "not all roots are real
3(i)		**B3**	**B1** Starts at (1, 0) **B1** Decreasing spiral **B1** All (essentially) correct
3(ii)	Area $= \frac{1}{2}\displaystyle\int_0^{2\pi} \frac{1}{(1+\theta)^2}\,\mathrm{d}\theta$	**M1**	Attempt to integrate $k(1 + \theta)^{-2}$
	$= \frac{1}{2}\left[\dfrac{-1}{1+\theta}\right]_0^{2\pi}$	**A1**	Correct integration
	$= \frac{1}{2}\left(1 - \dfrac{1}{1+2\pi}\right)$ or $\dfrac{\pi}{1+2\pi}$	**A1**	Correct answer

Question	Answer	Marks	Part Marks
4	$\dot{x} = t - \frac{1}{t}$ and $\dot{y} = 2$	B1	at least $\dot{x}$ correct
	$(\dot{x})^2 + (\dot{y})^2 = t^2 - 2 + \frac{1}{t^2} + 4$	M1	attempted
	$= \left(t + \frac{1}{t}\right)^2$	A1	Here or in the integral for S (2nd fraction of line below)
	$S = 2\pi \int_1^4 2t \, . \left(t + \frac{1}{t}\right) \mathrm{d}t$	M1	Use of formula (Ignore limits until final answer)
	$= 4\pi \int_1^4 \left(t^2 + 1\right) \mathrm{d}t$	A1	In a form ready to integrate
	$= 4\pi \left[\frac{t^3}{3} + t\right]_1^4$	B1	Correct integration (**FT** provided it is polynomial)
	$= 96\pi$	A1	
5(i)	$y = \tanh^{-1} x \Leftrightarrow \tanh y = x = \frac{\mathrm{e}^{2y} - 1}{\mathrm{e}^{2y} + 1}$	M1	
	$x\mathrm{e}^{2y} + x = \mathrm{e}^{2y} - 1 \Leftrightarrow 1 + x = \mathrm{e}^{2y}(1 - x)$	M1	Identifying e^{2y}
	$y = \tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$	A1	Legitimately obtained by taking logs Allow verification by substitution of given result
5(ii)	**Method I** $t + \frac{1}{t} = 4 \Rightarrow t^2 - 4t + 1 = 0$	M1	Creating a quadratic in tanh x
	$\Rightarrow t = 2 \pm \sqrt{3}$	M1	Solving
	Using $\frac{1}{2} \ln\left(\frac{1+t}{1-t}\right)$ with $t = 2 - \sqrt{3}$ and/or $2 + \sqrt{3}$	M1	(NB since $\lvert \tanh x \rvert < 1$, it must be $t = 2 - \sqrt{3}$)
	$x = \frac{1}{2} \ln\left(\frac{3 - \sqrt{3}}{-1 + \sqrt{3}} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}}\right) = \frac{1}{2} \ln\left(\sqrt{3}\right)$	M1	By rationalising denominator or direct observation (possibly from calculator use)
	$= \frac{1}{4} \ln(3)$	A1	Must be in this form

Question	Answer	Marks	Part Marks
5(ii)	**Method II** $\frac{\text{sh}}{\text{ch}}+\frac{\text{ch}}{\text{sh}}=4$	**M1**	
	$\Rightarrow \text{ch}^2+\text{sh}^2=4\text{sh.ch} \Rightarrow \cosh(2x)=2\sinh(2x)$	**M1**	Conversion to double-"angles"
	$\Rightarrow \tanh(2x)=\frac{1}{2}$	**A1**	
	$\Rightarrow 2x=\frac{1}{2}\ln\left(\frac{\frac{3}{2}}{\frac{1}{2}}\right)$	**M1**	Use of $\tanh^{-1}x$ formula from **(i)**
