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OCR ADVANCED SUBSIDIARY GCE
IN MATHEMATICS (3890, 3891 and 3892)

OCR ADVANCED GCE
IN MATHEMATICS (7890, 7891 and 7892)

Specimen Question Papers and Mark Schemes

These specimen question papers and mark schemes are intended to accompany the OCR Advanced Subsidiary GCE and Advanced GCE specifications in Mathematics for teaching from September 2004.

Centres are permitted to copy material from this booklet for their own internal use.

The specimen assessment material accompanying the new specifications is provided to give centres a reasonable idea of the general shape and character of the planned question papers in advance of the first operational examination.

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MATHEMATICS **4721**

Core Mathematics 1

Specimen Paper

Additional materials:
- Answer booklet
- Graph paper
- List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- **You are not permitted to use a calculator in this paper.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

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1 Write down the exact values of

(i) 4^{-2}, [1]

(ii) $(2\sqrt{2})^2$, [1]

(iii) $(1^3 + 2^3 + 3^3)^{\frac{1}{2}}$. [2]

2 **(i)** Express $x^2 - 8x + 3$ in the form $(x+a)^2 + b$. [3]

(ii) Hence write down the coordinates of the minimum point on the graph of $y = x^2 - 8x + 3$. [2]

3 The quadratic equation $x^2 + kx + k = 0$ has no real roots for x.

(i) Write down the discriminant of $x^2 + kx + k$ in terms of k. [2]

(ii) Hence find the set of values that k can take. [4]

4 Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in each of the following cases:

(i) $y = 4x^3 - 1$, [2]

(ii) $y = x^2(x^2 + 2)$, [3]

(iii) $y = \sqrt{x}$ [2]

5 **(i)** Solve the simultaneous equations

$$y = x^2 - 3x + 2, \qquad y = 3x - 7.$$ [5]

(ii) What can you deduce from the solution to part **(i)** about the graphs of $y = x^2 - 3x + 2$ and $y = 3x - 7$? [2]

(iii) Hence, or otherwise, find the equation of the normal to the curve $y = x^2 - 3x + 2$ at the point $(3, 2)$, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers. [4]

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6 **(i)** Sketch the graph of $y = \frac{1}{x}$, where $x \neq 0$, showing the parts of the graph corresponding to both positive and negative values of x. [2]

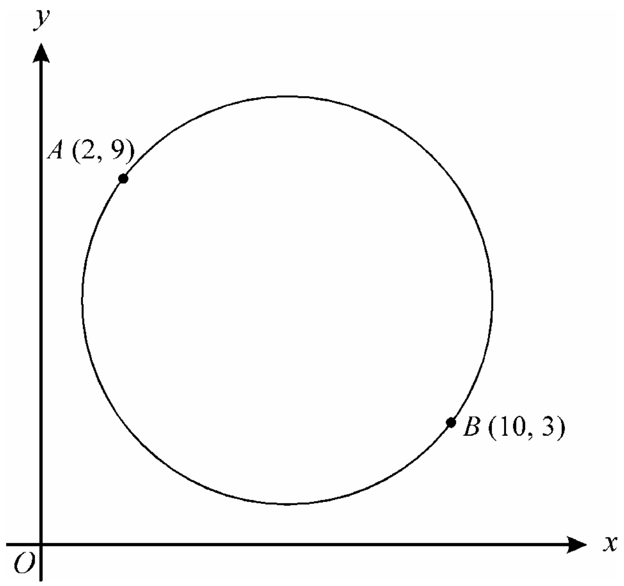
(ii) Describe fully the geometrical transformation that transforms the curve $y = \frac{1}{x}$ to the curve $y = \frac{1}{x+2}$.

Hence sketch the curve $y = \frac{1}{x+2}$. [5]

(iii) Differentiate $\frac{1}{x}$ with respect to x. [2]

(iv) Use parts **(ii)** and **(iii)** to find the gradient of the curve $y = \frac{1}{x+2}$ at the point where it crosses the y-axis. [3]

7



The diagram shows a circle which passes through the points $A\,(2, 9)$ and $B\,(10, 3)$. AB is a diameter of the circle.

(i) Calculate the radius of the circle and the coordinates of the centre. [4]

(ii) Show that the equation of the circle may be written in the form $x^2 + y^2 - 12x - 12y + 47 = 0$. [3]

(iii) The tangent to the circle at the point B cuts the x-axis at C. Find the coordinates of C. [6]

[Turn over

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8 **(i)** Find the coordinates of the stationary points on the curve $y = 2x^3 - 3x^2 - 12x - 7$. [6]

(ii) Determine whether each stationary point is a maximum point or a minimum point. [3]

(iii) By expanding the right-hand side, show that

$$2x^3 - 3x^2 - 12x - 7 = (x+1)^2(2x-7).$$ [2]

(iv) Sketch the curve $y = 2x^3 - 3x^2 - 12x - 7$, marking the coordinates of the stationary points and the points where the curve meets the axes. [3]

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MATHEMATICS **4721**

Core Mathematics 1

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

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1	**(i)** $\frac{1}{16}$	B1	**1**	For correct value (fraction or exact decimal)
	(ii) 8	B1	**1**	For correct value 8 only
	(iii) 6	M1		For $1^3+2^3+3^3=36$ seen or implied
		A1	**2**	For correct value 6 only
			$\boxed{4}$	
2	**(i)** $x^2-8x+3=(x-4)^2-13$	B1		For $(x-4)^2$ seen, or statement $a=-4$
	i.e. $a=-4, b=-13$	M1		For use of (implied) relation $a^2+b=3$
		A1	**3**	For correct value of b stated or implied
	(ii) Minimum point is $(4, -13)$	B1√		For x-coordinate equal to their $(-a)$
		B1√	**2**	For y-coordinate equal to their b
			$\boxed{5}$	
3	**(i)** Discriminant is k^2-4k	M1		For attempted use of the discriminant
		A1	**2**	For correct expression (in any form)
	(ii) For no real roots, $k^2-4k<0$	M1		For stating their $\Delta<0$
	Hence $k(k-4)<0$	M1		For factorising attempt (or other soln method)
	So $0<k<4$	A1		For both correct critical values 0 and 4 seen
		A1	**4**	For correct pair of inequalities
			$\boxed{6}$	
4	**(i)** $\frac{dy}{dx}=12x^2$	M1		For clear attempt at nx^{n-1}
		A1	**2**	For completely correct answer
	(ii) $y=x^4+2x^2$	B1		For correct expansion
	Hence $\frac{dy}{dx}=4x^3+4x$	M1		For correct differentiation of at least one term
		A1√	**3**	For correct differentiation of their 2 terms
	(iii) $\frac{dy}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$	M1		For clear differentiation attempt of $x^{\frac{1}{2}}$
		A1	**2**	For correct answer, in any form
			$\boxed{7}$	
5	**(i)** $x^2-3x+2=3x-7 \Rightarrow x^2-6x+9=0$	M1		For equating two expressions for y
		A1		For correct 3-term quadratic in x
	Hence $(x-3)^2=0$	M1		For factorising, or other solution method
	So $x=3$ and $y=2$	A1		For correct value of x
		A1	**5**	For correct value of y
	(ii) The line $y=3x-7$ is the tangent to the curve	B1		For stating tangency
	$y=x^2-3x+2$ at the point $(3, 2)$	B1	**2**	For identifying $x=3$, $y=2$ as coordinates
	(iii) Gradient of tangent is 3	B1		For stating correct gradient of given line
	Hence gradient of normal is $-\frac{1}{3}$	B1√		For stating corresponding perpendicular grad
	Equation of normal is $y-2=-\frac{1}{3}(x-3)$	M1		For appropriate use of straight line equation
	i.e. $x+3y-9=0$	A1	**4**	For correct equation in required form
			$\boxed{11}$	

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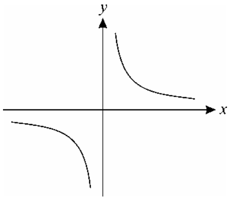
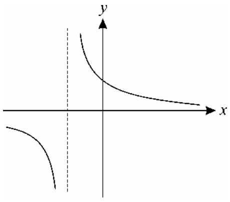
6	**(i)**		B1 B1	 **2**	For correct 1st quadrant branch For both branches correct and nothing else
	(ii)	Translation of 2 units in the negative x-direction	B1 B1 B1		For translation parallel to the x-axis For correct magnitude For correct direction
			B1√ B1	 **5**	For correct sketch of new curve For some indication of location, e.g. $\frac{1}{2}$ at y-intersection or -2 at asymptote
	(iii)	Derivative is $-x^{-2}$	M1 A1	 **2**	For correct power -2 in answer For correct coefficient -1
	(iv)	Gradient of $y=\dfrac{1}{x}$ at $x=2$ is required	B1		For correctly using the translation
		This is -2^{-2}, which is $-\frac{1}{4}$	M1		For substituting $x=2$ in their **(iii)**
			A1	**3**	For correct answer
				$\boxed{12}$	
7	**(i)**	$AB^2=(10-2)^2+(3-9)^2=100$	M1		For correct calculation method for AB^2
		Hence the radius is 5	A1		For correct value for radius
		Mid-point of AB is $\left(\dfrac{2+10}{2},\dfrac{9+3}{2}\right)$	M1		For correct calculation method for mid-point
		Hence centre is $(6, 6)$	A1	**4**	For both coordinates correct
	(ii)	Equation is $(x-6)^2+(y-6)^2=5^2$	M1		For using correct basic form of circle equn
		This is $x^2-12x+36+y^2-12y+36=25$	A1		For expanding at least one bracket correctly
		i.e. $x^2+y^2-12x-12y+47=0$, as required	A1	**3**	For showing given answer correctly
	(iii)	Gradient of AB is $\dfrac{3-9}{10-2}=-\dfrac{3}{4}$	M1		For finding the gradient of AB
			A1		For correct value $-\frac{3}{4}$ or equivalent
		Hence perpendicular gradient is $\dfrac{4}{3}$	A1√		For relevant perpendicular gradient
		Equation of tangent is $y-3=\frac{4}{3}(x-10)$	M1		For using their perp grad and B correctly
		Hence C is the point $(\frac{31}{4}, 0)$	M1		For substituting $y=0$ in their tangent eqn
			A1	**6**	For correct value $x=\frac{31}{4}$
				$\boxed{13}$	

[Turn over

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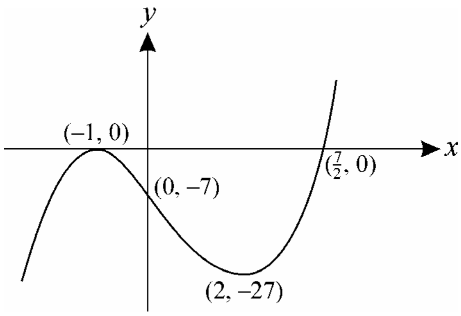
8 **(i)**	$\frac{dy}{dx} = 6x^2 - 6x - 12$	M1	For differentiation with at least 1 term OK
		A1	For completely correct derivative
	Hence $x^2 - x - 2 = 0$	M1	For equating their derivative to zero
	$(x-2)(x+1) = 0 \Rightarrow x = 2$ or -1	M1	For factorising or other solution method
		A1	For both correct x-coordinates
	Stationary points are $(2, -27)$ and $(-1, 0)$	A1 **6**	For both correct y-coordinates
(ii)	$\frac{d^2y}{dx^2} = 12x - 6 = \begin{cases} +18 \text{ when } x = 2 \\ -18 \text{ when } x = -1 \end{cases}$	M1	For attempt at second derivative and at least one relevant evaluation
	Hence $(2, -27)$ is a min and $(-1, 0)$ is a max	A1	For either one correctly identified
		A1 **3**	For both correctly identified (Alternative methods, e.g. based on gradients either side, are equally acceptable)
(iii)	$\text{RHS} = (x^2 + 2x + 1)(2x - 7)$ $= 2x^3 - 7x^2 + 4x^2 - 14x + 2x - 7$	M1	For squaring correctly and attempting complete expansion process
	$= 2x^3 - 3x^2 - 12x - 7$, as required	A1 **2**	For obtaining given answer correctly
(iv)		B1	For correct cubic shape
		B1	For maximum point lying on x-axis
		B1 **3**	For $x = \frac{7}{2}$ and $y = -7$ at intersections
		$\boxed{14}$	

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MATHEMATICS **4722**

Core Mathematics 2

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

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1 Expand $(1-2x)^4$ in ascending powers of x, simplifying the coefficients. [5]

2 **(i)** Find $\int \frac{1}{x^2}\,dx$. [3]

(ii) The gradient of a curve is given by $\frac{dy}{dx} = \frac{1}{x^2}$. Find the equation of the curve, given that it passes through the point $(1, 3)$. [3]

3 **(a)** Express each of the following in terms of $\log_2 x$:

(i) $\log_2(x^2)$, [1]

(ii) $\log_2(8x^2)$. [3]

(b) Given that $y^2 = 27$, find the value of $\log_3 y$. [3]

4 Records are kept of the number of copies of a certain book that are sold each week. In the first week after publication 3000 copies were sold, and in the second week 2400 copies were sold. The publisher forecasts future sales by assuming that the number of copies sold each week will form a geometric progression with first two terms 3000 and 2400. Calculate the publisher's forecasts for

(i) the number of copies that will be sold in the 20th week after publication, [3]

(ii) the total number of copies sold during the first 20 weeks after publication, [2]

(iii) the total number of copies that will ever be sold. [2]

5 **(i)** Show that the equation $15\cos^2\theta° = 13 + \sin\theta°$ may be written as a quadratic equation in $\sin\theta°$. [2]

(ii) Hence solve the equation, giving all values of θ such that $0 \leqslant \theta \leqslant 360$. [6]

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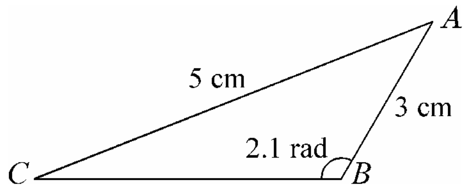


The diagram shows triangle ABC, in which $AB = 3\text{ cm}$, $AC = 5\text{ cm}$ and angle $ABC = 2.1$ radians. Calculate

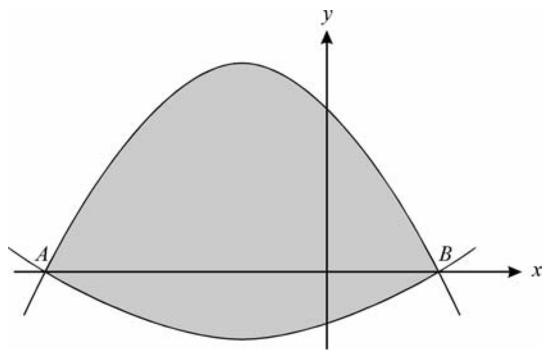
(i) angle ACB, giving your answer in radians, [2]

(ii) the area of the triangle. [3]

An arc of a circle with centre A and radius 3 cm is drawn, cutting AC at the point D.

(iii) Calculate the perimeter and the area of the sector ABD. [4]

7



The diagram shows the curves $y = -3x^2 - 9x + 30$ and $y = x^2 + 3x - 10$.

(i) Verify that the curves intersect at the points $A\,(-5, 0)$ and $B\,(2, 0)$. [2]

(ii) Show that the area of the shaded region between the curves is given by $\int_{-5}^{2} (-4x^2 - 12x + 40)\,dx$. [2]

(iii) Hence or otherwise show that the area of the shaded region between the curves is $228\frac{2}{3}$. [5]

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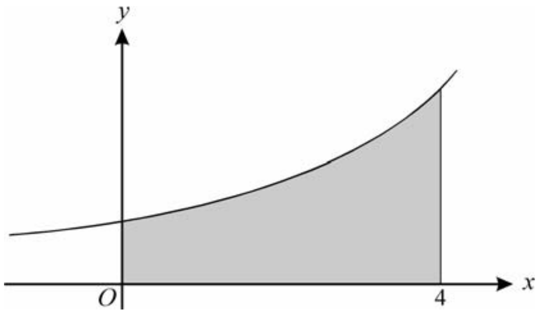
8



The diagram shows the curve $y = 1.25^x$.

(i) A point on the curve has y-coordinate 2. Calculate its x-coordinate. [3]

(ii) Use the trapezium rule with 4 intervals to estimate the area of the shaded region, bounded by the curve, the axes, and the line $x = 4$. [4]

(iii) State, with a reason, whether the estimate found in part **(ii)** is an overestimate or an underestimate. [2]

(iv) Explain briefly how the trapezium rule could be used to find a more accurate estimate of the area of the shaded region. [1]

9 The cubic polynomial $x^3 + ax^2 + bx - 6$ is denoted by $f(x)$.

(i) The remainder when $f(x)$ is divided by $(x-2)$ is equal to the remainder when $f(x)$ is divided by $(x+2)$. Show that $b = -4$. [3]

(ii) Given also that $(x-1)$ is a factor of $f(x)$, find the value of a. [2]

(iii) With these values of a and b, express $f(x)$ as a product of a linear factor and a quadratic factor. [3]

(iv) Hence determine the number of real roots of the equation $f(x) = 0$, explaining your reasoning. [3]

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MATHEMATICS **4722**

Core Mathematics 2

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

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1 $1-8x+24x^2-32x^3+16x^4$	B1 M1 M1 A1 A1 **5** $\boxed{5}$	For first two terms $1-8x$ For expansion in powers of $(-2x)$ For any correct use of binomial coefficients For any one further term correct For completely correct expansion
2 **(i)** $\int x^{-2}\,dx = -x^{-1}+c$	M1 A1 B1 **3**	For any attempt to integrate x^{-2} For correct expression $-x^{-1}$ (in any form) For adding an arbitrary constant
(ii) $y=-x^{-1}+c$ passes through $(1, 3)$, so $3=-1+c \Rightarrow c=4$ Hence curve is $y=-\frac{1}{x}+4$	 M1 A1√ A1 **3** $\boxed{6}$	 For attempt to use $(1, 3)$ to evaluate c For correct value from their equation For correct equation
3 **(a)** **(i)** $2\log_2 x$	B1 **1**	For correct answer
(ii) $\log_2(8x^2)=\log_2 8+\log_2 x^2$ $=3+2\log_2 x$	M1 M1 A1 **3**	For relevant sum of logarithms For relevant use of $8=2^3$ For correct simplified answer
(b) $2\log_3 y=\log_3 27$ Hence $\log_3 y=\frac{3}{2}$	M1 A1 A1 **3** $\boxed{7}$	For taking logs of both sides of the equation For any correct expression for $\log_3 y$ For correct simplified answer
4 **(i)** $r=\frac{2400}{3000}=0.8$ Forecast for week 20 is $3000\times0.8^{19}\approx 43$	B1 M1 A1 **3**	For the correct value of r For correct use of ar^{n-1} For correct (integer) answer
(ii) $\frac{3000(1-0.8^{20})}{1-0.8}=14\,827$	M1 A1 **2**	For correct use of $\frac{a(1-r^n)}{1-r}$ For correct answer (3sf is acceptable)
(iii) $\frac{3000}{1-0.8}=15\,000$	M1 A1 **2** $\boxed{7}$	For correct use of $\frac{a}{1-r}$ For correct answer
5 **(i)** LHS is $15(1-\sin^2\theta°)$ Hence equation is $15\sin^2\theta°+\sin\theta°-2=0$	M1 A1 **2**	For using the relevant trig identity For correct 3-term quadratic
(ii) $(5\sin\theta°+2)(3\sin\theta°-1)=0$ Hence $\sin\theta°=-\frac{2}{5}$ or $\frac{1}{3}$ So $\theta=19.5, 160.5, 203.6, 336.4$	M1 A1 M1 A1 A1√ A1√ **6** $\boxed{8}$	For factorising, or other solution method For both correct values For any relevant inverse sine operation For any one correct value For corresponding second value For both remaining values

