Cambridge Pre-U

Cambridge International Examinations
Cambridge Pre-U Certificate

0556521313

FURTHER MATHEMATICS (PRINCIPAL) **9795/02**

Paper 2 Further Applications of Mathematics **May/June 2018**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

This syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document consists of **5** printed pages and **3** blank pages.

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Section A: Probability (60 marks)

1 **(i)** The random variable X has the distribution $\mathrm{B}(200, 0.2)$. Use a suitable approximation to find $\mathrm{P}(X \leqslant 30)$. [4]

(ii) The random variable Y has the distribution $\mathrm{B}(200, 0.02)$. Use a suitable approximation to find $\mathrm{P}(Y \leqslant 3)$. [3]

2 Secret radio messages received under difficult conditions are subject to errors caused by random instantaneous breaks in transmission. The number of errors caused by breaks in transmission in a 10-minute period is denoted by B.

(i) State two conditions, other than randomness, needed for a Poisson distribution to be a suitable model for B. [2]

Assume now that $B \sim \mathrm{Po}(5)$.

(ii) Calculate the probability that in a 15-minute period there are between 6 and 10 errors, inclusive, caused by random breaks in transmission. [3]

Secret radio messages are also subject to errors caused by mistakes made by the sender. The number of errors caused by mistakes made by the sender in a 10-minute period, M, has the independent distribution $\mathrm{Po}(8)$.

(iii) Calculate the period of time, in seconds, for which the probability that a message contains no errors of either sort is 0.6. [4]

3 The moment generating function of a random variable X is $(1-2t)^{-3}$.

(i) Find the mean and variance of X. [5]

(ii) X_1 and X_2 are two independent observations of X. Find $\mathrm{E}\left[(X_1 + X_2)^3\right]$. [4]

4 The continuous random variable X has cumulative distribution function given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0, \\ \frac{1}{8}x^3 & 0 \leqslant x \leqslant 2, \\ 1 & x > 2. \end{cases}$$

(i) Find $\mathrm{E}(X)$. [3]

(ii) Find the probability density function of Y, where $Y = \dfrac{1}{X^2}$. [6]

5 A random sample of 12 seventeen-year-old boys and a random sample of 14 seventeen-year-old girls were given a certain task. The times, t minutes, taken to complete the task by the members of the two samples are summarised as follows.

	n	Σt	Σt^2
Boys	12	204	4236
Girls	14	312	7126

(i) Stating any necessary assumption(s), find a 95% symmetric confidence interval for the difference in the average times taken to complete the task by seventeen-year-old boys and seventeen-year-old girls. [9]

(ii) State with a reason whether the confidence interval calculated in part **(i)** suggests that there may in fact be no difference in the average times taken by seventeen-year-old boys and by seventeen-year-old girls. [1]

6 In a certain city there are N taxis. Each taxi displays a different licensing number which is an integer in the range 1 to N. A visitor to the city attempts to estimate the value of N, assuming that the licensing number of each taxi observed is equally likely to be any integer from 1 to N inclusive.

(i) The visitor observes one randomly chosen licensing number, X. Using standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that $\mathrm{E}(X) = \frac{1}{2}(N+1)$ and $\mathrm{Var}(X) = \frac{1}{12}(N^2-1)$. [6]

The mean of 40 independent observations of X is denoted by A.

(ii) Find an unbiased estimator E_1 of N based on A, and state the approximate distribution of E_1, giving the value(s) of any parameter(s). [5]

B is another random variable based on a random sample of 40 independent observations of X. It is given that $\mathrm{E}(B) = \frac{40}{27}N$ and that $\mathrm{Var}(B) = \alpha N^2$ where α is a constant.

(iii) Find an unbiased estimator E_2 of N based on B, and determine the set of values of α for which $\mathrm{Var}(E_2) > \mathrm{Var}(E_1)$ for all values of N. [5]

Section B: Mechanics (60 marks)

7 A car has mass 800 kg.

(i) The car accelerates from $10\,\mathrm{m\,s^{-1}}$ to $20\,\mathrm{m\,s^{-1}}$ in climbing a hill with a vertical height of 16 m. Ignoring resistive forces, find the work done by the engine. [3]

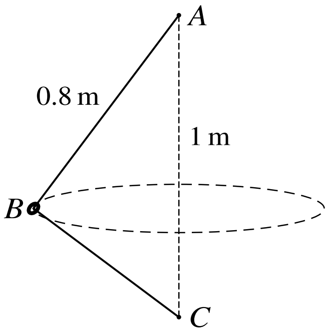
(ii) The engine produces a constant power output of 189 kW. The car now travels along horizontal ground. Modelling the resistive force as $7v^2$ N, where $v\,\mathrm{m\,s^{-1}}$ is the speed, find the value of v for which the speed of the car is constant. [3]

8 A light elastic string of natural length 0.2 m and modulus of elasticity 8 N has one end fixed to a point P on a horizontal ceiling. A particle of mass 0.4 kg is attached to the other end of the string.