	$\Rightarrow x=\frac{1}{4}\ln(3)$	**A1**	Must be in this form
	Method III $\frac{e^{2x}-1}{e^{2x}+1}+\frac{e^{2x}+1}{e^{2x}-1}=4$	**M1**	
	$\Rightarrow (e^{2x}-1)^2+(e^{2x}+1)^2=4(e^{2x}-1)(e^{2x}+1)$	**M1**	
	$\Rightarrow e^{4x}-2e^{2x}+1+e^{4x}+2e^{2x}+1=4(e^{4x}-1)$	**A2**	**A1** LHS **A1** RHS
	$\Rightarrow 6=2e^{4x} \Rightarrow x=\frac{1}{4}\ln(3)$	**A1**	Must be in this form
6(i)	HA $y=1$ VA $x=-1$	**B2**	**B1** for each
6(ii)	$y=\frac{x^2+1}{(x+1)^2}$ or $y=1-\frac{2x}{(x+1)^2}$ $\Rightarrow \frac{dy}{dx}=\frac{(x+1)^2(2x)-(x^2+1).2(x+1)}{(x+1)^4}$ or $-\frac{(x+1)^2.2-2x.2(x+1)}{(x+1)^4}=\frac{2(x-1)}{(x+1)^3}$	**M1A1**	Attempted; correct unsimplified
	$\Rightarrow \frac{dy}{dx}=0$ when $x=1,\ y=\frac{1}{2}$	**A2**	**A1** for each
6(iii)		**3**	**G1** for graph in 2 bits, separated by a (**FT**) vertical asymptote and all positive **G1** for y-intercept at (0, 1) and MIN. in (approx. **FT**) correct place **G1** for correct asymptotic behaviour

Question	Answer	Marks	Part Marks
7(i)	$y = kx\sin 2x \Rightarrow \frac{dy}{dx} = 2kx\cos 2x + k\sin 2x$	M1	attempt using the *Product Rule*
	and $\frac{d^2y}{dx^2} = kx.-4\sin 2x + 2k\cos 2x + 2k\cos 2x$	M1	attempt using the *Product Rule*
	$= -4y + 4k\cos 2x$	M1	for substn. into given d.e. or comparison
	$\Rightarrow k = 2$	A1	
7(ii)	Comp. Fn. from $m^2 + 4 = 0$	M1	
	$\Rightarrow y_C = A\cos 2x + B\sin 2x$	A1	Or $R\cos(2x - \alpha)$ etc.
	Gen. Soln. is thus $y = A\cos 2x + (B + 2x)\sin 2x$	B1	**FT**
	Then $\frac{dy}{dx} = -2A\sin 2x + 2(B + 2x)\cos 2x + 2\sin 2x$ OR $= 2(B + 2x)\cos 2x$ if found after A (correctly) evaluated	B1	
	Subst[g]. in given initial conditions	M1	
	$A = 1$ from $x = 0, y = 1$	A1	**FT** from an incorrect $x\sin 2x$ term in y
	$B = \frac{1}{2}$ from $x = 0, \frac{dy}{dx} = 1$ i.e. soln. is $y = \cos 2x + (2x + \frac{1}{2})\sin 2x$	A1	**FT** from an incorrect $x\cos 2x$ term in y' Withhold final A mark if in e^complex form
8(i)(a)	$\cos\theta = \frac{12+2+6}{3\times 7} = \frac{20}{21}$	M1A2	**A1** scalar product; **A1** both moduli Give B1s for correct scalar product; both moduli if $\sin\theta = \ldots$ used
8(i)(b)	Subst[g]. $(2\lambda, -\lambda, 2\lambda)$ into $6x - 2y + 3z = 35$	M1	
	$\Rightarrow \lambda = \frac{7}{4} \Rightarrow \mathbf{p} = \frac{7}{4}\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$	A1A1	Second **A1** is **FT**
8(i)(c)	SD O to $\Pi_1 = OP\cos\theta = \frac{7}{4} \times 3 \times \frac{20}{21} = 5$	M1A1	**A1FT**