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6	**(i)**	$\frac{3}{\sin C} = \frac{5}{\sin 2.1} \Rightarrow \sin C = \frac{3}{5}\sin 2.1$	M1	For any correct initial statement of the sine rule, together with an attempt to find $\sin C$
		Hence $C = 0.544$	A1 **2**	For correct value
	(ii)	Angle A is $\pi - 2.1 - 0.5444 = 0.4972$	M1	For calculation of angle A
		Area is $\frac{1}{2} \times 5 \times 3 \times \sin 0.4972$	M1	For any complete method for the area
		i.e. 3.58 cm^2	A1√ **3**	For correct value, following their C
	(iii)	Sector perimeter is $6 + 3 \times 0.4972$	M1	For using $r\theta$ with their A in radians
		i.e. 7.49 cm	A1t	For correct value, following their A
		Sector area is $\frac{1}{2} \times 3^2 \times 0.4972$	M1	For using $\frac{1}{2}r^2\theta$ with their A in radians
		i.e. 2.24 cm^2	A1√ **4**	For correct value, following their A
			$\boxed{9}$	
7	**(i)**	$-75 + 45 + 30 = 0,\ 25 - 15 - 10 = 0$	B1	For checking one point in both equations
		$-12 - 18 + 30 = 0,\ 4 + 6 - 10 = 0$	B1 **2**	For checking the other point in both
	(ii)	Area is $\int_{-5}^{2}\{(-3x^2 - 9x + 30) - (x^2 + 3x - 10)\}\,dx$	M1	For use of $\int (y_1 - y_2)\,dx$
		i.e. $\int_{-5}^{2}(-4x^2 - 12x + 40)\,dx$, as required	A1 **2**	For showing given answer correctly
	(iii)	*EITHER*: Area is $\left[-\frac{4}{3}x^3 - 6x^2 + 40x\right]_{-5}^{2}$	M1	For integration attempt with one term OK
			A1	For at least two terms correct
			A1	For completely correct indefinite integral
		$= (-\frac{32}{3} - 24 + 80) - (\frac{500}{3} - 150 - 200)$	M1	For correct use of limits
		$= 228\frac{2}{3}$	A1	For showing given answer correctly
		OR: Area under top curve is	M1	For complete evaluation attempt
			A1	For correct indefinite integration (allow for other curve if not earned here)
		$\left[-x^3 - \frac{9}{2}x^2 + 30x\right]_{-5}^{2} = 171\frac{1}{2}$	A1	For correct value
		Area above lower curve is		
		$-\left[\frac{1}{3}x^3 + \frac{3}{2}x^2 - 10x\right]_{-5}^{2} = 57\frac{1}{6}$	M1	For evaluation and sign change
		So area between is $171\frac{1}{2} + 57\frac{1}{6} = 228\frac{2}{3}$	A1 **5**	For showing given answer correctly
			$\boxed{9}$	
8	**(i)**	$1.25^x = 2 \Rightarrow x\log 1.25 = \log 2$	B1	For correct initial use of logs
		Hence $x = \frac{\log 2}{\log 1.25} = 3.11$	M1	For correct log expression for x
			A1 **3**	For correct numerical value
	(ii)	$\frac{1}{2}\{1.25^0 + 2(1.25^1 + 1.25^2 + 1.25^3) + 1.25^4\}$	B1	For correct recognition of $h = 1$
			M1	For any use of values 1.25^x for $x = 0, \ldots, 4$
			M1	For use of correct formula
		Area is 6.49	A1 **4**	For correct answer
	(iii)	The trapezia used in **(ii)** extend above the curve	M1	For stating or sketching trapezia above curve
		Hence the trapezium rule overestimates the area	A1 **2**	For stating overestimate with correct reason
	(iv)	Use more trapezia, with a smaller value of h	B1 **1**	For stating that more trapezia should be used
			$\boxed{10}$	

[Turn over

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9	**(i)** $8+4a+2b-6=-8+4a-2b-6$	M1	For equating $f(2)$ and $f(-2)$
		A1	For correct equation
	Hence $4b=-16 \Rightarrow b=-4$	A1 **3**	For showing given answer correctly
	(ii) $1+a-4-6=0$	M1	For equating $f(1)$ to 0 (*not* $f(-1)$)
	Hence $a=9$	A1 **2**	For correct value
	(iii) $f(x)=(x-1)(x^2+10x+6)$	M1	For quadratic factor with x^2 and/or $+6$ OK
		A1	For trinomial with both these terms correct
		A1 **3**	For completely correct factorisation
	(iv) The discriminant of the quadratic is 76	M1	For evaluating the discriminant
	Hence there are 3 real roots altogether	M1	For using positive discriminant to deduce that there are 2 roots from the quadratic factor
		A1 **3**	For completely correct explanation of 3 roots
			$\boxed{11}$

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OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
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MATHEMATICS 4723

Core Mathematics 3

Specimen Paper

Additional materials:
- Answer booklet
- Graph paper
- List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

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- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

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1 Solve the inequality $|2x+1| > |x-1|$. [5]

2 **(i)** Prove the identity

$$\sin(x+30°)+(\sqrt{3})\cos(x+30°) \equiv 2\cos x,$$

where x is measured in degrees. [4]

(ii) Hence express $\cos 15°$ in surd form. [2]

3 The sequence defined by the iterative formula

$$x_{n+1} = \sqrt[3]{(17-5x_n)},$$

with $x_1 = 2$, converges to α.

(i) Use the iterative formula to find α correct to 2 decimal places. You should show the result of each iteration. [3]

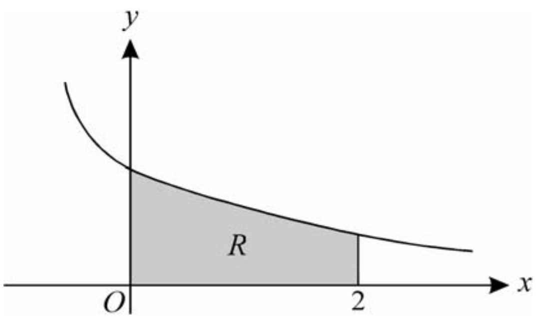
(ii) Find a cubic equation of the form

$$x^3 + cx + d = 0$$

which has α as a root. [2]

(iii) Does this cubic equation have any other real roots? Justify your answer. [2]

4



The diagram shows the curve

$$y = \frac{1}{\sqrt{(4x+1)}}.$$

The region R (shaded in the diagram) is enclosed by the curve, the axes and the line $x = 2$.

(i) Show that the exact area of R is 1. [4]

(ii) The region R is rotated completely about the x-axis. Find the exact volume of the solid formed. [4]

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5 At time t minutes after an oven is switched on, its temperature θ°C is given by

$$\theta = 200 - 180e^{-0.1t}.$$

(i) State the value which the oven's temperature approaches after a long time. [1]

(ii) Find the time taken for the oven's temperature to reach 150°C. [3]

(iii) Find the rate at which the temperature is increasing at the instant when the temperature reaches 150°C. [4]

6 The function f is defined by

$$\mathrm{f} : x \mapsto 1 + \sqrt{x} \qquad \text{for } x \geqslant 0.$$

(i) State the domain and range of the inverse function f^{-1}. [2]

(ii) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

(iii) By considering the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, show that the solution to the equation

$$\mathrm{f}(x) = \mathrm{f}^{-1}(x)$$

is $x = \frac{1}{2}(3 + \sqrt{5})$. [4]

7 **(i)** Write down the formula for $\tan 2x$ in terms of $\tan x$. [1]

(ii) By letting $\tan x = t$, show that the equation

$$4\tan 2x + 3\cot x \sec^2 x = 0$$

becomes

$$3t^4 - 8t^2 - 3 = 0.$$ [4]

(iii) Hence find all the solutions of the equation

$$4\tan 2x + 3\cot x \sec^2 x = 0$$

which lie in the interval $0 \leqslant x \leqslant 2\pi$. [4]

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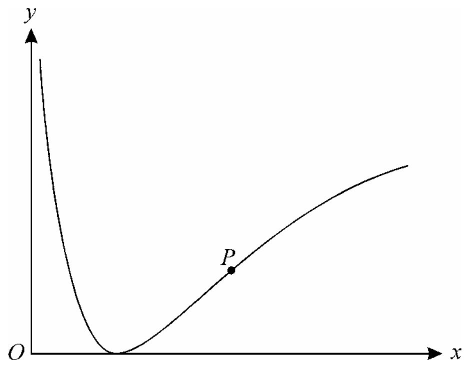
8

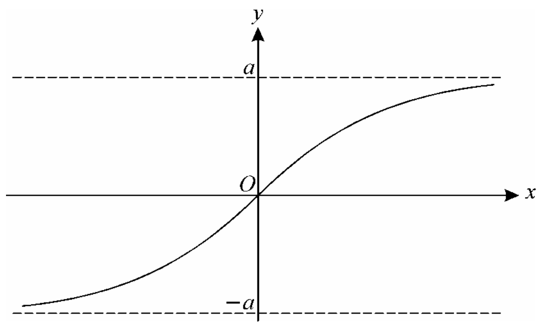


The diagram shows the curve $y = (\ln x)^2$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(ii) The point P on the curve is the point at which the gradient takes its maximum value. Show that the tangent at P passes through the point $(0, -1)$. [6]

9



The diagram shows the curve $y = \tan^{-1} x$ and its asymptotes $y = \pm a$.

(i) State the exact value of a. [1]

(ii) Find the value of x for which $\tan^{-1} x = \frac{1}{2}a$. [2]

The equation of another curve is $y = 2\tan^{-1}(x-1)$.

(iii) Sketch this curve on a copy of the diagram, and state the equations of its asymptotes in terms of a. [3]

(iv) Verify by calculation that the value of x at the point of intersection of the two curves is 1.54, correct to 2 decimal places. [2]

Another curve (which you are *not* asked to sketch) has equation $y = \left(\tan^{-1} x\right)^2$.

(v) Use Simpson's rule, with 4 strips, to find an approximate value for $\int_0^1 \left(\tan^{-1} x\right)^2 \, dx$. [3]

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Advanced Subsidiary General Certificate of Education
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MATHEMATICS **4723**

Core Mathematics 3

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

This mark scheme consists of 4 printed pages.

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1	*EITHER*: $4x^2+4x+1>x^2-2x+1$	M1	For squaring both sides
	i.e. $3x^2+6x>0$	A1	For reduction to correct quadratic
	So $x(x+2)>0$	M1	For factorising, or equivalent
	Hence $x<-2$ or $x>0$	A1	For both critical values correct
		A1	For completely correct solution set
	OR: Critical values where $2x+1=\pm(x-1)$	M1	For considering both cases, or from graphs
	i.e. where $x=-2$ and $x=0$	B1	For the correct value -2
		A1	For the correct value 0
	Hence $x<-2$ or $x>0$	M1	For any correct method for solution set using two critical values
		A1 **5**	For completely correct solution set
		[5]	
2	**(i)** $\sin x(\frac{1}{2}\sqrt{3})+\cos x(\frac{1}{2})+(\sqrt{3})(\cos x(\frac{1}{2}\sqrt{3})-\sin x(\frac{1}{2}))$	M1	For expanding both compound angles
		A1	For completely correct expansion
		M1	For using exact values of $\sin 30°$ and $\cos 30°$
	$=\frac{1}{2}\cos x+\frac{3}{2}\cos x=2\cos x$, as required	A1 **4**	For showing given answer correctly
	(ii) $\sin 45°+(\sqrt{3})\cos 45°=2\cos 15°$	M1	For letting $x=15°$ throughout
	Hence $\cos 15°=\dfrac{1+\sqrt{3}}{2\sqrt{2}}$	A1 **2**	For any correct exact form
		[6]	
3	**(i)** $x_2=\sqrt[3]{7}=1.9129...$	B1	For 1.91… seen or implied
	$x_3=1.9517...$, $x_4=1.9346...$	M1	For continuing the correct process
	$\alpha=1.94$ to 2dp	A1 **3**	For correct value reached, following x_5 and x_6 both 1.94 to 2dp
	(ii) $x=\sqrt[3]{(17-5x)} \Rightarrow x^3+5x-17=0$	M1	For letting $x_n=x_{n+1}=x$ (or α)
		A1 **2**	For correct equation stated
	(iii) *EITHER*: Graphs of $y=x^3$ and $y=17-5x$ only cross once	M1	For argument based on sketching a pair of graphs, or a sketch of the cubic by calculator
	Hence there is only one real root	A1√	For correct conclusion for a valid reason
	OR: $\dfrac{d}{dx}(x^3+5x-17)=3x^2+5>0$	M1	For consideration of the cubic's gradient
	Hence there is only one real root	A1√ **2**	For correct conclusion for a valid reason
		[7]	
4	**(i)** $\int_0^2 (4x+1)^{-\frac{1}{2}}\,dx=\left[\frac{1}{2}(4x+1)^{\frac{1}{2}}\right]_0^2=\frac{1}{2}(3-1)=1$	M1	For integral of the form $k(4x+1)^{\frac{1}{2}}$
		A1	For correct indefinite integral
		M1	For correct use of limits
		A1 **4**	For given answer correctly shown
	(ii) $\pi\int_0^2 \dfrac{1}{4x+1}\,dx=\pi\left[\frac{1}{4}\ln(4x+1)\right]_0^2=\frac{1}{4}\pi\ln 9$	M1	For integral of the form $k\ln(4x+1)$
		A1	For correct $\frac{1}{4}\ln(4x+1)$, with or without π
		M1	Correct use of limits and π
		A1 **4**	For correct (simplified) exact value
		[8]	

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5	**(i)**	200 °C	B1 **1**	For value 200
	(ii)	$150 = 200 - 180\mathrm{e}^{-0.1t} \Rightarrow \mathrm{e}^{-0.1t} = \frac{50}{180}$	M1	For isolating the exponential term
		Hence $-0.1t = \ln\frac{5}{18} \Rightarrow t = 12.8$	M1	For taking logs correctly
			A1 **3**	For correct value 12.8 (minutes)
	(iii)	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = 18\mathrm{e}^{-0.1t}$	M1	For differentiation attempt
			A1	For correct derivative
		Hence rate is $18\mathrm{e}^{-0.1\times 12.8} = 5.0$ °C per minute	M1	For using their value from **(ii)** in their $\dot{\theta}$
			A1 **4**	For value 5.0(0)
			$\boxed{8}$	
6	**(i)**	Domain of f^{-1} is $x \geqslant 1$	B1	For the correct set, in any notation
		Range is $x \geqslant 0$	B1 **2**	Ditto
	(ii)	If $y = 1 + \sqrt{x}$, then $x = (y-1)^2$	M1	For changing the subject, or equivalent
		Hence $\mathrm{f}^{-1}(x) = (x-1)^2$	A1 **2**	For correct expression in terms of x
	(iii)	The graphs intersect on the line $y = x$	B1	For stating or using this fact
		Hence x satisfies $x = (x-1)^2$	B1	For either $x = \mathrm{f}(x)$ or $x = \mathrm{f}^{-1}(x)$
		i.e. $x^2 - 3x + 1 = 0 \Rightarrow x = \frac{3 \pm \sqrt{5}}{2}$	M1	For solving the relevant quadratic equation
		So $x = \frac{1}{2}(3 + \sqrt{5})$ as x must be greater than 1	A1 **4**	For showing the given answer fully
			$\boxed{8}$	
7	**(i)**	$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$	B1 **1**	For correct RHS stated
	(ii)	$\frac{8t}{1-t^2} + 3 \times \frac{1}{t} \times (1 + t^2) = 0$	B1	For $\cot x = \frac{1}{t}$ seen
			B1	For $\sec^2 x = 1 + t^2$ seen
		Hence $8t^2 + 3(1-t^2)(1+t^2) = 0$	M1	For complete substitution in terms of t
		i.e. $3t^4 - 8t^2 - 3 = 0$, as required	A1 **4**	For showing given equation correctly
	(iii)	$(3t^2 + 1)(t^2 - 3) = 0$	M1	For factorising or other solution method
		Hence $t = \pm\sqrt{3}$	A1	For $t^2 = 3$ found correctly
		So $x = \frac{1}{3}\pi, \frac{2}{3}\pi, \frac{4}{3}\pi, \frac{5}{3}\pi$	A1	For any two correct angles
			A1 **4**	For all four correct and no others
			$\boxed{9}$	

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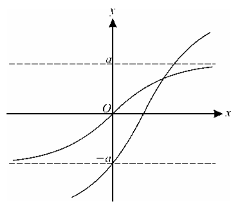
8	**(i)** $\frac{dy}{dx} = \frac{2\ln x}{x}$	M1	For relevant attempt at the chain rule
		A1	For correct result, in any form
	$\frac{d^2y}{dx^2} = \frac{x(2/x) - 2\ln x}{x^2} = \frac{2 - 2\ln x}{x^2}$	M1	For relevant attempt at quotient rule
		A1 **4**	For correct simplified answer
	(ii) For maximum gradient, $2 - 2\ln x = 0 \Rightarrow x = e$	M1	For equating second derivative to zero
		A1	For correct value e
	Hence P is $(e, 1)$	A1√	For stating or using the y-coordinate
	The gradient at P is $\frac{2}{e}$	A1√	For stating or using the gradient at P
	Tangent at P is $y - 1 = \frac{2}{e}(x - e)$	M1	For forming the equation of the tangent
	Hence, when $x = 0$, $y = -1$ as required	A1 **6**	For correct verification of $(0, -1)$
		10	
9	**(i)** $a = \frac{1}{2}\pi$	B1 **1**	For correct exact value stated
	(ii) $x = \tan(\frac{1}{4}\pi) = 1$	M1	For use of $x = \tan(\frac{1}{2}a)$
		A1√ **2**	For correct answer, following their a
	(iii) 	B1	For x-translation of (approx) $+1$
		B1	For y-stretch with (approx) factor 2
	Asymptotes are $y = \pm 2a$	B1 **3**	For correct statement of asymptotes
	x $\tan^{-1}x$ $2\tan^{-1}(x-1)$		
	(iv) 1.535 0.993 0.983	M1	For relevant evaluations at 1.535, 1.545
	1.545 0.996 0.998		
	Hence graphs cross between 1.535 and 1.545	A1 **2**	For correct details and explanation
	(v) Relevant values of $\left(\tan^{-1}x\right)^2$ are (approximately)	M1	For the relevant function values seen or
	0, 0.0600, 0.2150, 0.4141, 0.6169		implied; must be radians, not degrees
	$\frac{1}{12}\{0 + 4(0.0600 + 0.4141) + 2\times0.2150 + 0.6169\}$	M1	For use of correct formula with $h = \frac{1}{4}$
	Hence required approximation is 0.245	A1 **3**	For correct (2 or 3sf) answer
		4	

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MATHEMATICS 4724

Core Mathematics 4

Specimen Paper

Additional materials:
- Answer booklet
- Graph paper
- List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
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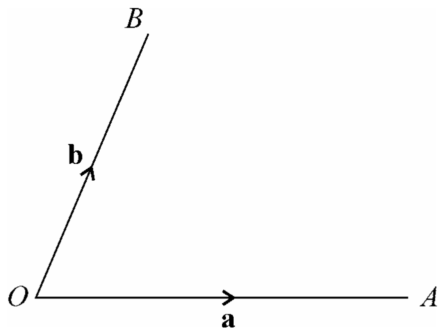
1 Find the quotient and remainder when $x^4 + 1$ is divided by $x^2 + 1$. [4]

2 **(i)** Expand $(1-2x)^{-\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^3. [4]

(ii) State the set of values for which the expansion in part **(i)** is valid. [1]

3 Find $\int_0^1 x\mathrm{e}^{-2x}\,\mathrm{d}x$, giving your answer in terms of e. [5]

4



As shown in the diagram the points A and B have position vectors **a** and **b** with respect to the origin O.

