Cambridge International Examinations
Cambridge Pre-U Certificate

FURTHER MATHEMATICS (SHORT COURSE) **1348/01**

Paper 1 Further Pure Mathematics **May/June 2018**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

This syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document consists of **4** printed pages.

1 **(i)** Express $\dfrac{3}{(3r-1)(3r+2)}$ in partial fractions. [2]

(ii) Using the method of differences, prove that $\displaystyle\sum_{r=1}^{n} \frac{3}{(3r-1)(3r+2)} = \frac{1}{2} - \frac{1}{3n+2}$. [2]

(iii) Deduce the value of $\displaystyle\sum_{r=1}^{\infty} \frac{1}{(3r-1)(3r+2)}$. [1]

2 **(i)** Determine the asymptotes and turning points of the curve with equation $y = \dfrac{x^2+3}{x+1}$. [7]

(ii) Sketch the curve. [3]

3 The complex numbers z_1 and z_2 are such that $|z_1| = 2$, $\arg(z_1) = \frac{7}{12}\pi$, $|z_2| = \sqrt{2}$ and $\arg(z_2) = -\frac{1}{8}\pi$.

(i) Find, in exact form, the modulus and argument of $\dfrac{z_1}{z_2}$. [3]

(ii) Let $z_3 = \left(\dfrac{z_1}{z_2}\right)^n$. It is given that n is the least positive integer for which z_3 is a positive real number. Find this value of n and the exact value of z_3. [4]

4 A curve has polar equation $r = \frac{3}{10}e^{\frac{3}{4}\theta}$ for $\theta \geqslant 0$. The length of the arc of this curve between $\theta = 0$ and $\theta = \alpha$ is denoted by $\mathrm{L}(\alpha)$.

(i) Show that $\mathrm{L}(\alpha) = \frac{1}{2}\left(e^{\frac{3}{4}\alpha} - 1\right)$. [5]

(ii) The point P on the curve corresponding to $\theta = \beta$ is such that $\mathrm{L}(\beta) = OP$, where O is the pole. Find the value of β. [2]

5 Find, in the form $y = \mathrm{f}(x)$, the solution of the differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}x} + y\tanh x = 2\cosh x$, given that $y = \frac{3}{4}$ when $x = \ln 2$. [8]

6 The cubic equation $4x^3 - 12x^2 + 9x - 16 = 0$ has roots r_1, r_2 and r_3. A second cubic equation, with integer coefficients, has roots $R_1 = \dfrac{r_2+r_3}{r_1}$, $R_2 = \dfrac{r_3+r_1}{r_2}$ and $R_3 = \dfrac{r_1+r_2}{r_3}$.

(i) Show that $1 + R_1 = \dfrac{3}{r_1}$ and write down the corresponding results for the other roots. [2]

(ii) Using a substitution based on this result, or otherwise, find this second cubic equation. [6]

7 The function y satisfies $\dfrac{d^2y}{dx^2} + x^2y = x$, and is such that $y = 1$ and $\dfrac{dy}{dx} = 1$ when $x = 1$.

(i) Using the given differential equation

(a) state the value of $\dfrac{d^2y}{dx^2}$ when $x = 1$, [1]

(b) find, by differentiation, the value of $\dfrac{d^3y}{dx^3}$ when $x = 1$. [2]

(ii) Hence determine the Taylor series for y about $x = 1$ up to and including the term in $(x-1)^3$ and deduce, correct to 4 decimal places, an approximation for y when $x = 1.1$. [3]

8 **(i)** Write down the values of the constants a and b for which $m^5 \equiv \frac{1}{6}m^3(am^2 + 2) - \frac{1}{12}m^2(bm)$. [1]

(ii) Prove by induction that $\displaystyle\sum_{r=1}^{n} r^5 = \frac{1}{6}n^3(n+1)^3 - \frac{1}{12}n^2(n+1)^2$ for all positive integers n. [7]

9 **(i)** Use de Moivre's theorem to prove that $\cos 3\theta = 4c^3 - 3c$, where $c = \cos\theta$. [3]

(ii) Solve the equation $2\cos 3\theta - \sqrt{3} = 0$ for $0 < \theta < \pi$, giving each answer in an exact form. [2]

(iii) Deduce, in trigonometric form, the three roots of the equation $x^3 - 3x - \sqrt{3} = 0$. [3]

10 **(i)** Let G be a group of order 10. Write down the possible orders of the elements of G and justify your answer. [2]

(ii) Let G_1 be the cyclic group of order 10 and let g be a generator of G_1 (that is, an element of order 10). List the ten elements of G_1 in terms of g and state the order of each element. [4]

(iii) The group G_2 is defined as the set of ordered pairs (x, y), where $x \in \{0, 1\}$ and $y \in \{0, 1, 2, 3, 4\}$, together with the binary operation $\oplus$ defined by

$$(x_1, y_1) \oplus (x_2, y_2) = (x_3, y_3),$$

where $x_3 = x_1 + x_2$ modulo 2 and $y_3 = y_1 + y_2$ modulo 5.

(a) List the elements of G_2 and state the order of each element. [3]

(b) State, with justification, whether G_1 and G_2 are isomorphic. [1]

11 Let $\mathbf{A}$ be the matrix $\begin{pmatrix} 17 & 12 \\ 12 & 10 \end{pmatrix}$.

(a) **(i)** Determine the integer n for which $27\mathbf{A} - \mathbf{A}^2 = n\mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix. [2]

(ii) **Hence** find $\mathbf{A}^{-1}$ in the form $p\mathbf{A} + q\mathbf{I}$ for rational numbers p and q. [2]

(b) The plane transformation T is defined by $\mathrm{T} : \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix}$. It is given that T is a stretch, with scale factor k, parallel to the line $y = mx$, where $m > 0$.

(i) Find the value of k. [2]

(ii) By considering $\mathbf{A}\begin{pmatrix} x \\ mx \end{pmatrix}$, or otherwise, determine the value of m. [4]

12 The curve C is given by $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$ for $2 \leqslant x \leqslant 8$.

(i) Find, in its simplest exact form, the length of C. [5]

(ii) When C is rotated through 2π radians about the x-axis, a surface of revolution is formed. Show that the area of this surface is $\pi\left(270 - 47\ln 2 - 2(\ln 2)^2\right)$. [10]

13 The planes Π_1 and Π_2 are both perpendicular to $\mathbf{n}$, where $\mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$. The points $A\,(0, -9, 13)$ and $B\,(8, 7, -3)$ lie in Π_1 and Π_2 respectively.

(i) Find the equations of Π_1 and Π_2 in the form $\mathbf{r}\,.\,\mathbf{n} = d$ and show that $\overrightarrow{AB}$ is parallel to $\mathbf{n}$. [4]

(ii) Calculate the perpendicular distance between Π_1 and Π_2. [2]

(iii) Write down two vectors which are perpendicular to $\mathbf{n}$ and hence find, in the form

$$\mathbf{r} = \mathbf{u} + \lambda\mathbf{v} + \mu\mathbf{w},$$

an equation for the plane Π_3 which is parallel to Π_1 and Π_2 and exactly half-way between them. [4]

(iv) The locus of all points P such that $AP = BP = 12\sqrt{2}$ is denoted by L.

(a) Give a full geometrical description of L. [4]

(b) Using the result of part **(iii)**, or otherwise, find a point on L which has integer coordinates. [4]