Question	Answer	Marks	Part Marks
8(i)(c)	**Alternative I** $(6\lambda, -2\lambda, 3\lambda)$ in plane $\Rightarrow 36\lambda + 4\lambda + 9\lambda = 35$	M1	$\Rightarrow \lambda = \frac{5}{7}$
	$\Rightarrow$ SD $= \lambda\sqrt{6^2+2^2+3^2} = 5$ cao	A1	
	Alternative II Quote formula: SD $= \left\|\dfrac{d}{\|\mathbf{n}\|}\right\| = \dfrac{35}{\sqrt{6^2+2^2+3^2}} = 5$ cao	M1A1	
8(ii)	Similar working gives $\lambda_1 = -\frac{21}{40}$	B1	
	Planes parallel, *and on opposite sides of O*, so total distance is $3\left(\dfrac{7}{4}+\dfrac{21}{40}\right)\cos\theta = \frac{13}{2}$	M1A1	
	Alternative I Π_2 has equation $\mathbf{r}\bullet\begin{pmatrix}6\\-2\\3\end{pmatrix} = -\dfrac{21}{2}$	B1	
	$\Rightarrow$ SD to Π_2 is $-\frac{3}{2}$	B1	
	Planes parallel, *and on opposite sides of O*, so distance between them is $5 - -\frac{3}{2} = \frac{13}{2}$	B1	FT
	Alternative II Quote Sh. Dist. formula for $P\left(\frac{7}{4}, -\frac{7}{2}, \frac{7}{2}\right)$ to Π_2	M1	or using distance from any point in Π_1 or Π_2 to other plane
	SD $= \left\|\dfrac{12(\frac{7}{4})-4(-\frac{7}{2})+6(\frac{7}{4})+21}{\sqrt{12^2+4^2+6^2}}\right\| = \dfrac{91}{14} = \frac{13}{2}$	A1A1	
9(i)	Full elimination of x: $I = \int \dfrac{1}{\cosh^2\theta.\sinh\theta}.\sinh\theta\, d\theta$	M1	
	$\Rightarrow I = \int \text{sech}^2\theta\ d\theta$	A1	
	$= \tanh\theta\ (+C)$	A1	
	$= \dfrac{\sqrt{x^2-1}}{x}\ (+C)$ from $\dfrac{\sinh\theta}{\cosh\theta}$	A1	(AG)

Question	Answer	Marks	Part Marks
9(ii)	$\sec y = x \Rightarrow \sec y \tan y \dfrac{\mathrm{d}y}{\mathrm{d}x} = 1$	M1A1	
	Use of $\tan y = \sqrt{\sec^2 y - 1}$	M1	
	to get $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{x\sqrt{x^2-1}}$	A1	AG Ignore lack of reason for taking the +ve sq.rt. (e.g. from +ve gradient of $\sec^{-1}$ curve)
9(iii)	$\int \sec^{-1} x \, . \dfrac{1}{x^2} \mathrm{d}x$ $= \sec^{-1} x \, . \dfrac{-1}{x} - \int \dfrac{-1}{x} \, . \dfrac{1}{x\sqrt{x^2-1}} \mathrm{d}x$ $= \dfrac{-\sec^{-1} x}{x} + \int \dfrac{1}{x^2\sqrt{x^2-1}} \mathrm{d}x$	M1A2	By parts
	$= \dfrac{-\sec^{-1} x}{x} + \dfrac{\sqrt{x^2-1}}{x} \; (+\,C)$	A1	using **(i)**
	Alternative Use $u = \sec^{-1} x \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{1}{x\sqrt{x^2-1}}$ $\Rightarrow \sec u \tan u \mathrm{d}u = \mathrm{d}x$	M1	
	$\Rightarrow \int \sec^{-1} x \, . \dfrac{1}{x^2} \mathrm{d}x = \int u \sin u \, \mathrm{d}u$	A1	
	2-stage integration by parts: $\int u \sin u \, \mathrm{d}u = -u \cos u + \int \cos u \, \mathrm{d}u$ $= -u \cos u + \sin u \; (+C)$	M1	