(i) Make a sketch of the diagram, and mark the points C, D and E such that $\overrightarrow{OC} = 2\mathbf{a}$, $\overrightarrow{OD} = 2\mathbf{a} + \mathbf{b}$ and $\overrightarrow{OE} = \frac{1}{3}\overrightarrow{OD}$. [3]

(ii) By expressing suitable vectors in terms of **a** and **b**, prove that E lies on the line joining A and B. [4]

5 **(i)** For the curve $2x^2 + xy + y^2 = 14$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [4]

(ii) Deduce that there are two points on the curve $2x^2 + xy + y^2 = 14$ at which the tangents are parallel to the x-axis, and find their coordinates. [4]

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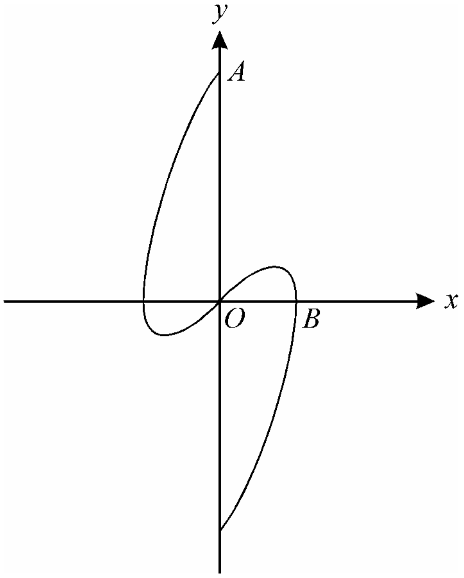
6



The diagram shows the curve with parametric equations

$$x = a\sin\theta, \quad y = a\theta\cos\theta,$$

where a is a positive constant and $-\pi \leqslant \theta \leqslant \pi$. The curve meets the positive y-axis at A and the positive x-axis at B.

(i) Write down the value of θ corresponding to the origin, and state the coordinates of A and B. [3]

(ii) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 - \theta\tan\theta$, and hence find the equation of the tangent to the curve at the origin. [6]

7 The line L_1 passes through the point $(3, 6, 1)$ and is parallel to the vector $2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$. The line L_2 passes through the point $(3, -1, 4)$ and is parallel to the vector $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

(i) Write down vector equations for the lines L_1 and L_2. [2]

(ii) Prove that L_1 and L_2 intersect, and find the coordinates of their point of intersection. [5]

(iii) Calculate the acute angle between the lines. [4]

8 Let $I = \displaystyle\int \frac{1}{x(1+\sqrt{x})^2}\,\mathrm{d}x$.

(i) Show that the substitution $u = \sqrt{x}$ transforms I to $\displaystyle\int \frac{2}{u(1+u)^2}\,\mathrm{d}u$. [3]

(ii) Express $\dfrac{2}{u(1+u)^2}$ in the form $\dfrac{A}{u} + \dfrac{B}{1+u} + \dfrac{C}{(1+u)^2}$. [5]

(ii) Hence find I. [4]

[Turn over

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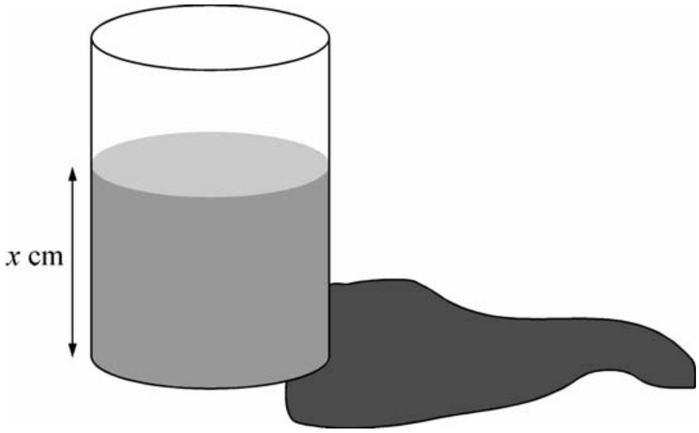
9



A cylindrical container has a height of 200 cm. The container was initially full of a chemical but there is a leak from a hole in the base. When the leak is noticed, the container is half-full and the level of the chemical is dropping at a rate of 1 cm per minute. It is required to find for how many minutes the container has been leaking. To model the situation it is assumed that, when the depth of the chemical remaining is x cm, the rate at which the level is dropping is proportional to $\sqrt{x}$.

Set up and solve an appropriate differential equation, and hence show that the container has been leaking for about 80 minutes. [11]

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MATHEMATICS **4724**

Core Mathematics 4

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

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[Turn over

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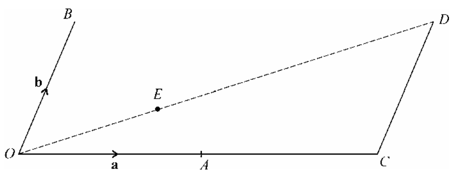
1	$\dfrac{x^4+1}{x^2+1} = x^2 - 1 + \dfrac{2}{x^2+1}$	B1	For correct leading term x^2 in quotient
		M1	For evidence of correct division process
		A1	For correct quotient $x^2 - 1$
		A1 **4**	For correct remainder 2
		$\boxed{4}$	
2	**(i)** $(1-2x)^{-\frac{1}{2}} = 1 + (-\tfrac{1}{2})(-2x) + \dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-2x)^2 + $		
	$+\dfrac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}(-2x)^3 + \ldots$	M1	For 2nd, 3rd or 4th term OK (unsimplified)
	$= 1 + x + \tfrac{3}{2}x^2 + \tfrac{5}{2}x^3$	A1	For $1 + x$ correct
		A1	For $+\tfrac{3}{2}x^2$ correct
		A1 **4**	For $+\tfrac{5}{2}x^3$ correct
	(ii) Valid for $\lvert x \rvert < \tfrac{1}{2}$	B1 **1**	For any correct expression(s)
		$\boxed{5}$	
3	$\int_0^1 x\mathrm{e}^{-2x}\,\mathrm{d}x = \left[-\tfrac{1}{2}x\mathrm{e}^{-2x}\right]_0^1 - \int_0^1 -\tfrac{1}{2}\mathrm{e}^{-2x}\,\mathrm{d}x$	M1	For attempt at 'parts' going the correct way
		A1	For correct terms $-\tfrac{1}{2}x\mathrm{e}^{-2x} - \int -\tfrac{1}{2}\mathrm{e}^{-2x}\,\mathrm{d}x$
	$= \left[-\tfrac{1}{2}x\mathrm{e}^{-2x} - \tfrac{1}{4}\mathrm{e}^{-2x}\right]_0^1$	M1	For consistent attempt at second integration
		M1	For correct use of limits throughout
	$= \tfrac{1}{4} - \tfrac{3}{4}\mathrm{e}^{-2}$	A1 **5**	For correct (exact) answer in any form
		$\boxed{5}$	
4	**(i)** 	B1	For C correctly located on sketch
		B1	For D correctly located on sketch
		B1$\surd$ **3**	For E correctly located wrt O and D
	(ii) $\overrightarrow{AE} = \tfrac{1}{3}(2\mathbf{a}+\mathbf{b}) - \mathbf{a} = \tfrac{1}{3}(\mathbf{b}-\mathbf{a})$	M1	For relevant subtraction involving $\overrightarrow{OE}$
		A1	For correct expression for $(\pm)\overrightarrow{AE}$ or $\overrightarrow{EB}$
	Hence AE is parallel to AB	A1	For correct recognition of parallel property
	i.e. E lies on the line joining A to B	A1 **4**	For complete proof of required result
		$\boxed{7}$	
5	**(i)** $4x + x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y + 2y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$	B1	For correct terms $x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y$
		B1	For correct term $2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$
	Hence $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{4x+y}{x+2y}$	M1	For solving for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$
		A1 **4**	For any correct form of expression
	(ii) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow y = -4x$	M1	For stating or using their $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$
	Hence $2x^2 + (-4x^2) + (-4x)^2 = 14$	M1	For solving simultaneously with curve equn
	i.e. $x^2 = 1$	A1	For correct value of x^2 (or y^2)
	So the two points are $(1, -4)$ and $(-1, 4)$	A1 **4**	For both correct points identified
		$\boxed{8}$	

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6 **(i)** $\theta = 0$ at the origin	B1	For the correct value
A is $(0, a\pi)$	B1	For the correct y-coordinate at A
B is $(a, 0)$	B1 **3**	For the correct x-coordinate at B
(ii) $\frac{dx}{d\theta} = a\cos\theta$	B1	For correct differentiation of x
$\frac{dy}{d\theta} = a(\cos\theta - \theta\sin\theta)$	M1	For differentiating y using product rule
Hence $\frac{dy}{dx} = \frac{\cos\theta - \theta\sin\theta}{\cos\theta} = 1 - \theta\tan\theta$	M1	For use of $\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta}$
	A1	For given result correctly obtained
Gradient of tangent at the origin is 1	M1	For using $\theta = 0$
Hence equation is $y = x$	A1 **6**	For correct equation
	$\boxed{9}$	
6 **(i)** $L_1 : \mathbf{r} = 3\mathbf{i} + 6\mathbf{j} + \mathbf{k} + s(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$	M1	For correct RHS structure for either line
$L_2 : \mathbf{r} = 3\mathbf{i} - \mathbf{j} + 4\mathbf{k} + t(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	A1 **2**	For both lines correct
(ii) $3 + 2s = 3 + t,\ 6 + 3s = -1 - 2t,\ 1 - s = 4 + t$	M1	For at least 2 equations with two parameters
First pair of equations give $s = -1, t = -2$	M1	For solving any relevant pair of equations
	A1	For both parameters correct
Third equation checks: $1 + 1 = 4 - 2$	A1	For explicit check in unused equation
Point of intersection is $(1, 3, 2)$	A1 **5**	For correct coordinates
(iii) $2\times1 + 3\times(-2) + (-1)\times1 = (\sqrt{14})(\sqrt{6})\cos\theta$	B1	For scalar product of correct direction vectors
	B1	For correct magnitudes $\sqrt{14}$ and $\sqrt{6}$
	M1	For correct process for $\cos\theta$ with *any* pair of vectors relevant to these lines
Hence acute angle is $56.9°$	A1 **4**	For correct acute angle
	$\boxed{11}$	
8 **(i)** $I = \int \frac{1}{u^2(1+u)^2} \times 2u\,du = \int \frac{2}{u(1+u)^2}\,du$	M1	For any attempt to find $\frac{dx}{du}$ or $\frac{du}{dx}$
	A1	For '$dx = 2u\,du$' or equivalent correctly used
	A1 **3**	For showing the given result correctly
(ii) $2 \equiv A(1+u)^2 + Bu(1+u) + Cu$	M1	For correct identity stated
$A = 2$	B1	For correct value stated
$C = -2$	B1	For correct value stated
$0 = A + B$ (e.g.)	A1	For any correct equation involving B
$B = -2$	A1 **5**	For correct value
(iii) $2\ln u - 2\ln(1+u) + \frac{2}{1+u}$	B1$\surd$	For $A\ln u + B\ln(1+u)$ with their values
	B1$\surd$	For $-C(1+u)^{-1}$ with their value
Hence $I = \ln x - 2\ln(1+\sqrt{x}) + \frac{2}{1+\sqrt{x}} + c$	M1	For substituting back
	A1 **4**	For completely correct answer (excluding c)
	$\boxed{12}$	

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9	$\frac{dx}{dt} = -k\sqrt{x}$	M1		For use of derivative for rate of change
		A1		For correct equation (neg sign optional here)
	$x = 100$ and $\frac{dx}{dt} = -1 \Rightarrow k = 0.1$	M1		For use of data and their DE to find k
	Hence equation is $\frac{dx}{dt} = -0.1\sqrt{x}$	A1		For any form of correct DE
	$\int x^{-\frac{1}{2}}\,dx = -0.1\int dt \Rightarrow 2x^{\frac{1}{2}} = -0.1t + c$	M1		For separation and integration of both sides
		A1		For $2x^{\frac{1}{2}}$ correct
		A1$\sqrt{}$		For $(\pm)kt$ correct (the numerical evaluation of k may be delayed until after the DE is solved)
		B1		For one arbitrary constant included (or equivalent statement of both pairs of limits)
	$x = 200, t = 0 \Rightarrow c = 2\sqrt{200}$	M1		For evaluation of c
	So when $x = 100$, $2\sqrt{100} = -0.1t + 2\sqrt{200}$	M1		For evaluation of t
	i.e. $t = 82.8$	A1	**11**	For correct value 82.8 (minutes)
			$\boxed{11}$	

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Advanced Subsidiary General Certificate of Education
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MATHEMATICS **4725**

Further Pure Mathematics 1

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

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- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

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[Turn over

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1 Use formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} r(r+1) = \tfrac{1}{3}n(n+1)(n+2).$$ [5]

2 The cubic equation $x^3 - 6x^2 + kx + 10 = 0$ has roots $p - q$, p and $p + q$, where q is positive.

(i) By considering the sum of the roots, find p. [2]

(ii) Hence, by considering the product of the roots, find q. [3]

(iii) Find the value of k. [3]

3 The complex number $2 + \mathrm{i}$ is denoted by z, and the complex conjugate of z is denoted by z^*.

(i) Express z^2 in the form $x + \mathrm{i}y$, where x and y are real, showing clearly how you obtain your answer. [2]

(ii) Show that $4z - z^2$ simplifies to a real number, and verify that this real number is equal to zz^*. [3]

(iii) Express $\frac{z+1}{z-1}$ in the form $x + \mathrm{i}y$, where x and y are real, showing clearly how you obtain your answer. [3]

4 A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_n = 3^{2n} - 1.$$

(i) Write down the value of u_1. [1]

(ii) Show that $u_{n+1} - u_n = 8 \times 3^{2n}$. [3]

(iii) Hence prove by induction that each term of the sequence is a multiple of 8. [4]

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5 **(i)** Show that

$$\frac{1}{2r-1}-\frac{1}{2r+1}=\frac{2}{4r^2-1}.$$ [2]

(ii) Hence find an expression in terms of n for

$$\frac{2}{3}+\frac{2}{15}+\frac{2}{35}+\ldots+\frac{2}{4n^2-1}.$$ [4]

(iii) State the value of

(a) $\displaystyle\sum_{r=1}^{\infty}\frac{2}{4r^2-1}$, [1]

(b) $\displaystyle\sum_{r=n+1}^{\infty}\frac{2}{4r^2-1}$. [1]

6 In an Argand diagram, the variable point P represents the complex number $z = x + \mathrm{i}\,y$, and the fixed point A represents $a = 4 - 3\mathrm{i}$.

(i) Sketch an Argand diagram showing the position of A, and find $|a|$ and $\arg a$. [4]

(ii) Given that $|z-a| = |a|$, sketch the locus of P on your Argand diagram. [3]

(iii) Hence write down the non-zero value of z corresponding to a point on the locus for which

(a) the real part of z is zero, [1]

(b) $\arg z = \arg a$. [2]

7 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$.

(i) Draw a diagram showing the unit square and its image under the transformation represented by **A**. [3]

(ii) The value of $\det \mathbf{A}$ is 5. Show clearly how this value relates to your diagram in part **(i)**. [3]

A represents a sequence of two elementary geometrical transformations, one of which is a rotation R.

(iii) Determine the angle of R, and describe the other transformation fully. [3]

(iv) State the matrix that represents R, giving the elements in an exact form. [2]

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8 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} a & 2 & -1 \\ 2 & 3 & -1 \\ 2 & -1 & 1 \end{pmatrix}$, where a is a constant.

(i) Show that the determinant of $\mathbf{M}$ is $2a$. [2]

(ii) Given that $a \neq 0$, find the inverse matrix $\mathbf{M}^{-1}$. [4]

(iii) Hence or otherwise solve the simultaneous equations

$$\begin{aligned} x + 2y - z &= 1, \\ 2x + 3y - z &= 2, \\ 2x - y + z &= 0. \end{aligned}$$

[3]

(iv) Find the value of k for which the simultaneous equations

$$\begin{aligned} 2y - z &= k, \\ 2x + 3y - z &= 2, \\ 2x - y + z &= 0, \end{aligned}$$

have solutions. [3]

(v) Do the equations in part **(iv)**, with the value of k found, have a solution for which $x = z$? Justify your answer. [2]

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MATHEMATICS **4725**

Further Pure Mathematics 1

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

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1	$\sum_{r=1}^{n} r(r+1) = \sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r = \frac{1}{6}n(n+1)(2n+1) + \frac{1}{2}n(n+1)$	M1		For considering the two separate sums
		A1		For either correct sum formula stated
		A1		For completely correct expression
	$= \frac{1}{6}n(n+1)(2n+1+3) = \frac{1}{3}n(n+1)(n+2)$	M1		For factorising attempt
		A1	**5**	For showing given answer correctly
			$\boxed{5}$	
2	**(i)** $(p-q)+p+(p+q)=6 \Rightarrow p=2$	M1		For use of $\Sigma\alpha = -b/a$
		A1	**2**	For correct answer
	(ii) $2(2-q)(2+q)=-10$	B1√		For use of $\alpha\beta\gamma = -d/a$
	Hence $4-q^2=-5 \Rightarrow q=3$	M1		For expanding and solving for q^2
		A1	**3**	For correct answer
	(iii) *EITHER*: Roots are $-1, 2, 5$	B1√		For stating or using three numerical roots
	$-1\times2+2\times5+-1\times5=k$	M1		For use of $\Sigma\alpha\beta = c/a$
	i.e. $k=3$	A1√		For correct answer from their roots
	OR: Roots are $-1, 2, 5$	B1√		For stating or using three numerical roots
	Equation is $(x+1)(x-2)(x-5)=0$	M1		For stating and expanding factorised form
	Hence $k=3$	A1√	**3**	For correct answer from their roots
			$\boxed{8}$	
3	**(i)** $z^2=(2+\mathrm{i})^2=4+4\mathrm{i}+\mathrm{i}^2=3+4\mathrm{i}$	M1		For showing 3-term or 4-term expansion
		A1	**2**	For correct answer
	(ii) $4z-z^2=8+4\mathrm{i}-3-4\mathrm{i}=5$	B1		For correct value 5
	$zz^*=(2+\mathrm{i})(2-\mathrm{i})=5$	B1		For stating or using $z^*=2-\mathrm{i}$
		B1	**3**	For correct verification of given restult
	(iii) $\frac{z+1}{z-1}=\frac{3+\mathrm{i}}{1+\mathrm{i}}=\frac{(3+\mathrm{i})(1-\mathrm{i})}{(1+\mathrm{i})(1-\mathrm{i})}=\frac{4-2\mathrm{i}}{2}=2-\mathrm{i}$	B1		For correct initial form $\frac{3+\mathrm{i}}{1+\mathrm{i}}$
		M1		For multiplying top and bottom by $1-\mathrm{i}$
		A1	**3**	For correct answer $2-\mathrm{i}$
			$\boxed{8}$	
4	**(i)** $u_1=8$	B1	**1**	For correct value stated
	(ii) $3^{2(n+1)}-1-(3^{2n}-1)=9\times3^{2n}-3^{2n}=8\times3^{2n}$	B1		For stating or using $u_{n+1}=3^{2(n+1)}-1$
		M1		For relevant manipulation of indices in u_{n+1}
		A1	**3**	For showing given answer correctly
	(iii) u_1 is divisible by 8, from **(i)**	B1		For explicit check for u_1
	Suppose u_k is divisible by 8, i.e. $u_k=8a$	M1		For induction hypothesis u_k is mult. of 8
	Then $u_{k+1}=u_k+8\times3^{2k}=8(a+3^{2k})=8b$	M1		For obtaining and simplifying expr. for u_{k+1}
	i.e. u_{k+1} is also divisible by 8, and result follows by the induction principle	A1	**4**	For correct conclusion, stated and justified
			$\boxed{8}$	