(i) Find the extension of the string when the particle hangs in equilibrium vertically below P. [2]

(ii) The particle is held at rest, with the string stretched, at a point x m vertically below P and is then released. Find the smallest value of x for which the particle will reach the ceiling. [5]

9



A light inextensible string of length 1.4 m has its ends attached to two points A and C, where A is 1 m vertically above C. A smooth bead B of mass 0.2 kg is threaded on the string and rotates in a horizontal circle with the string taut. The distance BA is 0.8 m (see diagram). Find

(i) the tension in the string, [4]

(ii) the time taken for the bead to perform one complete circle. [5]

10 A particle P is attached to one end of a light inextensible string of length 1.4 m. The other end of the string is fixed to the ceiling at C. The angle between CP and the vertical is θ radians. The particle is held with the string taut with $\theta = 0.3$ and is then released.

(i) **(a)** Show that the motion of the system is approximately simple harmonic, and state its period. [4]

(b) Hence find an approximation for the speed of P when $\theta = 0.2$. [4]

(ii) Find the speed of P when $\theta = 0.2$ using an energy method, and hence find the percentage error in the answer to part **(i) (b)**. [4]

11 A particle of mass 0.2 kg is projected so that it hits a smooth sloping plane Π that makes an angle of $\sin^{-1} 0.6$ above the horizontal. The path of the particle is in a vertical plane containing a line of greatest slope of Π. Immediately before the first impact between the particle and Π, the particle is moving horizontally with speed $10\,\mathrm{m\,s^{-1}}$. The coefficient of restitution between the particle and Π is 0.5.

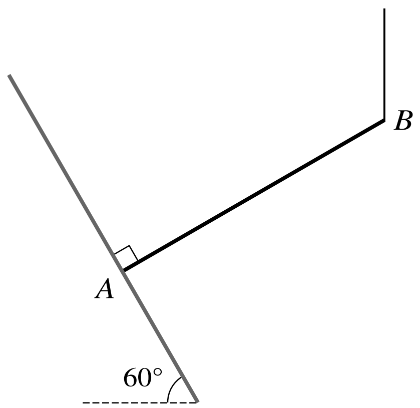
(i) Find the magnitude of the impulse on the particle from Π at the first impact, and state the direction of this impulse. [5]

(ii) Find the distance between the points on Π where the first and second impacts occur. [4]

(iii) Find the time taken between the first and third impacts. [4]

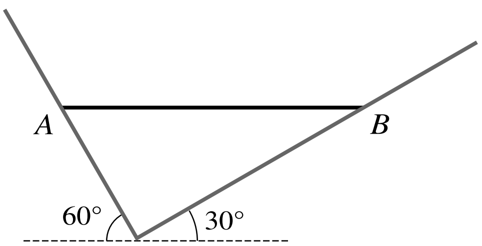
12 A uniform rod AB has mass 5 kg and length 4 m.

(i)



The rod rests with A on a rough plane that makes an angle of 60° to the horizontal. A string is attached to B and the rod is in equilibrium in the vertical plane containing the line of greatest slope of the plane, with the string vertical and AB perpendicular to the plane (see diagram). Find the magnitude of the frictional force at A and the tension in the string. [5]

(ii)



The rod now rests horizontally with A in contact with a rough plane that makes an angle of 60° with the horizontal and B in contact with a rough plane that makes an angle of 30° with the horizontal (see diagram). The rod and the lines of greatest slope of the two planes are all in the same vertical plane. The coefficients of friction at A and B are μ_A and μ_B respectively. Friction is limiting at both A and B, with A on the point of slipping downwards. Show that $\mu_B = \dfrac{1 - \alpha\mu_A}{\alpha + \mu_A}$ where α is an irrational number to be found. [8]

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