	Correctly turning this back into $= \dfrac{-\sec^{-1} x}{x} + \dfrac{\sqrt{x^2-1}}{x} \; (+\,C)$	A1	
10(i)	$\dfrac{1}{(k-1)k(k+1)} \equiv \dfrac{A}{k-1} + \dfrac{B}{k} + \dfrac{C}{k+1}$	M1	Correct form
	Equating terms / substn. / cover-up	M1	Method for determining constants
	$\equiv \dfrac{\frac{1}{2}}{k-1} - \dfrac{1}{k} + \dfrac{\frac{1}{2}}{k+1}$	A1	

Question	Answer	Marks	Part Marks
10(ii)	$\sum_{k=3}^{n}\frac{1}{(k-1)k(k+1)} \equiv \frac{1}{2}\sum_{k=3}^{n}\frac{1}{k-1}+\frac{1}{2}\sum_{k=3}^{n}\frac{1}{k+1}-\sum_{k=3}^{n}\frac{1}{k}$	M1	Splitting up
	$\equiv \frac{1}{2}\left\{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{n-1}\right\}+\frac{1}{2}\left\{\frac{1}{4}+\ldots+\frac{1}{n-1}+\frac{1}{n}+\frac{1}{n+1}\right\}$ $-\left\{\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{n-1}+\frac{1}{n}\right\}$	M1	Attempt at cancelling of terms
	$\equiv \frac{1}{2}\left\{\frac{1}{2}+\frac{1}{3}\right\}+\frac{1}{2}\left\{\frac{1}{n}+\frac{1}{n+1}\right\}-\left\{\frac{1}{3}+\frac{1}{n}\right\}$	A1	Correct ones clearly identifed
	$\equiv \frac{1}{12}-\frac{1}{2}\left\{\frac{1}{n}-\frac{1}{n+1}\right\}\equiv\frac{1}{12}-\frac{1}{2n(n+1)}$	A1	Legitimately shown **(AG)**
	Limit (S_n) as $n\to\infty$ is $S=\frac{1}{12}$	B1	**FT**
	Alternative $\sum_{k=3}^{n}\frac{1}{(k-1)k(k+1)}\equiv\frac{1}{2}\sum_{k=3}^{n}\frac{1}{k(k-1)}-\frac{1}{2}\sum_{k=3}^{n}\frac{1}{k(k+1)}$	M1	
	$=\frac{1}{2}\left(\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\ldots+\frac{1}{n(n-1)}\right)-$ $\frac{1}{2}\left(\frac{1}{12}+\frac{1}{20}+\ldots+\frac{1}{n(n-1)}+\frac{1}{n(n+1)}\right)$	M1	Clear listing of terms
	All correct and ready to cancel	A1	
	$=\frac{1}{12}-\frac{1}{2n(n+1)}$	A1	Legitimately shown **(AG)**
	Limit (S_n) as $n\to\infty$ is $S=\frac{1}{12}$	B1	**FT**
10(iii)	$k^3>k^3-k=k(k-1)(k+1)$ $\Rightarrow \frac{1}{k^3}<\frac{1}{(k-1)k(k+1)}$	B1	
10(iv)	$\sum_{k=1}^{\infty}\frac{1}{k^3}>1+\frac{1}{8}=\frac{9}{8}=\frac{27}{24}$	B1	Given result justified
	$\sum_{k=1}^{\infty}\frac{1}{k^3}=1+\frac{1}{8}+\sum_{k=3}^{\infty}\frac{1}{k^3}<1+\frac{1}{8}+$ $\sum_{k=3}^{n}\frac{1}{(k-1)k(k+1)}$	M1	
	$=1+\frac{1}{8}+\frac{1}{12}=\frac{29}{24}$	A1	Given result justified
11(i)(a)	$\mathbf{AB}=\begin{pmatrix}ae+bg & af+bh\\ ce+dg & cf+dh\end{pmatrix}$	B1	
	det $\mathbf{A}=ad-bc$ and det $\mathbf{B}=eh-fg$	B1	

Question	Answer	Marks	Part Marks