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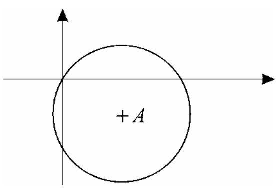
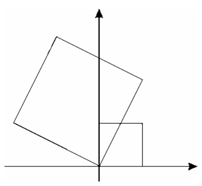
5	(i) LHS $= \dfrac{2r+1-(2r-1)}{(2r-1)(2r+1)} = \dfrac{2}{4r^2-1} =$ RHS	M1		For correct process for adding fractions
		A1	2	For showing given result correctly
	(ii) Sum is $\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\ldots+\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$	M1		For expressing terms as differences using **(i)**
		A1		For at least first two and last terms correct
	This is $1-\dfrac{1}{2n+1}$	M1		For cancelling pairs of terms
		A1	4	For any correct form
	(iii) **(a)** Sum to infinity is 1	B1√	1	For correct value; follow their **(ii)** if cnvgt
	(b) Required sum is $\dfrac{1}{2n+1}$	B1√	1	For correct difference of their **(iii)(a)** and **(ii)**
			$\boxed{8}$	
6	(i) (See diagram in part **(ii)** below)	B1		For point A correctly located
	$\lvert a \rvert = \sqrt{(3^2+4^2)} = 5$	B1		For correct value for the modulus
	$\arg a = -\tan^{-1}\left(\frac{3}{4}\right) = -0.644$	M1		For any correct relevant trig statement
		A1	4	For correct answer (radians or degrees)
	(ii) 	B1		For any indication that locus is a circle
		B1		For any indication that the centre is at A
		B1	3	For a completely correct diagram
	(iii) **(a)** $z = -6\text{i}$	B1	1	For correct answer
	(b) $z = 8-6\text{i}$	M1		For identification of end of diameter thru A
		A1	2	For correct answer
			$\boxed{10}$	
7	(i) $\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 & -1 \\ 0 & 2 & 1 & 3 \end{pmatrix}$	M1		For at least one correct image
		A1		For all vertices correct
		A1	3	For correct diagram
	(ii) The area scale-factor is 5	B1		For identifying det as area scale factor
	The transformed square has side of length $\sqrt{5}$	M1		For calculation method relating to large sq.
	So its area is 5 times that of the unit square	A1	3	For a complete explanataion
	(iii) Angle is $\tan^{-1}(2) = 63.4°$	B1		For $\tan^{-1}(2)$, or equivalent
	Enlargement with scale factor $\sqrt{5}$	B1		For stating 'enlargement'
		B1	3	For correct (exact) scale factor
	(iv) $\begin{pmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$	M1		For correct $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ pattern
		A1	2	For correct matrix in exact form
			$\boxed{11}$	

[Turn over

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8	(i)	$\det \mathbf{M} = a(3-1) - 2(2-(-2)) - 1(-2-6)$	M1	For correct expansion process
		$= 2a$	A1 **2**	For showing given answer correctly
	(ii)	$\mathbf{M}^{-1} = \frac{1}{2a}\begin{pmatrix} 2 & -1 & 1 \\ -4 & a+2 & a-2 \\ -8 & a+4 & 3a-4 \end{pmatrix}$	M1	For correct process for adjoint entries
			A1	For at least 4 correct entries in adjoint
			B1	For dividing by the determinant
			A1 **4**	For completely correct inverse
	(iii)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, with $a = 1$	B1	For correct statement involving inverse
		So $x = 0,\ y = 1,\ z = 1$	M1	For carrying out the correct multiplication
			A1 **3**	For all three correct values
	(iv)	Eliminating x gives $4y - 2z = 2$	M1	For eliminating x from 2nd and 3rd equns
		So for consistency with 1st equn, $k = 1$	M1	For comparing two y-z equations
			A1 **3**	For correct value for k
	(v)	Solving $x + 3y = 2,\ 3x - y = 0$ gives $x = \frac{1}{5},\ y = \frac{3}{5}$	M1	For using $x = z$ to solve a pair of equns
		These values check in $2y - x = 1$, so soln exists	A1 **2**	For a completely correct demonstration
			14	

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MATHEMATICS **4726**

Further Pure Mathematics 2

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

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- The total number of marks for this paper is 72.
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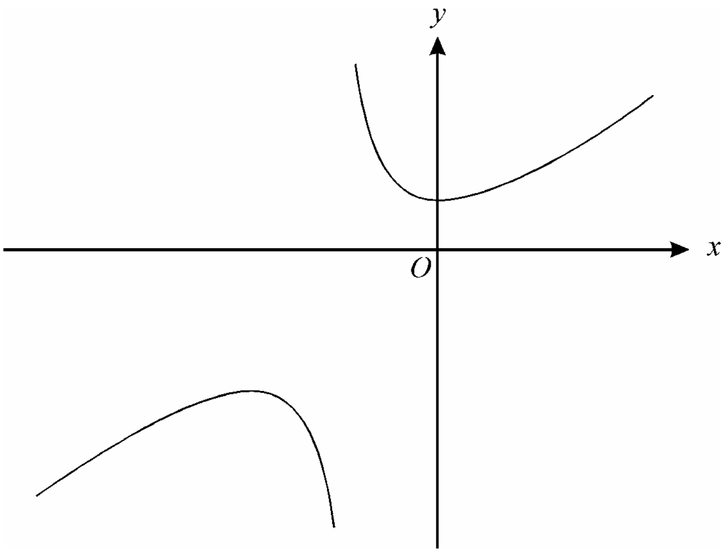
1 **(i)** Starting from the definition of $\cosh x$ in terms of e^x, show that $\cosh 2x = 2\cosh^2 x - 1$. [2]

(ii) Given that $\cosh 2x = k$, where $k > 1$, express each of $\cosh x$ and $\sinh x$ in terms of k. [4]

2



The diagram shows the graph of

$$y = \frac{2x^2 + 3x + 3}{x + 1}.$$

(i) Find the equations of the asymptotes of the curve. [3]

(ii) Prove that the values of y between which there are no points on the curve are -5 and 3. [4]

3 **(i)** Find the first three terms of the Maclaurin series for $\ln(2 + x)$. [4]

(ii) Write down the first three terms of the series for $\ln(2 - x)$, and hence show that, if x is small, then

$$\ln\left(\frac{2+x}{2-x}\right) \approx x.$$ [3]

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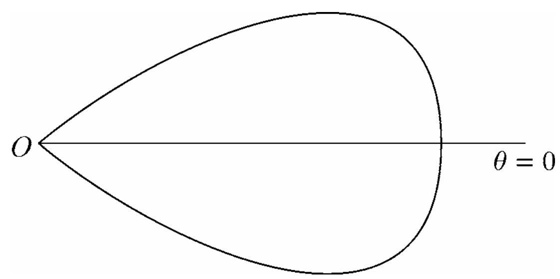
4 The equation of a curve, in polar coordinates, is

$$r = 2\cos 2\theta \qquad (-\pi < \theta \leqslant \pi).$$

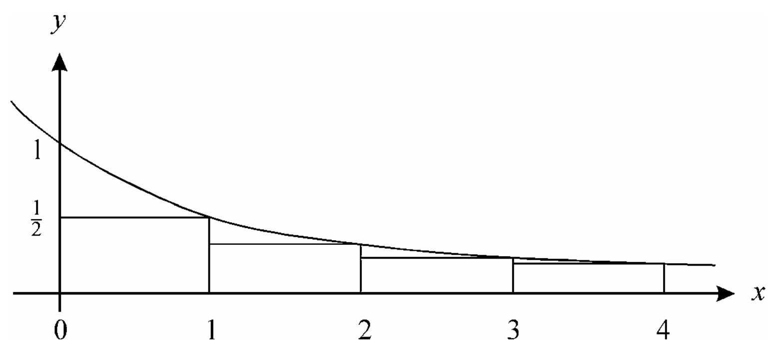
(i) Find the values of θ which give the directions of the tangents at the pole. [3]

One loop of the curve is shown in the diagram.



(ii) Find the exact value of the area of the region enclosed by the loop. [5]

5



The diagram shows the curve $y = \dfrac{1}{x+1}$ together with four rectangles of unit width.

(i) Explain how the diagram shows that

$$\tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \tfrac{1}{5} < \int_0^4 \frac{1}{x+1}\,dx.$$ [2]

The curve $y = \dfrac{1}{x+2}$ passes through the top left-hand corner of each of the four rectangles shown.

(ii) By considering the rectangles in relation to this curve, write down a second inequality involving $\tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \tfrac{1}{5}$ and a definite integral. [2]

(iii) By considering a suitable range of integration and corresponding rectangles, show that

$$\ln(500.5) < \sum_{r=2}^{1000} \frac{1}{r} < \ln(1000).$$ [4]

[Turn over

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6 **(i)** Given that $I_n = \int_0^1 x^n \sqrt{(1-x)}\ \mathrm{d}x$, prove that, for $n \geqslant 1$,

$$(2n+3)I_n = 2nI_{n-1}.$$ [6]

(ii) Hence find the exact value of I_2. [4]

7 The curve with equation

$$y = \frac{x}{\cosh x}$$

has one stationary point for $x > 0$.

(i) Show that the x-coordinate of this stationary point satisfies the equation $x \tanh x - 1 = 0$. [2]

The positive root of the equation $x \tanh x - 1 = 0$ is denoted by α.

(ii) Draw a sketch showing (for positive values of x) the graph of $y = \tanh x$ and its asymptote, and the graph of $y = \frac{1}{x}$. Explain how you can deduce from your sketch that $\alpha > 1$. [3]

(iii) Use the Newton-Raphson method, taking first approximation $x_1 = 1$, to find further approximations x_2 and x_3 for α. [5]

(iv) By considering the approximate errors in x_1 and x_2, estimate the error in x_3. [3]

8 **(i)** Use the substitution $t = \tan\frac{1}{2}x$ to show that

$$\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos x}{1+\sin x}}\ \mathrm{d}x = 2\sqrt{2}\int_0^1 \frac{t}{(1+t)(1+t^2)}\ \mathrm{d}t.$$ [4]

(ii) Express $\frac{t}{(1+t)(1+t^2)}$ in partial fractions. [5]

(iii) Hence find $\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos x}{1+\sin x}}\ \mathrm{d}x$, expressing your answer in an exact form. [4]

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MATHEMATICS **4726**

Further Pure Mathematics 2

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

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1	**(i)**	$\text{RHS} = 2\left(\frac{1}{2}(e^x + e^{-x})\right)^2 - 1 = \frac{1}{2}(e^{2x} + e^{-2x}) = \text{LHS}$	M1		For correct squaring of $(e^x + e^{-x})$
			A1	**2**	For completely correct proof
	(ii)	$2\cosh^2 x - 1 = k \Rightarrow \cosh x = \sqrt{\left(\frac{1}{2}(1+k)\right)}$	M1		For use of **(i)** and solving for $\cosh x$
			A1		For correct positive square root only
		$2\sinh^2 x + 1 = k \Rightarrow \sinh x = \pm\sqrt{\left(\frac{1}{2}(k-1)\right)}$	M1		For use of $\cosh^2 x - \sinh^2 x = 1$, or equivalent
			A1	**4**	For both correct square roots
				$\boxed{6}$	
2	**(i)**	$x = -1$ is an asymptote	B1		For correct equation of vertical asymptote
		$y = 2x + 1 + \dfrac{2}{x+1}$	M1		For algebraic division, or equivalent
		Hence $y = 2x + 1$ is an asymptote	A1	**3**	For correct equation of oblique asymptote
	(ii)	*EITHER*: Quadratic $2x^2 + (3-y)x + (3-y) = 0$	M1		For using discriminant of relevant quadratic
		has no real roots if $(3-y)^2 < 8(3-y)$	A1		For correct inequality or equation in y
		Hence $(3-y)(-5-y) < 0$	M1		For factorising, or equivalent
		So required values are 3 and -5	A1		For given answer correctly shown
		OR: $\dfrac{dy}{dx} = 2 - \dfrac{2}{(x+1)^2} = 0$	M1		For differentiating and equating to zero
		Hence $(x+1)^2 = 1$	A1		For correct simplified quadratic in x
		So $x = -2$ and $0 \Rightarrow y = -5$ and 3	M1		For solving for x and substituting to find y
			A1	**4**	For given answer correctly shown
				$\boxed{7}$	
3	**(i)**	*EITHER*: If $f(x) = \ln(x+2)$, then $f'(x) = \dfrac{1}{2+x}$	M1		For at least one differentiation attempt
		and $f''(x) = -\dfrac{1}{(2+x)^2}$	A1		For correct first and second derivatives
		$f(0) = \ln 2,\ f'(0) = \frac{1}{2},\ f''(0) = -\frac{1}{4}$	A1√		For all three evaluations correct
		Hence $\ln(x+2) = \ln 2 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots$	A1		For three correct terms
		OR: $\ln(2+x) = \ln[2(1+\frac{1}{2}x)]$	M1		For factorising in this way
		$= \ln 2 + \ln(1 + \frac{1}{2}x)$	A1		For using relevant log law correctly
		$= \ln 2 + \frac{1}{2}x - \dfrac{(\frac{1}{2}x)^2}{2} + \ldots$	M1		For use of standard series expansion
		$= \ln 2 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots$	A1	**4**	For three correct terms
	(ii)	$\ln(2-x) \approx \ln 2 - \frac{1}{2}x - \frac{1}{8}x^2$	B1√		For replacing x by $-x$
		$\ln\left(\dfrac{2+x}{2-x}\right) \approx (\ln 2 + \frac{1}{2}x - \frac{1}{8}x^2) - (\ln 2 - \frac{1}{2}x - \frac{1}{8}x^2)$	M1		For subtracting the two series
		$\approx x$, as required	A1	**3**	For showing given answer correctly
				$\boxed{7}$	

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4	(i) $r=0 \Rightarrow \cos 2\theta = 0 \Rightarrow \theta = \pm\frac{1}{4}\pi, \pm\frac{3}{4}\pi$	M1		For equating r to zero and solving for θ
		A1		For any two correct values
		A1	3	For all four correct values and no others
	(ii) Area is $\frac{1}{2}\int_{-\frac{1}{4}\pi}^{\frac{1}{4}\pi} 4\cos^2 2\theta \, d\theta$	M1		For us of correct formula $\frac{1}{2}\int r^2 \, d\theta$
		B1√		For correct limits from **(i)**
	i.e. $\int_{-\frac{1}{4}\pi}^{\frac{1}{4}\pi} 1+\cos 4\theta \, d\theta = \left[\theta + \frac{1}{4}\sin 4\theta\right]_{-\frac{1}{4}\pi}^{\frac{1}{4}\pi} = \frac{1}{2}\pi$	M1		For using double-angle formula
		A1		For $\theta + \frac{1}{4}\sin 4\theta$ correct
		A1	5	For correct (exact) answer
			$\boxed{8}$	
5	(i) LHS is the total area of the four rectangles	B1		For identifying rectangle areas (not heights)
	RHS is the corresponding area under the curve, which is clearly greater	B1	2	For correct explanation
	(ii) $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5} > \int_0^4 \frac{1}{x+2} \, dx$	M1		For attempt at relevant new inequality
		A1	2	For correct statement
	(iii) Sum is the area of 999 rectangles	M1		For considering the sum as an area again
	Bounds are $\int_0^{999} \frac{1}{x+2} \, dx$ and $\int_0^{999} \frac{1}{x+1} \, dx$	M1		For stating either integral as a bound
	So lower bound is $[\ln(x+2)]_0^{999} = \ln(500.5)$	A1		For showing the given value correctly
	and upper bound is $[\ln(x+1)]_0^{999} = \ln(1000)$	A1	4	Ditto
			$\boxed{8}$	
6	(i) $I_n = \left[-\frac{2}{3}x^n(1-x)^{\frac{3}{2}}\right]_0^1 + \frac{2}{3}n\int_0^1 x^{n-1}(1-x)^{\frac{3}{2}} \, dx$	M1		For using integration by parts
		A1		For correct first stage result
	$= \frac{2}{3}n\int_0^1 x^{n-1}(1-x)\sqrt{(1-x)} \, dx$	M1		For use of limits in integrated term
		M1		For splitting the remaining integral up
	$= \frac{2}{3}n(I_{n-1} - I_n)$	A1		For correct relation between I_n and I_{n-1}
	Hence $(2n+3)I_n = 2nI_{n-1}$, as required	A1	6	For showing given answer correctly
	(ii) $I_2 = \frac{4}{7}I_1 = \frac{4}{7}\times\frac{2}{5}I_0$	M1		For two uses of the recurrence relation
		A1		For correct expression in terms of I_0
	Hence $I_2 = \frac{8}{35}\left[-\frac{2}{3}(1-x)^{\frac{3}{2}}\right]_0^1 = \frac{16}{105}$	M1		For evaluation of I_0
		A1	4	For correct answer
			$\boxed{10}$	

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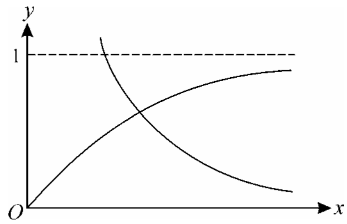
7	(i)	$\dfrac{dy}{dx} = \dfrac{\cosh x - x\sinh x}{\cosh^2 x}$	M1	For differentiating and equating to zero
		Max occurs when $\cosh x = x\sinh x$, i.e. $x\tanh x = 1$	A1 **2**	For showing given result correctly
	(ii)		B1 B1 B1 **3**	For correct sketch of $y = \tanh x$ For identification of asymptote $y = 1$ For correct explanation of $\alpha > 1$ based on intersection $(1, 1)$ of $y = 1/x$ with $y = 1$
	(iii)	$x_{n+1} = x_n - \dfrac{x_n \tanh x_n - 1}{\tanh x_n + x_n \operatorname{sech}^2 x_n}$	M1	For correct Newton-Raphson structure
			A1	For all details in $x - \dfrac{f(x)}{f'(x)}$ correct
		$x_1 = 1 \Rightarrow x_2 = 1.20177...$	M1	For using Newton-Raphson at least once
			A1	For x_2 correct to at least 3sf
		$x_3 = 1.1996785...$	A1 **5**	For x_3 correct to at least 4sf
	(iv)	$e_1 \approx 0.2,\ e_2 \approx -0.002$	B1√	For both magnitudes correct
		$\dfrac{e_3}{e_2^2} \approx \dfrac{e_2}{e_1^2} \Rightarrow e_3 \approx -2\times10^{-7}$	M1	For use of quadratic convergence property
			A1 **3**	For answer of correct magnitude
			$\boxed{13}$	
8	(i)	$\dfrac{dt}{dx} = \frac{1}{2}(1+t^2)$	B1	For this relation, stated or used
		$\displaystyle\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos}{1+\sin x}}\,dx = \int_0^1 \sqrt{\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}}}.\frac{2}{1+t^2}\,dt$	M1	For complete substitution for x in integrand
			B1	For justification of limits 0 and 1 for t
		$\displaystyle = \int_0^1 \sqrt{\frac{2t^2}{(1+t)^2}}.\frac{2}{1+t^2}\,dt = 2\sqrt{2}\int_0^1 \frac{t}{(1+t)(1+t^2)}\,dt$	A1 **4**	For correct simplification to given answer
	(ii)	$\dfrac{t}{(1+t)(1+t^2)} = \dfrac{A}{1+t} + \dfrac{Bt+C}{1+t^2}$	B1	For statement of correct form of pfs
		Hence $t \equiv A(1+t^2) + (Bt+C)(1+t)$	M1	For any use of the identity involving B or C
		From which $A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}$	B1	For correct value of A
			A1	For correct value of B
			A1 **5**	For correct value of C
	(iii)	Int is $2\sqrt{2}\left[-\frac{1}{2}\ln(1+t) + \frac{1}{4}\ln(1+t^2) + \frac{1}{2}\tan^{-1} t\right]_0^1$	B1√	For both logarithm terms correct
			B1√	For the inverse tan term correct
		$= \frac{1}{4}(\pi - 2\ln 2)\sqrt{2}$	M1	For use of appropriate limits
			A1 **4**	For correct (exact) answer in any form
			$\boxed{13}$	

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OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
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MATHEMATICS **4727**

Further Pure Mathematics 3

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

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[Turn over

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1 Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{y}{x} = x,$$

giving y in terms of x in your answer. [5]

2 The set $S = \{a, b, c, d\}$ under the binary operation $*$ forms a group G of order 4 with the following operation table.