11(i)(b)	det(**AB**) = $(ae+bg)(cf+dh)-(af+bh)(ce+dg)$ and some attempt to multiply out	M1	
	$= acef + adeh + bcfg + bdgh - acef - bceh - adfg - bdgh$ $= adeh - bceh - adfg + bcfg$ $= (ad-bc)(eh-fg)$	A1	Legitimately shown
11(ii)	*CLOSURE*: $\mathbf{A}, \mathbf{B} \in S \Rightarrow \det \mathbf{A} = \det \mathbf{B} = 1$	M1	Attempted
	and above result $\Rightarrow \det \mathbf{AB} = 1 \Rightarrow \mathbf{AB} \in S$ (*ASSOCIATIVITY*: given)	A1	Convincing
	IDENTITY: $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \in S$ since $\det \mathbf{I} = 1.1 - 0.0 = 1$	B1	Must show why $\mathbf{I} \in S$ and not just say that **I** is the identity
	INVERSES: $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in S \Rightarrow \mathbf{A}^{-1}$ $= \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \in S$	B1	for stating $\mathbf{A}^{-1}$ (or explaining that it exists)
	Since $da - (-b)(-c) = ad - bc = 1$ Hence $(S, \times_{\mathbf{M}})$ is a group, G.	B1	for justifying its membership of S
11(iii)(a)	$\det \mathbf{K} = 1.0 - \text{i.i} = -\text{i}^2 = 1$ (so $\mathbf{K} \in S$)	B1	
11(iii)(b)	Attempt at powers of **K**; $\mathbf{K}^2$ & $\mathbf{K}^3$	M1	
	$\mathbf{K}^2 = \begin{pmatrix} 0 & i \\ i & -1 \end{pmatrix}$ and $\mathbf{K}^3 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	A1	
	NB $\mathbf{K}^4 = \begin{pmatrix} -1 & -i \\ -i & 0 \end{pmatrix}$ and $\mathbf{K}^5 = \begin{pmatrix} 0 & -i \\ -i & 1 \end{pmatrix}$ $\Rightarrow \mathbf{K}^6 = \mathbf{I}$ and H has order $n = 6$	A1	
11(iii)(c)	e.g. The set of rotations about O through multiples of 60° OR $(\mathbf{K}^*)$ = group generated by $\begin{pmatrix} 1 & -i \\ -i & 0 \end{pmatrix}$	B1	**FT** for any n
	Justifying the two are isomorphic	B1	e.g. stating both are cyclic, etc.

Question	Answer	Marks	Part Marks
12(i)	**Method I** $\mathrm{F}_{n+2}(\theta) - \frac{1}{4}\sin^2(2\theta)\ \mathrm{F}_{n+1}(\theta)$ $\equiv (c^2+s^2)(c^{2n+4}+s^{2n+4})$ $-\frac{1}{4}(2sc)^2(c^{2n+2}+s^{2n+2})$	M2	**M1** all F_n terms **M1** $\sin 2\theta$ form
	$\equiv c^{2n+6}+c^2s^{2n+4}+s^2c^{2n+4}+s^{2n+6}$ $-c^2s^2(c^{2n+2}+s^{2n+2})$	A1	
	$\equiv c^{2n+6}+s^{2n+6} \equiv \mathrm{F}_{n+3}(\theta)$	A1	**AG**
	Method II $\equiv c^{2n+4}+s^{2n+4} - s^2c^2(c^{2n+2}+s^{2n+2})$	M1	Use of $\sin 2\theta$ form
	$\equiv c^{2n+4}+s^{2n+4} - s^2c^{2n+4} - c^2s^{2n+4}$	A1	
	$\equiv (1-s^2)c^{2n+4}+(1-c^2)s^{2n+4}$	M1	
	$\equiv c^{2n+6}+s^{2n+6} \equiv \mathrm{F}_{n+3}(\theta)$	A1	**AG**
12(ii)(a)	Use of $z = c + \mathrm{i}s$ and $z^{-1} = c - \mathrm{i}s$	M1	
	$z + z^{-1} = 2c$ and $z - z^{-1} = 2\mathrm{i}s$	A2	**A1** for each
12(ii)(b)	**Method I** $(2c)^6 = (z+z^{-1})^6 = z^6+6z^4+15z^2+20$ $+15z^{-2}+6z^{-4}+z^{-6}$	M1	
	$= 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$	A1	
	$-(2s)^6 = (z-z^{-1})^6 = z^6-6z^4+15z^2-20$ $+15z^{-2}-6z^{-4}+z^{-6}$ $= 2\cos 6\theta - 12\cos 4\theta + 30\cos 2\theta - 20$	B1	**FT** (Must have – sign)
	Subtracting: $64(c^6+s^6) = 12(z^4+z^{-4})+40$ $= 12\ .\ 2\cos 4\theta + 40$	M1	
	Dividing by 8: $8(c^6+s^6) = 3\cos 4\theta + 5$	A1	**AG**
	Use of $\cos 4\theta = 2\cos^2 2\theta - 1$ and $1 = \cos^2 2\theta + \sin^2 2\theta$	M1	
	$\Rightarrow c^6+s^6 = \frac{3}{8}(2\cos^2 2\theta) + (-\frac{3}{8}+\frac{5}{8})(\cos^2 2\theta + \sin^2 2\theta)$ $= \cos^2 2\theta + \frac{1}{4}\sin^2 2\theta$	A1	**AG**

Question	Answer	Marks	Part Marks
12(ii)(b)	**Method II** $\cos 4\theta = \text{Re}(c + \text{i}s)^4$	M1	
	$= c^4 - 6c^2s^2 + s^4 = c^4 - 6c^2(1-c^2) + (1-c^2)^2$ $= 8c^4 - 8c^2 + 1$	A1	
	$c^6 + s^6 = c^6 + (1-c^2)^3 = c^6 + 1 - 3c^2 + 3c^4 - c^6$	M1	
	$= 3c^4 - 3c^2 + 1$	A1	
	so that $8(c^6 + s^6) = 3\cos 4\theta + 5$	A1	AG
	Use of $\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$ and $1 = \cos^2 2\theta + \sin^2 2\theta$	M1	
	$\Rightarrow 8(c^6 + s^6) = 3\cos 4\theta + 5$ $= 3(\cos^2 2\theta - \sin^2 2\theta) + 5(\cos^2 2\theta + \sin^2 2\theta)$ $\Rightarrow c^6 + s^6 = \cos^2 2\theta + \frac{1}{4}\sin^2 2\theta$	A1	AG
12(iii)	Case for $n = 1$ established in (ii) (b):	B1	noted explicitly (possibly at end)
	Assume $c^{2k+4} + s^{2k+4} \leqslant \cos^2 2\theta + \frac{1}{2^{k+1}}\sin^2 2\theta$	B1	i.e. the case for $n = k$
	A clear statement of the result must be given, possibly within what follows Then $c^{2k+6} + s^{2k+6} =$ $c^{2k+4} + s^{2k+4} - \frac{1}{4}\sin^2 2\theta(c^{2k+2} + s^{2k+2})$	M1	attempt at $n = k + 1$ case using **(i)**'s identity
	$\leqslant \cos^2 2\theta + \frac{1}{2^{k+1}}\sin^2 2\theta - \frac{1}{4}\sin^2 2\theta(c^{2k+2} + s^{2k+2})$	M1	use of the induction hypothesis (i.e. the $n = k$ case)
	$= \cos^2 2\theta + \frac{1}{2^{k+2}}\sin^2 2\theta - \frac{1}{4}\sin^2 2\theta\left(c^{2k+2} + s^{2k+2} - \frac{1}{2^k}\right)$	M1A1	splitting up the $\sin^2 2\theta$ term into two equal parts
	$\leqslant \cos^2 2\theta + \frac{1}{2^{k+2}}\sin^2 2\theta$ Proof follows by induction since $\sin^2 2\theta \geqslant 0$ and given result that $c^{2k+2} + s^{2k+2} \geqslant \frac{1}{2^k}$	A1	