$*$	a	b	c	d
a	d	a	b	c
b	a	b	c	d
c	b	c	d	a
d	c	d	a	b

(i) Find the order of each element of G. [3]

(ii) Write down a proper subgroup of G. [1]

(iii) Is the group G cyclic? Give a reason for your answer. [1]

(iv) State suitable values for each of a, b, c and d in the case where the operation $*$ is multiplication of complex numbers. [1]

3 The planes Π_1 and Π_2 have equations $\mathbf{r}.(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) = 1$ and $\mathbf{r}.(2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = 3$ respectively. Find

(i) the acute angle between Π_1 and Π_2, correct to the nearest degree, [4]

(ii) the equation of the line of intersection of Π_1 and Π_2, in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [4]

4 In this question, give your answers exactly in polar form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$.

(i) Express $4((\sqrt{3}) - \mathrm{i})$ in polar form. [2]

(ii) Find the cube roots of $4((\sqrt{3}) - \mathrm{i})$ in polar form. [4]

(iii) Sketch an Argand diagram showing the positions of the cube roots found in part **(ii)**. Hence, or otherwise, prove that the sum of these cube roots is zero. [3]

5 The lines l_1 and l_2 have equations

$$\frac{x-5}{1} = \frac{y-1}{-1} = \frac{z-5}{-2} \quad \text{and} \quad \frac{x-1}{-4} = \frac{y-11}{-14} = \frac{z-2}{2}.$$

(i) Find the exact value of the shortest distance between l_1 and l_2. [5]

(ii) Find an equation for the plane containing l_1 and parallel to l_2 in the form $ax + by + cz = d$. [4]

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6 The set S consists of all non-singular 2×2 real matrices $\mathbf{A}$ such that $\mathbf{AQ} = \mathbf{QA}$, where

$$\mathbf{Q} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

(i) Prove that each matrix $\mathbf{A}$ must be of the form $\begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$. [4]

(ii) State clearly the restriction on the value of a such that $\begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$ is in S. [1]

(iii) Prove that S is a group under the operation of matrix multiplication. (You may assume that matrix multiplication is associative.) [5]

7 **(i)** Prove that if $z = e^{i\theta}$, then $z^n + \frac{1}{z^n} = 2\cos n\theta$. [2]

(ii) Express $\cos^6 \theta$ in terms of cosines of multiples of θ, and hence find the exact value of

$$\int_0^{\frac{1}{3}\pi} \cos^6 \theta \, d\theta.$$ [8]

8 **(i)** Find the value of the constant k such that $y = kx^2 e^{-2x}$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 2e^{-2x}.$$ [4]

(ii) Find the solution of this differential equation for which $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$. [7]

(iii) Use the differential equation to determine the value of $\frac{d^2y}{dx^2}$ when $x = 0$. Hence prove that $0 < y \leqslant 1$ for $x \geqslant 0$. [4]

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OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
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MATHEMATICS **4727**

Further Pure Mathematics 3

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

This mark scheme consists of 4 printed pages.

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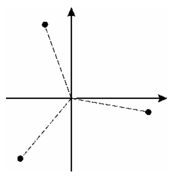
1	Integrating factor is $e^{\int -x^{-1}\,dx} = e^{-\ln x} = \frac{1}{x}$	M1 A1	For finding integrating factor For correct simplified form
	$\frac{d}{dx}\left(\frac{y}{x}\right) = 1 \Rightarrow \frac{y}{x} = \int 1\,dx \Rightarrow y = x^2 + cx$	M1 B1 A1 **5**	For using integrating factor correctly For arbitrary constant introduced correctly For correct answer in required form
		$\boxed{5}$	
2	**(i)** b is the identity and so has order 1 $d * d = b$, so d has order 2 $a * a = c * c = d$, so a and c each have order 4	B1 B1 B1 **3**	For identifying b as the identity element For stating the order of d is 2 For both orders stated
	(ii) $\{b, d\}$	B1 **1**	For stating this subgroup
	(iii) G is cyclic because it has an element of order 4	B1 **1**	For correct answer with justification
	(iv) $b = 1,\ d = -1,\ a = i,\ c = -i$ (or *vice versa* for a, c)	B1 **1**	For all four correct values
		$\boxed{6}$	
3	**(i)** Normals are $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$	B1	For identifying both normal vectors
	Acute angle is $\cos^{-1}\left(\frac{\lvert 2-4-2 \rvert}{3\times 3}\right) \approx 64°$	M1 M1 A1 **4**	For using the scalar product of the normals For completely correct process for the angle For correct answer
	(ii) Direction of line is $(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) \times (2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$, i.e. $-2\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$	M1 A1	For using vector product of normals For correct vector for **b**
	$x - 2y + 2z = 1,\ 2x + 2y - z = 3 \Rightarrow 3x + z = 4$, so a common point is $(1, 1, 1)$, for example Hence line is $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + t(-2\mathbf{i} + 5\mathbf{j} + 6\mathbf{k})$	M1 A1 **4**	For complete method to find a suitable **a** For correct equation of line (Other methods are possible)
		$\boxed{8}$	
4	**(i)** $4\left((\sqrt{3}) - i\right) = 8e^{-\frac{1}{6}\pi i}$	B1 B1 **2**	For $r = 8$ For $\theta = -\frac{1}{6}\pi$
	(ii) One cube root is $2e^{-\frac{1}{18}\pi i}$ Others are found be multiplying by $e^{\pm\frac{2}{3}\pi i}$	B1√ M1	For modulus and argument both correct For multiplication by either cube root of 1 (or equivalent use of symmetry)
	Giving $2e^{\frac{11}{18}\pi i}$ and $2e^{-\frac{13}{18}\pi i}$	A1 A1 **4**	For either one of these roots For both correct
	(iii) 	B1√	For correct diagram from their **(ii)**
	The roots have equal modulus and args differing by $\frac{2}{3}\pi$, so adding them geometrically makes a closed equilateral triangle; i.e. sum is zero	M1 A1 **3**	For geometrical interpretation of addition For a correct proof (or via components, etc)
		$\boxed{9}$	

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5	(i)	$(\mathbf{i}-\mathbf{j}-2\mathbf{k})\times(-4\mathbf{i}-14\mathbf{j}+2\mathbf{k})=-30\mathbf{i}+6\mathbf{j}-18\mathbf{k}$	M1		For vector product of direction vectors
		So common perp is parallel to $5\mathbf{i}-\mathbf{j}+3\mathbf{k}$	A1		For correct vector for common perp
		$(5\mathbf{i}+\mathbf{j}+5\mathbf{k})-(\mathbf{i}+11\mathbf{j}+2\mathbf{k})=4\mathbf{i}-10\mathbf{j}+3\mathbf{k}$	B1		For calculating the difference of positions
		$d=\dfrac{\lvert(4\mathbf{i}-10\mathbf{j}+3\mathbf{k}).(5\mathbf{i}-\mathbf{j}+3\mathbf{k})\rvert}{\lvert 5\mathbf{i}-\mathbf{j}+3\mathbf{k}\rvert}=\dfrac{39}{\sqrt{35}}$	M1		For calculation of the projection
			A1	5	For correct exact answer
	(ii)	Normal vector for plane is $5\mathbf{i}-\mathbf{j}+3\mathbf{k}$	B1√		For stating or using the normal vector
		Point on plane is $5\mathbf{i}+\mathbf{j}+5\mathbf{k}$	B1		For using any point of l_1
		Equation is $5x-y+3z=25-1+15$	M1		For using relevant direction and point
		i.e. $5x-y+3z=39$	A1	4	For a correct equation
				$\boxed{9}$	
6	(i)	$\mathbf{AQ}=\mathbf{QA}\Rightarrow\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}1&1\\0&1\end{pmatrix}=\begin{pmatrix}1&1\\0&1\end{pmatrix}\begin{pmatrix}a&b\\c&d\end{pmatrix}$	M1		For considering $\mathbf{AQ}=\mathbf{QA}$ with general $\mathbf{A}$
		i.e. $\begin{pmatrix}a&a+b\\c&c+d\end{pmatrix}=\begin{pmatrix}a+c&b+d\\c&d\end{pmatrix}$	A1		For correct simplified equation
		Hence $a=a+c$ and $a+b=b+d$	M1		For equating corresponding entries
		i.e. $c=0$ and $d=a$	A1	4	For complete proof
	(ii)	To be non-singular, $a\neq 0$	B1	1	For stating that a is non-zero
	(iii)	Identity is $\begin{pmatrix}1&0\\0&1\end{pmatrix}$ as usual, since this is in S	B1		For justifying the identity correctly
		Inverse of $\begin{pmatrix}a&b\\0&a\end{pmatrix}$ is $\begin{pmatrix}1/a&-b/a^2\\0&1/a\end{pmatrix}$, as $a\neq 0$	B1		For statement of correct inverse
			B1		For justification via non-zero a
		$\begin{pmatrix}a_1&b_1\\0&a_1\end{pmatrix}\begin{pmatrix}a_2&b_2\\0&a_2\end{pmatrix}=\begin{pmatrix}a_1a_2&a_1b_2+b_1a_2\\0&a_1a_2\end{pmatrix}$	M1		For considering a general product
		This is in S, since $a_1a_2\neq 0$, so all necessary group properties are shown	A1	5	For complete proof
				$\boxed{10}$	
7	(i)	$z^n=\cos n\theta+\mathrm{i}\sin n\theta$	B1		For applying de Moivre's theorem
		$z^{-n}=\cos n\theta-\mathrm{i}\sin n\theta$, hence $z^n+z^{-n}=2\cos n\theta$	B1	2	For complete proof
	(ii)	$2^6\cos^6\theta=(z+z^{-1})^6$	M1		For considering $(z+z^{-1})^6$
		$=(z^6+z^{-6})+6(z^4+z^{-4})+15(z^2+z^{-2})+20$	M1		For expanding and grouping terms
		$=2\cos 6\theta+12\cos 4\theta+30\cos 2\theta+20$	A1		For correct substitution of multiple angles
		Hence $\cos^6\theta=\frac{1}{32}(\cos 6\theta+6\cos 4\theta+15\cos 2\theta+10)$	A1		For correct answer
		Integral is $\frac{1}{32}\left[\frac{1}{6}\sin 6\theta+\frac{3}{2}\sin 4\theta+\frac{15}{2}\sin 2\theta+10\theta\right]_0^{\frac{1}{3}\pi}$	M1		For integrating multiple angle expression
			A1√		For correct terms
		$=\frac{1}{32}\left(\frac{1}{6}\times 0+\frac{3}{2}\times(-\frac{1}{2}\sqrt{3})+\frac{15}{2}\times(\frac{1}{2}\sqrt{3})+10\times\frac{1}{3}\pi\right)$	M1		For use of limits
		$=\frac{1}{32}\left(3\sqrt{3}+\frac{10}{3}\pi\right)$	A1	8	For correct answer
				$\boxed{10}$	

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8	**(i)**	$y = kx^2 e^{-2x} \Rightarrow y' = 2kx e^{-2x} - 2kx^2 e^{-2x}$ and	M1	For differentiation at least once
		$y'' = 2k e^{-2x} - 8kx e^{-2x} + 4kx^2 e^{-2x}$	A1	For both y' and y'' correct
		$(2k - 8kx + 4kx^2 + 8kx - 8kx^2 + 4kx^2)e^{-2x} \equiv 2e^{-2x}$	M1	For substituting completely in D.E.
		Hence $k = 1$	A1 **4**	For correct value of k
	(ii)	Auxiliary equation is $m^2 + 4m + 4 = 0 \Rightarrow m = -2$	B1	For correct repeated root
		Hence C.F. is $(A + Bx)e^{-2x}$	B1	For correct form of C.F.
		G.S. is $y = (A + Bx)e^{-2x} + x^2 e^{-2x}$	B1√	For sum of C.F. and P.I.
		$x = 0,\ y = 1 \Rightarrow 1 = A$	M1	For using given values of x and y in G.S.
		$y' = Be^{-2x} - 2(A + Bx)e^{-2x} + 2xe^{-2x} - 2x^2 e^{-2x}$	M1	For differentiating the G.S.
		$x = 0,\ y' = 0 \Rightarrow 0 = B - 2A \Rightarrow B = 2$	M1	For using given values of x and y' in G.S.
		Hence solution is $y = (1 + x)^2 e^{-2x}$	A1 **7**	For correct answer
	(iii)	$\dfrac{d^2y}{dx^2} = 2 - 4 = -2$ when $x = 0$	B1	For correct value -2
		Hence $(0, 1)$ is a maximum point	B1	For statement of maximum at $x = 0$
		$\dfrac{dy}{dx} = 2(1 + x)e^{-2x} - 2(1 + x)^2 e^{-2x} = -2x(1 + x)e^{-2x}$,		
		so there are no turning points for $x > 0$	M1	For investigation of turning points, or equiv
		Hence $0 < y \leqslant 1$, since $y \to 0$ as $x \to \infty$	A1 **4**	For complete proof of given result
			$\boxed{15}$	

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OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
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MATHEMATICS **4728**

Mechanics 1

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use 9.8 m s^{-2}.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

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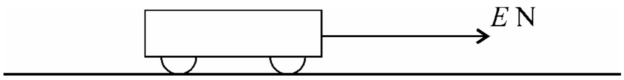
1



An engine pulls a truck of mass 6000 kg along a straight horizontal track, exerting a constant horizontal force of magnitude E newtons on the truck (see diagram). The resistance to motion of the truck has magnitude 400 N, and the acceleration of the truck is 0.2 m s^{-2}. Find the value of E. [4]

2

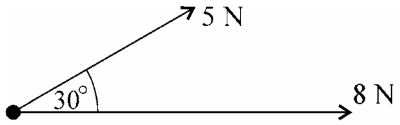


Fig. 1

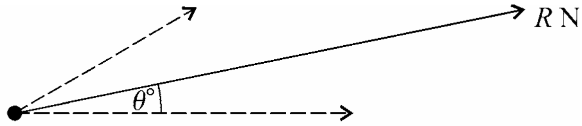


Fig. 2

Forces of magnitudes 8 N and 5 N act on a particle. The angle between the directions of the two forces is $30°$, as shown in Fig. 1. The resultant of the two forces has magnitude R N and acts at an angle $\theta°$ to the force of magnitude 8 N, as shown in Fig. 2. Find R and θ. [7]

3 A particle is projected vertically upwards, from the ground, with a speed of 28 m s^{-1}. Ignoring air resistance, find

(i) the maximum height reached by the particle, [2]

(ii) the speed of the particle when it is 30 m above the ground, [3]

(iii) the time taken for the particle to fall from its highest point to a height of 30 m, [3]

(iv) the length of time for which the particle is more than 30 m above the ground. [2]

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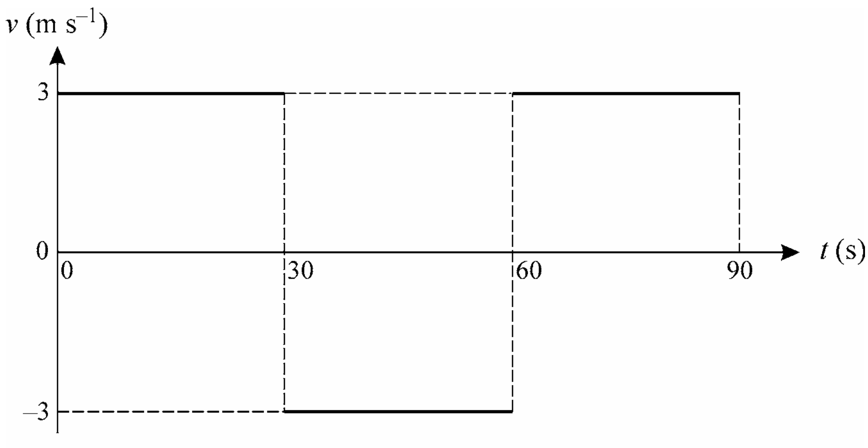

Fig. 1

A woman runs from A to B, then from B to A and then from A to B again, on a straight track, taking 90 s. The woman runs at a constant speed throughout. Fig. 1 shows the (t, v) graph for the woman.

(i) Find the total distance run by the woman. [3]

(ii) Find the distance of the woman from A when $t = 50$ and when $t = 80$, [3]

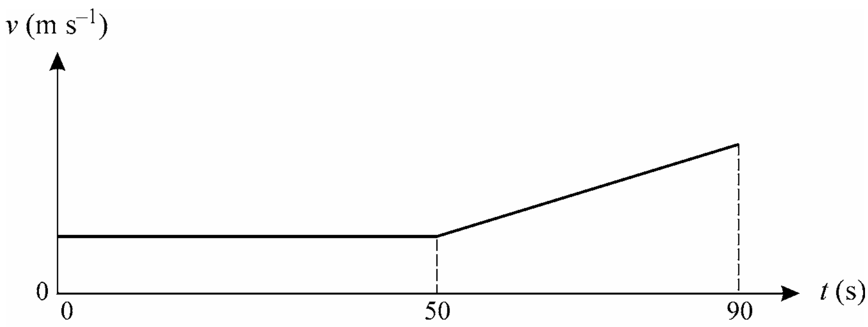

Fig. 2

At time $t = 0$, a child also starts to move, from A, along AB. The child walks at a constant speed for the first 50 s and then at an increasing speed for the next 40 s. Fig. 2 shows the (t, v) graph for the child; it consists of two straight line segments.

(iii) At time $t = 50$, the woman and the child pass each other, moving in opposite directions. Find the speed of the child during the first 50 s. [3]

(iv) At time $t = 80$, the woman overtakes the child. Find the speed of the child at this instant. [3]

5 A particle P moves in a straight line so that, at time t seconds after leaving a fixed point O, its acceleration is $-\frac{1}{10}t$ m s^{-2}. At time $t = 0$, the velocity of P is V m s^{-1}.

(i) Find, by integration, an expression in terms of t and V for the velocity of P. [4]

(ii) Find the value of V, given that P is instantaneously at rest when $t = 10$. [2]

(iii) Find the displacement of P from O when $t = 10$. [4]

(iv) Find the speed with which the particle returns to O. [3]

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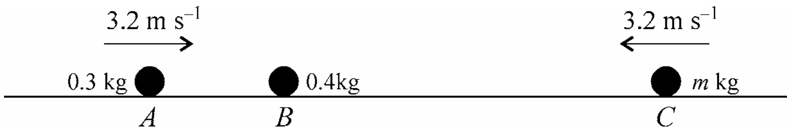

Three uniform spheres A, B and C have masses 0.3 kg, 0.4 kg and m kg respectively. The spheres lie in a smooth horizontal groove with B between A and C. Sphere B is at rest and spheres A and C are each moving with speed 3.2 m s^{-1} towards B (see diagram). Air resistance may be ignored.

(i) A collides with B. After this collision A continues to move in the same direction as before, but with speed 0.8 m s^{-1}. Find the speed with which B starts to move. [4]

(ii) B and C then collide, after which they both move towards A, with speeds of 3.1 m s^{-1} and 0.4 m s^{-1} respectively. Find the value of m. [4]

(iii) The next collision is between A and B. Explain briefly how you can tell that, after this collision, A and B cannot both be moving towards C. [1]

(iv) When the spheres have finished colliding, which direction is A moving in? What can you say about its speed? Justify your answers. [4]

7 A sledge of mass 25 kg is on a plane inclined at $30°$ to the horizontal. The coefficient of friction between the sledge and the plane is 0.2.

(i)

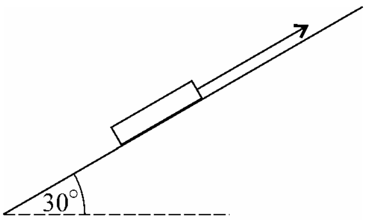

Fig. 1

The sledge is pulled up the plane, with constant acceleration, by means of a light cable which is parallel to a line of greatest slope (see Fig. 1). The sledge starts from rest and acquires a speed of 0.8 m s^{-1} after being pulled for 10 s. Ignoring air resistance, find the tension in the cable. [6]

(ii)

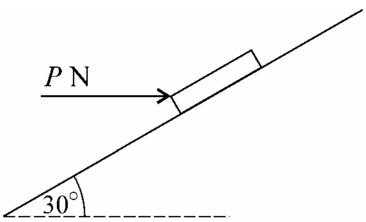

Fig. 2

On a subsequent occasion the cable is not in use and two people of total mass 150 kg are seated in the sledge. The sledge is held at rest by a horizontal force of magnitude P newtons, as shown in Fig. 2. Find the least value of P which will prevent the sledge from sliding down the plane. [7]

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MATHEMATICS **4728**

Mechanics 1

MARK SCHEME

Specimen Paper

MAXIMUM MARK	**72**

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1	$E - 400 = 6000 \times 2$	B1		For resultant force $E - 400$ stated or implied
		M1		For use of Newton II for the truck
		A1√		For the correct equation
	Hence $E = 1600$	A1	**4**	For correct answer 1600
			$\boxed{4}$	
2	*EITHER*: $R\cos\theta = 8 + 5\cos 30°$	M1		For attempt at resolving ∥ or ⊥ to 8 N force
		A1		For one completely correct equation
	$R\sin\theta = 5\sin 30°$	A1		For a second correct equation
	Hence $R^2 = (12.33...)^2 + 2.5^2$	M1		For correct method for either unknown
	$R = 12.6$	A1√		For correct value
	$\tan\theta = \dfrac{2.5}{12.33...}$	M1		For correct method for second unknown
	$\theta = 11.5$	A1√		For correct value
	OR: Triangle of forces has 5, 8, R and $150°$	M1		For considering any triangle with 5, 8, R
		A1		For correct triangle drawn or used
	$R^2 = 8^2 + 5^2 - 2\times5\times8\times\cos150°$	M1		For use of cosine formula attempted
		A1		For correct expression for R^2
	Hence $R = 12.6$	A1√		For correct value
	$\sin\theta = \dfrac{5\sin150°}{12.58...} = 0.1987...$	M1		For use of sine formula with numerical R
	Hence $\theta = 11.5$	A1√	**7**	For correct value
			$\boxed{7}$	
3	**(i)** $0 = 28^2 - 2\times9.8\times h$	M1		For use of const acc formula(s) to find h
	Hence maximum height is 40 m	A1	**2**	For correct value 40
	(ii) $v^2 = 28^2 - 2\times9.8\times30$	M1		For use of const acc formula(s) to find v
		A1		For correct equation in v
	Hence speed is 14 m s^{-1}	A1	**3**	For correct value 14
	(iii) $10 = \frac{1}{2}\times9.8t^2$	M1		For use of const acc formula(s) to find t
		A1√		For correct equation in t
	Hence time is $\frac{10}{7} \approx 1.43$ s	A1	**3**	For correct value $\frac{10}{7}$ or equivalent
	(iv) Length of time is $2\times\frac{10}{7} = \frac{20}{7}$ s	M1		For doubling, or equiv longer method
		A1√	**2**	For correct value, i.e. double their **(iii)**
			$\boxed{10}$	
4	**(i)** Total distance is $3\times30 + 3\times30 + 3\times30 = 270$ m	M1		For any calculation of a rectangular area
		M1		For addition of three positive areas
		A1	**3**	For correct value 270
	(ii) Distance at $t = 50$ is $90 - 60 = 30$ m	M1		For correct use of signed areas
		A1		For correct value 30
	Distance at $t = 80$ is 60 m	A1	**3**	For correct value 60
	(iii) Child's speed is $\dfrac{30}{50} = 0.6 \text{ m s}^{-1}$	B1√		For distance 30 m
		M1		For dividing by 50
		A1	**3**	For correct value 0.6
	(iv) Child walks $60 - 30 = 30$ m in next 30 s	B1√		For child's distance gone from $t = 50$ to 80
	Hence $30 = \frac{1}{2}(0.6 + v)\times30$	M1		For suitable use of $s = \frac{1}{2}(u+v)t$ or equiv
	i.e. child's speed is 1.4 m s^{-1}	A1	**3**	For correct value 1.4
			$\boxed{12}$	

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5 **(i)** $v = \int -\frac{1}{10}t \; dt = -\frac{1}{20}t^2 + c$	M1	For integrating the acceleration formula
	A1	For $v = -\frac{1}{20}t^2$, with or without c
$V = 0 + c$	M1	For using $v = V$ when $t = 0$ to find c
Hence $v = V - \frac{1}{20}t^2$	A1 **4**	For correct equation for v in terms of t and V
(ii) $0 = V - \frac{10^2}{20} \Rightarrow V = 5$	M1	For use of given values to find V
	A1 **2**	For correct value 5
(iii) $s = \int (5 - \frac{1}{20}t^2) \; dt = 5t - \frac{1}{60}t^3 + k$	M1	For any attempt to integrate velocity
	A1√	For correct integration (ignoring k)
Hence displacement is $50 - \frac{1000}{60} = 33\frac{1}{3}$ m	M1	For evaluation of s when $t = 10$
	A1√ **4**	For correct value $33\frac{1}{3}$; allow omission of k
(iv) Returns to O when $0 = -\frac{1}{60}t^3 + 5t \Rightarrow t^2 = 300$	M1	For attempting non-zero root of $s = 0$
When $t^2 = 300$, $v = -\frac{1}{20} \times 300 + 5$	M1	For consequent evaluation of v
i.e. speed is 10 m s^{-1}	A1 **3**	For correct value 3 (allow negative here)
	$\boxed{13}$	
6 **(i)** $0.3 \times 3.2 = 0.3 \times 0.8 + 0.4 \times b$	M1	For using conservation of momentum
	A1	For correct LHS
	A1	For correct RHS
Hence $b = 1.8$ so B's speed is 1.8 m s^{-1}	A1 **4**	For correct value 1.8 correctly obtained
(ii) $0.4 \times 1.8 - 3.2m = -0.4 \times 3.1 - 0.4m$	M1	For momentum equn with at least one relevant negative sign
	A1	For correct LHS
	A1	For correct RHS
Hence $m = 0.7$	A1 **4**	For correct value 0.4 correctly obtained
(iii) $0.4 \times 3.1 > 0.3 \times 0.8$, so net momentum of A and B is towards the left and therefore they can't both move towards the right after the impact	B1 **1**	For correctly explained application of momentum conservation.
(iv) Total momentum of all three particles is leftwards	M1	For reasoning based on the total momentum
Hence A ends up moving left, as if it moves right after all collisions so do B and C	A1	For correct conclusion regarding direction
Total momentum left is at most $1.4a$	M1	For use of the idea that $a \geqslant b \geqslant c$
Hence $1.4a \geqslant 0.7 \times 3.2 - 0.3 \times 3.2$, so the speed of A is at least 0.914 m s^{-1}	A1 **4**	For correct conclusion
	$\boxed{13}$	

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7	**(i)** Acceleration is $\frac{0.8}{10} = 0.08 \text{ m s}^{-2}$	B1		For $0.8 \div 10$ stated or implied
	$R = 25g\cos 30°$	B1		For correct resolving $\perp$ plane
	$T - 25g\sin 30° - 0.2 \times 25g\cos 30° = 25 \times 0.08$	M1		For attempting Newton II $\parallel$ plane
		B1		For upwards force $T - 25g\sin 30° - F$
		B1$\sqrt{}$		For $F = 0.2 \times 25g\cos 30°$
	Hence the tension is 167 N	A1	**6**	For correct value 167
	(ii) $R' = P\sin 30° + 175g\cos 30°$	M1		For resolving $\perp$ plane, with 3 forces
		A1		For correct equation
	$P\cos 30° + 0.2R' = 175g\sin 30°$	M1		For resolving $\parallel$ plane, with 3 forces
		A1		For correct equation
	$P(\cos 30° + 0.2\sin 30°) = 175g(\sin 30° - 0.2\cos 30°)$	M1		For attempting elimination of R'
	Hence $P = \frac{175g(\sin 30° - 0.2\cos 30°)}{\cos 30° + 0.2\sin 30°} = 580$	M1		For solving a relevant equation for P
		A1	**7**	For correct value 580
			$\boxed{13}$	

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MATHEMATICS **4729**

Mechanics 2

Specimen Paper

Additional materials:
- Answer booklet
- Graph paper
- List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use 9.8 m s^{-2}.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

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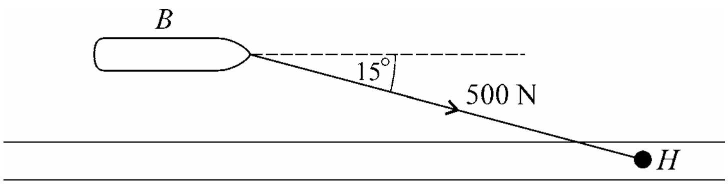
1



A barge B is pulled along a canal by a horse H, which is on the tow-path. The barge and the horse move in parallel straight lines and the tow-rope makes a constant angle of 15° with the direction of motion (see diagram). The tow-rope remains taut and horizontal, and has a constant tension of 500 N.

(i) Find the work done on the barge by the tow-rope, as the barge travels a distance of 400 m. [3]

The barge moves at a constant speed and takes 10 minutes to travel the 400 m.

(ii) Find the power applied to the barge. [2]

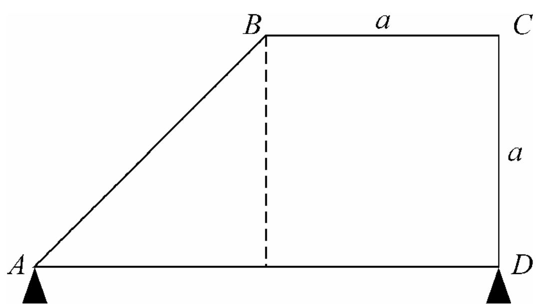
2 A uniform circular cylinder, of radius 6 cm and height 15 cm, is in equilibrium on a fixed inclined plane with one of its ends in contact with the plane.

(i) Given that the cylinder is on the point of toppling, find the angle the plane makes with the horizontal. [3]

The cylinder is now placed on a horizontal board with one of its ends in contact with the board. The board is then tilted so that the angle it makes with the horizontal gradually increases.

(ii) Given that the coefficient of friction between the cylinder and the board is $\frac{3}{4}$, determine whether or not the cylinder will slide before it topples, justifying your answer. [4]

3



A uniform lamina $ABCD$ has the shape of a square of side a adjoining a right-angled isosceles triangle whose equal sides are also of length a. The weight of the lamina is W. The lamina rests, in a vertical plane, on smooth supports at A and D, with AD horizontal (see diagram).

(i) Show that the centre of mass of the lamina is at a horizontal distance of $\frac{11}{9}a$ from A. [4]

(ii) Find, in terms of W, the magnitudes of the forces on the supports at A and D. [4]

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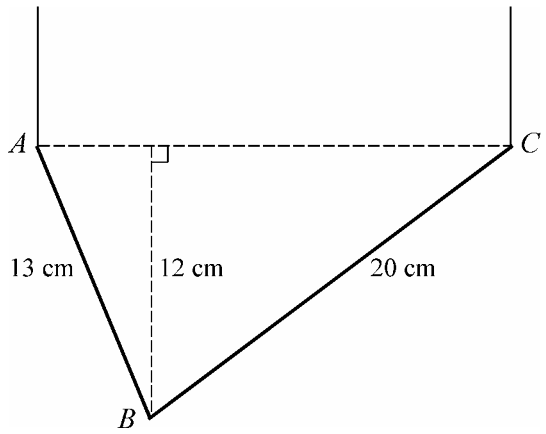
4



A rigid body ABC consists of two uniform rods AB and BC, rigidly joined at B. The lengths of AB and BC are 13 cm and 20 cm respectively, and their weights are 13 N and 20 N respectively. The distance of B from AC is 12 cm. The body hangs in equilibrium, with AC horizontal, from two vertical strings attached at A and C. Find the tension in each string. [8]

5 A cyclist and his machine have a combined mass of 80 kg. The cyclist ascends a straight hill AB of constant slope, starting from rest at A and reaching a speed of 5 m s^{-1} at B. The level of B is 4 m above the level of A.

(i) Find the gain in kinetic energy and the gain in gravitational potential energy of the cyclist and his machine. [3]

During the ascent the resistance to motion is constant and has magnitude 70 N.

(ii) Given that the work done by the cyclist in ascending the hill is 8000 J, find the distance AB. [3]

At B the cyclist is working at 720 watts and starts to move in a straight line along horizontal ground. The resistance to motion has the same magnitude of 70 N as before.

(iii) Find the acceleration with which the cyclist starts to move horizontally. [4]

6 An athlete 'puts the shot' with an initial speed of 19 m s^{-1} at an angle of $11°$ above the horizontal. At the instant of release the shot is 1.53 m above the horizontal ground. By treating the shot as a particle and ignoring air resistance, find

(i) the maximum height, above the ground, reached by the shot, [4]

(ii) the horizontal distance the shot has travelled when it hits the ground. [6]

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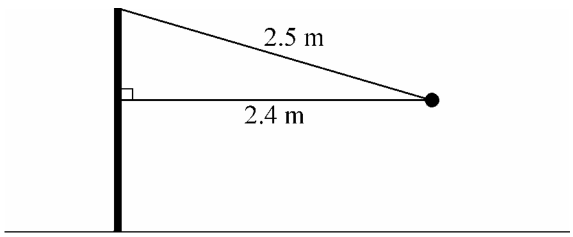
7



A ball of mass 0.08 kg is attached by two strings to a fixed vertical post. The strings have lengths 2.5 m and 2.4 m, as shown in the diagram. The ball moves in a horizontal circle, of radius 2.4 m, with constant speed v m s^{-1}. Each string is taut and the lower string is horizontal. The modelling assumptions made are that both strings are light and inextensible, and that there is no air resistance.

(i) Find the tension in each string when $v = 10.5$. [7]

(ii) Find the least value of v for which the lower string is taut. [4]

8 Two uniform smooth spheres, A and B, have the same radius. The mass of A is 0.24 kg and the mass of B is m kg. Sphere A is travelling in a straight line on a horizontal table, with speed 8 m s^{-1}, when it collides directly with sphere B, which is at rest. As a result of the collision, sphere A continues in the same direction with a speed of 6 m s^{-1}.

(i) Find the magnitude of the impulse exerted by A on B. [3]

(ii) Show that $m \leqslant 0.08$. [3]

It is given that $m = 0.06$.

(iii) Find the coefficient of restitution between A and B. [3]

On another occasion A and B are travelling towards each other, each with speed 4 m s^{-1}, when they collide directly.

(iv) Find the speeds of A and B immediately after the collision. [4]

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MATHEMATICS **4729**

Mechanics 2

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

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1 **(i)** Work done is $500\cos 15° \times 400 \approx 193\,000$ J	M1	For attempt to use Force × distance
	A1	For correct unsimplified product
	A1 **3**	For correct answer 193 000
(ii) Power applied is $\frac{193185}{600} \approx 322$ W	M1	For relevant use of $\frac{\text{work}}{\text{time}}$ or force × velocity
	A1 **2**	For correct answer 322
	$\boxed{5}$	
2 **(i)** CM is vertically above lowest point of base	B1	For stating or implying correct geometry
Hence $\tan\alpha = \frac{6}{7.5} \Rightarrow \alpha = 38.7°$	M1	For appropriate trig calculation
	A1 **3**	For correct answer 38.7
(ii) Cylinder slides when $\tan\theta = \frac{3}{4}$	B1	For stating or implying limiting friction case
But $\frac{3}{4} < 0.8$, so $\theta < \alpha$	M1	For comparing $\tan\alpha$ to $\tan\theta$, or equivalent
	A1	For correct comparison of the angles
Hence it slides first (at inclination 36.9°)	A1 **4**	For correct conclusion of sliding first
	$\boxed{7}$	
3 **(i)** CG of triangle is $\frac{2}{3}a$ horizontally from A	B1	
Moments: $\frac{1}{3}W \times \frac{2}{3}a + \frac{2}{3}W \times \frac{3}{2}a = W \times \bar{x}$	M1	For equating moments about A, or equivalent
	A1	For a correct unsimplified equation
Hence $\bar{x} = \frac{11}{9}a$	A1 **4**	Given answer correctly shown
(ii) $R_A \times 2a = W \times \frac{7}{9}a \Rightarrow R_A = \frac{7}{18}W$	M1	For one moments equation
	A1	For one correct answer
$R_A + R_D = W \Rightarrow R_D = \frac{11}{18}W$	M1	For resolving, or a second moments equation
	A1√ **4**	For a second correct answer
	$\boxed{8}$	
4 Horiz distances of B from A and C are 5 cm and 16 cm	M1	For appropriate use of Pythagoras
	A1	For both distances correct
$21T_A = 13 \times 18.5 + 20 \times 8$	M1	For any moments equation for the system
	A1√	For any one relevant term correct
	A1√	For a completely correct equation
$T_A + T_C = 33$	M1	For resolving, or using another moments eqn
Hence $T_A = 19.1$ N and $T_C = 13.9$ N	A1	For correct answer 19.1
	A1 **8**	For correct answer 13.9
	$\boxed{8}$	
5 **(i)** Gain in KE is $\frac{1}{2} \times 80 \times 5^2 = 1000$ J	M1	For use of formula $\frac{1}{2}mv^2$
Gain in PE is $80 \times 9.8 \times 4 = 3136$ J	M1	For use of formula mgh
	A1 **3**	For both answers 1000 and 3136 correct
(ii) $8000 = 1000 + 3136 + 70d$	M1	For equating work done to energy change
	M1	For relevant use of force × distance
Hence distance AB is 55.2 m	A1 **3**	For correct answer 55.2
(iii) $\frac{720}{5} - 70 = 80a$	B1	For driving force $\frac{720}{5}$
	M1	For use of Newton II with 3-term equation
	A1	For a completely correct equation
Hence acceleration is 0.925 m s^{-2}	A1 **4**	For correct answer 0.925
	$\boxed{10}$	

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6 **(i)** $0=(19\sin 11°)^2-2gh$	M1	For use of relevant const acc equation for h
	B1	For correct vertical component $19\sin 11°$
Hence max height is $\dfrac{(19\sin 11°)^2}{19.6}+1.53=2.20$ m	A1	For correct expression for h (≈ 0.67)
	A1 **4**	For correct answer 2.20
(ii) *EITHER*: Time to top point is $\dfrac{19\sin 11°}{g}\approx 0.3699$	M1	For use of relevant const acc equation for t_{up}
Time to fall is $\sqrt{\dfrac{2\times 2.20}{9.8}}\approx 0.6701$	M1	For use of relevant const acc eqn for t_{down}
	A1	For a correct expression for t_{down}
Total time of flight is 1.04	A1	For correct value (or expression)
Horiz dist is $19\cos 11°\times 1.04\approx 19.4$ m	M1	For any use of $x=(19\cos 11°)t$
	A1	For correct answer 19.4 [Alternative approaches for the first four marks are equally acceptable; e.g. the use of $s=ut-\frac{1}{2}gt^2$ to find $t=1.04$]
OR: $-1.53=x\tan 11°-\dfrac{gx^2}{2\times(19\cos 11°)^2}$	M1	For relevant use of trajectory equation
	B1	For $y=-1.53$ correctly substituted
	A1	For completely correct equation for x
Hence $x=19.4$	M1	For attempt to solve relevant quadratic
	A2 **6**	For correct answer 19.4
	10	
7 **(i)** $T_1\times\frac{7}{25}=0.08g$	M1	For resolving vertically
	B1	For $\frac{7}{25}$ or $\sin 16.3°$ or equivalent
Hence tension in upper string is 2.8 N	A1	For correct value 2.8
$T_1\times\frac{24}{25}+T_2=0.08\times\dfrac{10.5^2}{2.4}$	M1	For correct use of Newton II horizontally
	B1	For any use of $\dfrac{10.5^2}{2.4}$, or equivalent
	A1	For correct horizontal equation
Hence tension in horizontal string is 0.987 N	A1 **7**	For correct value 0.987
(ii) $2.8\times\dfrac{2.4}{2.5}=0.08\times\dfrac{v^2}{2.4}$	M1	For new horizontal equation with $T_2=0$
	A1√	For correct equation for v
Hence $v=8.98$	M1	For solving for v correctly
	A1 **4**	For correct value 8.98
	11	

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8	**(i)**	Change of momentum of A is 0.24×2	M1	For considering momentum of A
			A1	For correct expression for change in mom
		Hence magnitude of impulse is 0.48 N s	A1 **3**	For correct answer 0.48
	(ii)	$mv_B = 0.48$	M1	For considering momentum of B
		$v_B \geqslant 6$	M1	For using the inequality $v_B \geqslant v_A$
		Hence $m \leqslant \frac{0.48}{6} = 0.08$	A1 **3**	For showing given answer correctly
	(iii)	$m = 0.06 \Rightarrow v_B = 8$	B1	For correct speed of B
		Hence $8 - 6 = e(8 - 0)$	M1	For correct use of Newton's law
		i.e. $e = \frac{1}{4}$	A1 **3**	For correct answer $\frac{1}{4}$ or equivalent
	(iv)	$0.24 \times 4 - 0.06 \times 4 = 0.24a + 0.06b$	B1	For a correct momentum equation
		$b - a = \frac{1}{4}(4 + 4)$	B1√	For a correct restitution equation
		Hence speeds of A and B are 2 m s^{-1} and 4 m s^{-1}	M1	For solution of relevant simultaneous equns
			A1 **4**	For both answers correct
			$\boxed{13}$	

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MATHEMATICS 4730

Mechanics 3

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use 9.8 m s^{-2}.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

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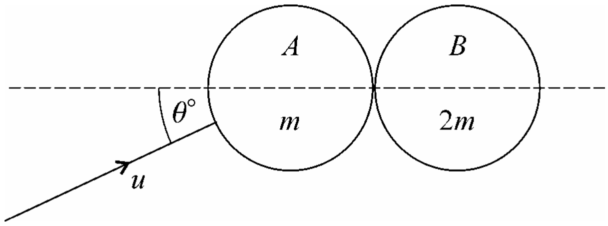
1 A particle is moving with simple harmonic motion in a straight line. The period is 0.2 s and the amplitude of the motion is 0.3 m. Find the maximum speed and the maximum acceleration of the particle. [6]

2



A sphere A of mass m, moving on a horizontal surface, collides with another sphere B of mass $2m$, which is at rest on the surface. The spheres are smooth and uniform, and have equal radius. Immediately before the collision, A has velocity u at an angle $\theta°$ to the line of centres of the spheres (see diagram). Immediately after the collision, the spheres move in directions that are perpendicular to each other.

(i) Find the coefficient of restitution between the spheres. [4]

(ii) Given that the spheres have equal speeds after the collision, find θ. [3]

3 An aircraft of mass $80\,000$ kg travelling at 90 m s^{-1} touches down on a straight horizontal runway. It is brought to rest by braking and resistive forces which together are modelled by a horizontal force of magnitude $(27\,000 + 50v^2)$ newtons, where $v \text{ m s}^{-1}$ is the speed of the aircraft. Find the distance travelled by the aircraft between touching down and coming to rest. [8]

4 For a bungee jump, a girl is joined to a fixed point O of a bridge by an elastic rope of natural length 25 m and modulus of elasticity 1320 N. The girl starts from rest at O and falls vertically. The lowest point reached by the girl is 60 m vertically below O. The girl is modelled as a particle, the rope is assumed to be light, and air resistance is neglected.

(i) Find the greatest tension in the rope during the girl's jump. [2]

(ii) Use energy considerations to find

(a) the mass of the girl, [4]

(b) the speed of the girl when she has fallen half way to the lowest point. [3]

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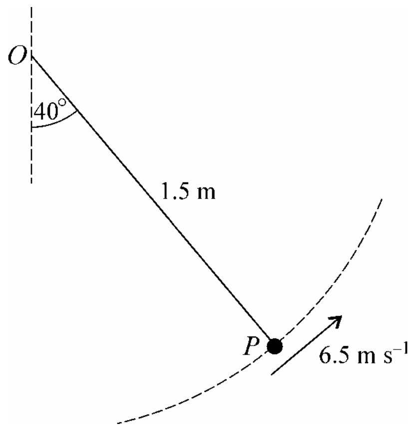
5



A particle P of mass 0.3 kg is moving in a vertical circle. It is attached to the fixed point O at the centre of the circle by a light inextensible string of length 1.5 m. When the string makes an angle of $40°$ with the downward vertical, the speed of P is $6.5\ \mathrm{m\,s^{-1}}$ (see diagram). Air resistance may be neglected.

(i) Find the radial and transverse components of the acceleration of P at this instant. [2]

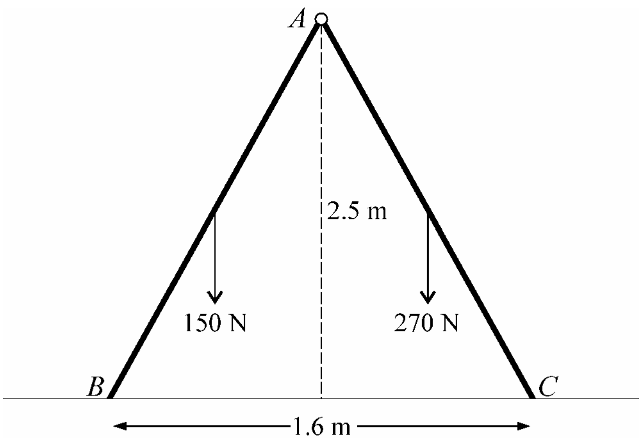
In the subsequent motion, with the string still taut and making an angle $\theta°$ with the downward vertical, the speed of P is $v\ \mathrm{m\,s^{-1}}$

(ii) Use conservation of energy to show that $v^2 \approx 19.7 + 29.4\cos\theta°$. [4]

(iii) Find the tension in the string in terms of θ. [4]

(iv) Find the value of v at the instant when the string becomes slack. [3]

6



A step-ladder is modelled as two uniform rods AB and AC, freely jointed at A. The rods are in equilibrium in a vertical plane with B and C in contact with a rough horizontal surface. The rods have equal lengths; AB has weight 150 N and AC has weight 270 N. The point A is 2.5 m vertically above the surface, and $BC = 1.6$ m (see diagram).

(i) Find the horizontal and vertical components of the force acting on AC at A. [8]

(ii) The coefficient of friction has the same value μ at B and at C, and the step-ladder is on the point of slipping. Giving a reason, state whether the equilibrium is limiting at B or at C, and find μ. [6]

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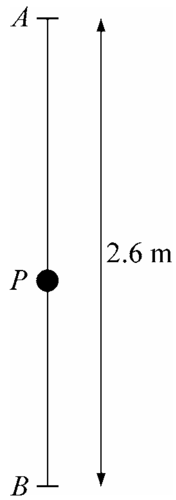
7



Two points A and B lie on a vertical line with A at a distance 2.6 m above B. A particle P of mass 10 kg is joined to A by an elastic string and to B by another elastic string (see diagram). Each string has natural length 0.8 m and modulus of elasticity 196 N. The strings are light and air resistance may be neglected.

(i) Verify that P is in equilibrium when P is vertically below A and the length of the string PA is 1.5 m. [4]

The particle is set in motion along the line AB with both strings remaining taut. The displacement of P below the equilibrium position is denoted by x metres.

(ii) Show that the tension in the string PA is $245(0.7+x)$ newtons, and the tension in the string PB is $245(0.3-x)$ newtons. [3]

(iii) Show that the motion of P is simple harmonic. [3]

(iv) Given that the amplitude of the motion is 0.25 m, find the proportion of time for which P is above the mid-point of AB. [5]

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MATHEMATICS 4730

Mechanics 3

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

This mark scheme consists of 4 printed pages.

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1 $0.2=\frac{2\pi}{\omega}\Rightarrow\omega=10\pi$	M1	For relevant use of $\frac{2\pi}{\omega}$
	A1	For correct value 10π
Hence maximum speed is $0.3\times 10\pi=3\pi\approx 9.42$ m s^{-1}	M1	For relevant use of $v=a\omega$
	A1√	For correct value 3π or 9.42
Maximum acc is $0.3\times(10\pi)^2=30\pi^2\approx 296$ m s^{-2}	M1	For relevant use of $a\omega^2$
	A1√ 6	For correct value 30π or 296
	$\boxed{6}$	
3 **(i)** A and B move off $\perp$ and $\parallel$ resp. to line of centres	M1	For correct directions of motion after impact
$2mv_B=mu\cos\theta$	A1	For correct momentum equation
$v_B=eu\cos\theta$	A1	For correct restitution equation
Hence $e=0.5$	A1 4	For correct answer 0.5
(ii) $v_A=u\sin\theta$	B1	For correct equation
Hence $v_A=v_B\Rightarrow u\sin\theta=0.5u\cos\theta$	M1	For forming the relevant equation for θ
So $\theta=\tan^{-1}0.5\approx 26.6°$	A1 3	For correct value 26.6
	$\boxed{7}$	
3 $80\,000v\frac{\mathrm{d}v}{\mathrm{d}x}=-(27\,000+50v^2)$	M1	For using Newton II to form a DE
	A1	For correct equation including $v\frac{\mathrm{d}v}{\mathrm{d}x}$
Hence $x=-\int\frac{1600v}{540+v^2}\,\mathrm{d}v$	M1	For separation of variables
$=-800\ln(540+v^2)+k$	M1	For logarithmic form of integral
	A1√	For correct integration of (their) $\frac{av}{b+cv^2}$
$v=90$ when $x=0\Rightarrow k=800\ln 8640$	M1	For use of initial condition to find k
Hence when $v=0$, $x=800\ln 16$	M1	For evaluation of required distance
		(The previous two M marks can equivalently be earned by using definite integration)
So distance is 2220 m approximately	A1 8	For correct value 2220
	$\boxed{8}$	
4 **(i)** Greatest tension $=\frac{1320\times 35}{25}=1848$ N	M1	For use of $\frac{\lambda x}{l}$ at lowest point
	A1 2	For correct answer 1848
(ii) **(a)** $mg\times 60=\frac{1320}{2\times 25}(60-25)^2$	M1	For use of correct EPE formula $\frac{\lambda x^2}{2l}$
	A1	For correct unsimplified expression for EPE
Hence the girl's mass is 55 kg	M1	For use of equation involving EPE and GPE
	A1 4	For correct answer 55
(b) $55g\times 30=\frac{1}{2}\times 55v^2+\frac{1320}{2\times 25}\times(30-25)^2$	M1	For energy equation with KE, GPE and EPE
	A1√	For equation with all terms correct
So $v^2=564$, hence speed is 23.7 m s^{-1}	A1 3	For correct answer 24.3
	$\boxed{9}$	

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5 **(i)** Radial acc is $\frac{6.5^2}{1.5} = 28.2 \text{ m s}^{-2}$	B1		For correct value 28.2
Transverse acc is $g \sin 40° = 6.30 \text{ m s}^{-2}$	B1	**2**	For correct value 6.30
(ii) $\frac{1}{2} \times 0.3 \times (6.5^2 - v^2) = 0.3 \times 9.8 \times 1.5(\cos 40° - \cos\theta°)$	M1		For equating PE gain to KE loss, or equiv
	B1		For correct expression for PE gain
	B1		For correct expression for KE loss
Hence $42.25 - v^2 = 29.4(\cos 40° - \cos\theta°)$			
i.e. $v^2 \approx 19.7 + 29.4\cos\theta°$	A1	**4**	For showing given answer correctly
(iii) $T - 0.3g\cos\theta° = 0.3 \times \frac{v^2}{1.5}$	M1		For use of Newton II, including $\frac{mv^2}{r}$ term
	A1		For correct (unsimplified) equation
Hence $T = 2.94\cos\theta° + 0.2(19.7 + 29.4\cos\theta°)$	M1		For substitution, to obtain expression for T
$= 3.95 + 8.82\cos\theta°$	A1	**4**	For correct answer
(iv) $T = 0$ when $3.95 + 8.82\cos\theta° = 0$	M1		For equating T to zero to find $\cos\theta$
Hence $v^2 = 19.7 + 29.4 \times \left(-\frac{3.95}{8.82}\right) \Rightarrow v \approx 2.56$	M1		For using this $\cos\theta$ to find v
	A1	**3**	For correct answer 2.56
		$\boxed{13}$	
6 **(i)** Mom @ B for BAC: $V_C \times 1.6 = 150 \times 0.4 + 270 \times 1.2$	M1		For suitable moments equation for BAC
Hence $V_C = 240$	A1		For correct value for V_C (or equivalent)
Mom @ C for AC: $V_A \times 0.8 + H_A \times 2.5 = 270 \times 0.4$	M1		For a moments equation for one rod with all required forces included
	A1		For a correct equation
Res $\updownarrow$ for AC: $V_A + V_C = 270$	M1		For another equation leading to V_A
Hence $V_A = 270 - 240 = 30$ N (upwards)	A1		For correct magnitude and direction
and $2.5H_A = 108 - 0.8 \times 30 \Rightarrow H_A = 33.6$ N (right)	M1		For substituting back to find H_A
	A1	**8**	For correct magnitude and direction
(ii) $V_B = 270 + 150 - V_C = 180$	M1		For finding all of V_B, H_B and H_C
	A1√		For correct V_B
$H_B = H_C = H_A = 33.6$	A1√		For both H_B and H_C correct
$\frac{H_B}{V_B} = \frac{33.6}{180} = 0.187$, $\frac{H_C}{V_C} = \frac{33.6}{240} = 0.14$	M1		For considering ratios at B and C, or equiv
Hence friction is limiting at B	A1√		For identifying point with larger ratio
Value of μ is 0.187	A1√	**6**	For identifying the larger ratio as μ
		$\boxed{14}$	

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7 (i)	$T_{AP} = \frac{196}{0.8} \times (1.5 - 0.8) = 171.5$	M1		For using Hook's law to find either tension
	$T_{BP} = \frac{196}{0.8} \times (2.6 - 1.5 - 0.8) = 73.5$	A1		For both tensions correct
	$T_{AP} - T_{BP} = 98 = 10g$, hence equilibrium	M1		For considering $T_{AP} = mg + T_{BP}$, or equiv
		A1	4	For showing given result correctly
(ii)	Extension of *PA* is $1.5 + x - 0.8 = 0.7 + x$	M1		For finding either extension in terms of x
	Hence $T_{AP} = \frac{196}{0.8}(0.7 + x) = 245(0.7 + x)$	A1		For showing one given answer correctly
	and $T_{BP} = \frac{196}{0.8}(1.1 - x - 0.8) = 245(0.3 - x)$	A1	3	For showing the other given answer correctly
(iii)	$245(0.3 - x) + 10g - 245(07. + x) = 10\ddot{x}$	M1		For use of Newton II, at a general position
		A1		For a correct equation
	Hence $\ddot{x} = -49x$, so the motion is SHM	A1	3	For showing the given result correctly
(iv)	$0.2 = 0.25\cos(7t)$	M1		For use of ± 0.2 in SHM equation involving t
		A1		For a correct equation for a relevant time
	Hence half of time above mid-pt is $t = 0.0919...$	A1		For correct value for a relevant time
	Proportion is $\frac{t}{\pi / \omega} = 0.205$	M1		For relating t to period of oscillation
		A1	5	For correct proportion 0.205
			$\boxed{15}$	

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MATHEMATICS **4731**

Mechanics 4

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use 9.8 m s^{-2}.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

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- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

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1 A circular flywheel of radius 0.2 m is rotating freely about a fixed axis through its centre and perpendicular to its plane. The moment of inertia of the flywheel about the axis is 0.37 kg m^2. When the angular speed of the flywheel is 8 rad s^{-1} a particle of mass 0.75 kg, initially at rest, sticks to a point on the circumference of the flywheel. Find

(i) the angular speed of the flywheel immediately after the particle has stuck to it, [4]

(ii) the loss of energy that results when the particle sticks to the flywheel. [2]

2 A uniform solid sphere, of mass 4 kg and radius 0.1 m, is rotating freely about a fixed axis with angular speed 20 rad s^{-1}. The axis is a diameter of the sphere. A couple, having constant moment 0.36 N m about the axis and acting in the direction of rotation, is then applied for 6 seconds. For this time interval, find

(i) the angular acceleration of the sphere, [3]

(ii) the angle through which the sphere turns, [2]

(iii) the work done by the couple. [2]

3 The region bounded by the x-axis, the y-axis, and the curve $y = 4 - x^2$ for $0 \leqslant x \leqslant 2$, is occupied by a uniform lamina of mass 35 kg. The unit of length is the metre. Show that the moment of inertia of the lamina about the y-axis is 28 kg m^2. [8]

4 A straight rod AB of length a has variable density, and at a distance x from A its mass per unit length is $k\left(1 + \dfrac{x^2}{a^2}\right)$, where k is a constant.

(i) Find the distance of the centre of mass of the rod from A. [6]

You are given that the moment of inertia of the rod about a perpendicular axis through A is $\frac{8}{15}ka^3$.

(ii) Show that the period of oscillation of the rod as a compound pendulum, when freely pivoted at the other end B, is $2\pi\sqrt{\dfrac{22a}{35g}}$. [5]

5 A uniform rod AB, of mass m and length $2a$, is free to rotate in a vertical plane about a fixed horizontal axis through A. The rod is released from rest with AB horizontal. Air resistance may be neglected. For the instant when the rod has rotated through an angle $\frac{1}{6}\pi$,

(i) show that the angular acceleration of the rod is $\dfrac{(3\sqrt{3})g}{8a}$, [2]

(ii) find the angular speed of the rod, [3]

(iii) show that the force acting on the rod at A has magnitude $\dfrac{\sqrt{103}}{8}mg$. [7]

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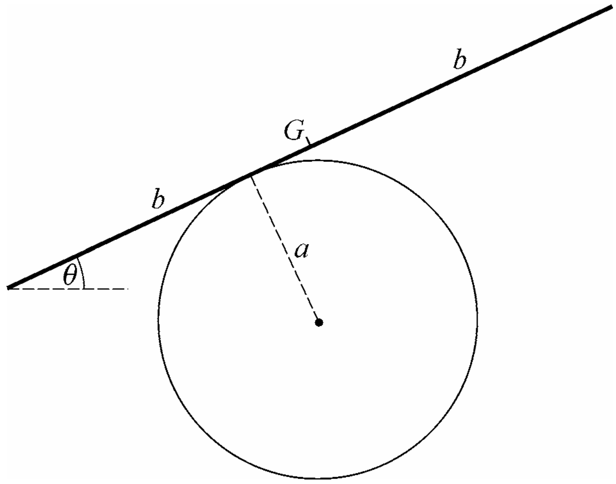
6



A cylinder with radius a is fixed with its axis horizontal. A uniform rod, of mass m and length $2b$, moves in a vertical plane perpendicular to the axis of the cylinder, maintaining contact with the cylinder and not slipping (see diagram). When the rod is horizontal, its mid-point G is in contact with the cylinder. You are given that, when the rod makes an angle θ with the horizontal, the height of G above the axis of the cylinder is $a(\theta \sin \theta + \cos \theta)$.

(i) By considering the potential energy of the rod, show that $\theta = 0$ is a position of stable equilibrium. [6]

(ii) You are also given that, when θ is small, the kinetic energy of the rod is approximately $\frac{1}{6} m b^2 \dot{\theta}^2$.

Show that the approximate period of small oscillations about the position $\theta = 0$ is $\dfrac{2\pi b}{\sqrt{(3ga)}}$. [7]

7 An unidentified object U is flying horizontally due east at a constant speed of 220 m s^{-1}. An aircraft is 15 000 m from U and is at the same height as U. The bearing of U from the aircraft is $310°$.

(i) Assume that the aircraft flies in a straight line at a constant speed of 160 m s^{-1}.

(a) Find the bearings of the two possible directions in which the aircraft can fly to intercept U. [6]

(b) Given that the interception occurs in the shorter of the two possible times, find the time taken to make the interception. [5]

(ii) Assuming instead that the aircraft flies in a straight line at a constant speed of 130 m s^{-1}, show that the nearest the aircraft can come to U is approximately 988 m. [4]

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MATHEMATICS **4731**

Mechanics 4

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

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1	(i) MI with particle is $0.37 + 0.75 \times 0.2^2 = 0.4$	M1 A1	For 0.75×0.2^2 For correct MI, stated or implied
	$0.4\omega = 0.37 \times 8$	M1	For relevant use of cons. of ang. mom.
	Hence angular speed is 7.4 rad s^{-1}	A1 **4**	For correct value 7.4
	(ii) K.E. loss $\frac{1}{2} \times 0.37 \times 8^2 - \frac{1}{2} \times 0.4 \times 7.4^2 = 0.888$ J	M1	For an correct relevant use of $\frac{1}{2} I\omega^2$
		A1√ **2**	For correct value for the KE loss
		$\boxed{6}$	
2	(i) $I = \frac{2}{5} \times 4 \times 0.1^2 = 0.016$	B1	For correct use of $\frac{2}{5}mr^2$
	$0.36 = 0.016\alpha$	M1	For use of $C = I\alpha$ to find α
	Hence angular acceleration is 22.5 rad s^{-2}	A1 **3**	For correct value 22.5
	(ii) $\theta = 20 \times 6 + \frac{1}{2} \times 22.5 \times 6^2$	M1	For use of $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ to find θ
	Angle turned through is 525 radians	A1√ **2**	For correct answer 525
	(iii) Work done $= 0.36 \times 525 = 189$ J	M1	For use of $C\theta$, or increase in $\frac{1}{2} I\omega^2$
		A1√ **2**	For correct answer 189
		$\boxed{7}$	
3	*EITHER*: Area is $\int_0^2 (4 - x^2)\,dx = \left[4x - \frac{1}{3}x^3\right]_0^2 = \frac{16}{3}$	M1	For evaluation of $\int_0^2 y\,dx$
		A1	For correct value $\frac{16}{3}$
	Hence $\frac{16}{3}\rho = 35 \Rightarrow \rho = \frac{105}{16}$	B1√	For correct density
	$I = \int_0^2 \rho x^2 y\,dx = \frac{105}{16}\int_0^2 x^2(4 - x^2)\,dx$	M1	For use of $\int x^2 y\,dx$
		A1√	For correct expression for I
	$= \frac{105}{16}\left[\frac{4}{3}x^3 - \frac{1}{5}x^5\right]_0^2 = \frac{105}{16} \times \frac{64}{15} = 28$	A1	For correct indefinite integral $\frac{4}{3}x^3 - \frac{1}{5}x^5$
		A1√	For correct numerical expression $\frac{64}{15}\rho$
		A1	For obtaining given answer 28 correctly
	OR: Area is $\int_0^4 (4 - y)^{\frac{1}{2}}\,dy = \left[-\frac{2}{3}(4 - y)^{\frac{3}{2}}\right]_0^4 = \frac{16}{3}$	M1	For evaluation of $\int_0^4 x\,dy$
		A1	For correct value $\frac{16}{3}$
	Hence $\frac{16}{3}\rho = 35 \Rightarrow \rho = \frac{105}{16}$	B1√	For correct density
	$I = \frac{1}{3}\rho\int_0^4 x^3\,dy = \frac{35}{16}\int_0^4 (4 - y)^{\frac{3}{2}}\,dy$	M1	For use of $\frac{1}{3}\int x^3\,dy$
		A1√	For correct expression for I
	$= \frac{35}{16}\left[-\frac{2}{5}(4 - y)^{\frac{5}{2}}\right]_0^4 = \frac{35}{16} \times \frac{64}{5} = 28$	A1	For correct indefinite integral $-\frac{2}{5}(4 - y)^{\frac{5}{2}}$
		A1√	For correct numerical expression $\frac{1}{3}\rho \times \frac{64}{5}$
		A1 **8**	For obtaining given answer 28 correctly
		$\boxed{8}$	

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4 **(i)** Moment @ $A = \int_0^a kx\left(1+\frac{x^2}{a^2}\right)dx = k\left[\frac{x^2}{2}+\frac{x^4}{4a^2}\right]_0^a$	M1	For attempted integration of ρx with limits
$= \frac{3}{4}ka^2$	A1	For correct MI $\frac{3}{4}ka^2$
Mass of rod is $\int_0^a k\left(1+\frac{x^2}{a^2}\right)dx = k\left[x+\frac{x^3}{3a^2}\right]_0^a$	M1	For attempted integration of ρ with limits
$= \frac{4}{3}ka$	A1	For correct mass $\frac{4}{3}ka$
Hence $\frac{4}{3}ka\bar{x} = \frac{3}{4}ka^2 \Rightarrow \bar{x} = \frac{9}{16}a$	M1	For moments equation for $\bar{x}$
	A1 **6**	For correct answer $\frac{9}{16}a$
(ii) $I_G = I_A - m(\bar{x})^2 = \frac{8}{15}ka^3 - \frac{4}{3}ka\left(\frac{9}{16}a\right)^2 = \frac{107}{960}ka^3$	B1	For stating correct relation $I_G = I_A - m(\bar{x})^2$
$I_B = I_G + m(a-\bar{x})^2 = \frac{107}{960}ka^3 + \frac{4}{3}ka\left(\frac{7}{16}a\right)^2 = \frac{11}{30}ka^3$	M1	For correct use of || axes to find I_B
	A1	For correct value $\frac{11}{30}ka^3$, or equivalent
Period is $2\pi\sqrt{\frac{\frac{11}{30}ka^3}{(\frac{4}{3}ka)g(\frac{7}{16}a)}} = 2\pi\sqrt{\frac{22a}{35g}}$	M1	For correct use of $2\pi\sqrt{\frac{I}{mgh}}$
	A1 **5**	For showing given answer correctly
	$\boxed{11}$	
5 **(i)** $mga\cos\frac{1}{6}\pi = \frac{4}{3}ma^2\alpha$	M1	For use of $C = I_A\alpha$
Hence $\alpha = \frac{(3\sqrt{3})g}{8a}$	A1 **2**	For obtaining given answer correctly
(ii) $\frac{1}{2}\times\frac{4}{3}ma^2\times\omega^2 = mga\sin\frac{1}{6}\pi$	M1	For relevant use of conservation of energy
	A1	For correct equation
Hence $\omega = \sqrt{\left(\frac{3g}{4a}\right)}$	A1 **3**	For correct answer
(iii) Res || rod: $R - mg\sin\frac{1}{6}\pi = ma\omega^2$	M1	For Newton II equation with 3 terms
Hence $R = \frac{1}{2}mg + \frac{3}{4}mg = \frac{5}{4}mg$	A1√	For correct component
Res ⊥ rod: $mg\cos\frac{1}{6}\pi - S = ma\alpha$	M1	For Newton II equation with 3 terms
	A1	For correct equation
Hence $S = \left(\frac{1}{2}\sqrt{3}\right)mg - \left(\frac{3}{8}\sqrt{3}\right)mg = \left(\frac{1}{8}\sqrt{3}\right)mg$	A1	For correct component
Magnitude is $\sqrt{(R^2+S^2)} = \frac{1}{8}mg\sqrt{(10^2+3)}$	M1	For correct method for resultant
$= \frac{\sqrt{103}}{8}mg$	A1 **7**	For obtaining given answer correctly
	$\boxed{12}$	

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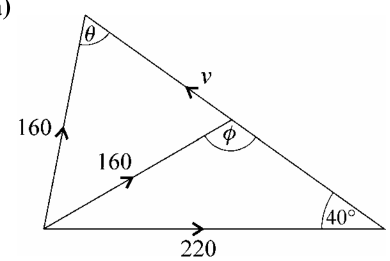
6 **(i)**	$V = mga(\theta\sin\theta + \cos\theta)$, so			
	$\dfrac{dV}{d\theta} = mga(\theta\cos\theta + \sin\theta - \sin\theta) = mga\theta\cos\theta$	M1		For differentiation using product rule
		A1		For correct derivative
	Hence equilibrium at $\theta = 0$, since $\dfrac{dV}{d\theta} = 0$	A1		For showing the given result correctly
	$\dfrac{d^2V}{d\theta^2} = mga(\cos\theta - \theta\sin\theta)$	M1		For differentiating again using product rule
		A1		For correct second derivative
	When $\theta = 0$, $\dfrac{d^2V}{d\theta^2} = mga > 0$, so equm is stable	A1	6	For showing the given result correctly
(ii)	$mga(\theta\sin\theta + \cos\theta) + \frac{1}{6}mb^2\dot{\theta}^2 = K$	B1		For correct statement of energy equation
	Hence $(mga\theta\cos\theta)\dot{\theta} + \frac{1}{3}mb^2\dot{\theta}\ddot{\theta} = 0$	M1		For attempt to differentiate w.r.t. t
		A1√		For correct derivative of PE term
		A1		For correct derivative of KE term
	For small θ, $mga\theta + \frac{1}{3}mb^2\ddot{\theta} \approx 0 \Rightarrow \ddot{\theta} \approx -\dfrac{3ga}{b^2}\theta$	M1		For use of $\cos\theta \approx 1$ and simplifying
	Motion is approximate SHM with period $\dfrac{2\pi b}{\sqrt{(3ga)}}$	M1		For use of $\dfrac{2\pi}{\omega}$ from standard SHM form
		A1	7	For showing the given answer correctly
			$\boxed{13}$	
7 **(i)** **(a)**		B1		For correct triangle for at least one case
		B1		For both triangle (together or separately)
	$\dfrac{\sin\theta}{220} = \dfrac{\sin 40°}{160}$	M1		For a method for finding a relevant angle
	Hence $\theta = 62.1°$, $\phi = 117.9°$	A1		For either angle correct
	Required bearings are $012.1°$ and $067.9°$	A1		For one correct bearing
		A1	6	For the other correct bearing
(b)	Shorter time occurs for $\theta = 62.1°$	B1√		For selecting the appropriate case
	$\dfrac{v}{\sin 77.9°} = \dfrac{160}{\sin 40°} \Rightarrow v = 243.4$	M1		For finding the relative speed, or equivalent
		A1		For correct value 243.4
	Hence time is $\dfrac{15000}{243.4} = 61.6$ s	M1		For calculation of the time taken
		A1√	5	For correct value 61.6
(ii)	For closest approach, $\sin\alpha = \dfrac{130}{220} \Rightarrow \alpha = 36.2°$	M1		For use of correct velocity triangle
		A1		For correct angle
	Hence min distance is $15000\sin(40 - \alpha) \approx 988$ m	M1		For use of correct displacement triangle
		A1	4	For showing given answer correctly
			$\boxed{15}$	

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OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4732

Probability and Statistics 1

Specimen Paper

Additional materials:
- Answer booklet
- Graph paper
- List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

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1 Janet and John wanted to compare their daily journey times to work, so they each kept a record of their journey times for a few weeks.

(i) Janet's daily journey times, x minutes, for a period of 25 days, were summarised by $\Sigma x = 2120$ and $\Sigma x^2 = 180\,044$. Calculate the mean and standard deviation of Janet's journey times. [3]

(ii) John's journey times had a mean of 79.7 minutes and a standard deviation of 6.22 minutes. Describe briefly, in everyday terms, how Janet and John's journey times compare. [2]

2 Two independent assessors awarded marks to each of 5 projects. The results were as shown in the table.

Project	*A*	*B*	*C*	*D*	*E*
First assessor	38	91	62	83	61
Second assessor	56	84	41	85	62

(i) Calculate Spearman's rank correlation coefficient for the data. [5]

(ii) Show, by sketching a suitable scatter diagram, how two assessors might have assessed 5 projects in such a way that Spearman's rank correlation coefficient for their marks was $+1$ while the product moment correlation coefficient for their marks was not $+1$. (Your scatter diagram need not be drawn accurately to scale.) [2]

3 Five friends, Ali, Bev, Carla, Don and Ed, stand in a line for a photograph.

(i) How many different possible arrangements are there if Ali, Bev and Carla stand next to each other? [2]

(ii) How many different possible arrangements are there if none of Ali, Bev and Carla stand next to each other? [3]

(iii) If all possible arrangements are equally likely, find the probability that two of Ali, Bev and Carla are next to each other, but the third is not next to either of the other two. [3]

4 Each packet of the breakfast cereal Fizz contains one plastic toy animal. There are five different animals in the set, and the cereal manufacturers use equal numbers of each. Without opening a packet it is impossible to tell which animal it contains. A family has already collected four different animals at the start of a year and they now need to collect an elephant to complete their set. The family is interested in how many packets they will need to buy before they complete their set.

(i) Name an appropriate distribution with which to model this situation. State the value(s) of any parameter(s) of the distribution, and state also any assumption(s) needed for the distribution to be a valid model. [3]

(ii) Find the probability that the family will complete their set with the third packet they buy after the start of the year. [2]

(iii) Find the probability that, in order to complete their collection, the family will need to buy more than 4 packets after the start of the year. [3]

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5 A sixth-form class consists of 7 girls and 5 boys. Three students from the class are chosen at random. The number of boys chosen is denoted by the random variable X. Show that

(i) $P(X = 0) = \frac{7}{44}$, [2]

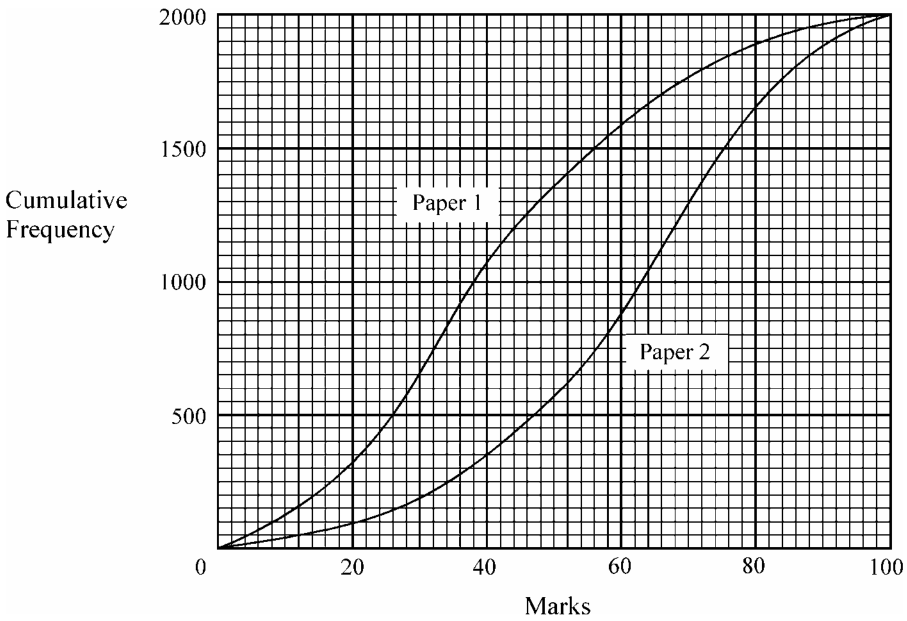
(ii) $P(X = 2) = \frac{7}{22}$. [3]

The complete probability distribution of X is shown in the following table.

x	0	1	2	3
$P(X = x)$	$\frac{7}{44}$	$\frac{21}{44}$	$\frac{7}{22}$	$\frac{1}{22}$

(iii) Calculate $E(X)$ and $Var(X)$. [5]

6



The diagram shows the cumulative frequency graphs for the marks scored by the candidates in an examination. The 2000 candidates each took two papers; the upper curve shows the distribution of marks on paper 1 and the lower curve shows the distribution on paper 2. The maximum mark on each paper was 100.

(i) Use the diagram to estimate the median mark for each of paper 1 and paper 2. [3]

(ii) State with a reason which of the two papers you think was the easier one. [2]

(iii) To achieve grade A on paper 1 candidates had to score 66 marks out of 100. What mark on paper 2 gives equal proportions of candidates achieving grade A on the two papers? What is this proportion? [4]

(iv) The candidates' marks for the two papers could also be illustrated by means of a pair of box-and whisker plots. Give two brief comments comparing the usefulness of cumulative frequency graphs and box-and-whisker plots for representing the data. [2]

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7 Items from a production line are examined for any defects. The probability that any item will be found to be defective is 0.15, independently of all other items.

(i) A batch of 16 items is inspected. Using tables of cumulative binomial probabilities, or otherwise, find the probability that

(a) at least 4 items in the batch are defective, [2]

(b) exactly 4 items in the batch are defective. [2]

(ii) Five batches, each containing 16 items, are taken.

(a) Find the probability that at most 2 of these 5 batches contain at least 4 defective items. [4]

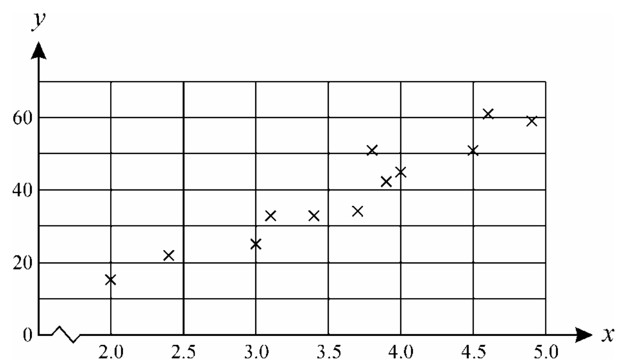
(b) Find the expected number of batches that contain at least 4 defective items. [2]

8 An experiment was conducted to see whether there was any relationship between the maximum tidal current, $y\,\text{cm}\,\text{s}^{-1}$, and the tidal range, x metres, at a particular marine location. [The *tidal range* is the difference between the height of high tide and the height of low tide.] Readings were taken over a period of 12 days, and the results are shown in the following table.

x	2.0	2.4	3.0	3.1	3.4	3.7	3.8	3.9	4.0	4.5	4.6	4.9
y	15.2	22.0	25.2	33.0	33.1	34.2	51.0	42.3	45.0	50.7	61.0	59.2

$$[\Sigma x = 43.3, \Sigma y = 471.9, \Sigma x^2 = 164.69, \Sigma y^2 = 20\,915.75, \Sigma xy = 1837.78.]$$

The scatter diagram below illustrates the data.



(i) Calculate the product moment correlation coefficient for the data, and comment briefly on your answer with reference to the appearance of the scatter diagram. [4]

(ii) Calculate the equation of the regression line of maximum tidal current on tidal range. [4]

(iii) Estimate the maximum tidal current on a day when the tidal range is 4.2 m, and comment briefly on how reliable you consider your estimate is likely to be. [3]

(iv) It is suggested that the equation found in part **(ii)** could be used to predict the maximum tidal current on a day when the tidal range is 15 m. Comment briefly on the validity of this suggestion. [2]

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Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS **4732**

Probability and Statistics 1

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

This mark scheme consists of 4 printed pages.

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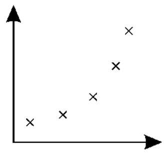
1	(i)	Mean is 84.8 minutes	B1		For correct value 84.8
		Standard deviation $= \sqrt{\dfrac{180044}{25} - 84.8^2}$	M1		For correct formula or calculator use
		$= 3.27$ minutes	A1	3	For correct value 3.27
	(ii)	John's average time is about 5 minutes less than Janet's	B1√		For correct comparison of averages
		John's times are more variable than Janet's	B1√	2	For correct comparison of variability
				$\boxed{5}$	
2	(i)	Ranks are: $\begin{matrix} 1 & 5 & 3 & 4 & 2 \\ 2 & 4 & 1 & 5 & 3 \end{matrix}$	B2		For correct ranks (or reversed); B1 if 1 error
		Values of d are $-1,\ 1,\ 2,\ -1,\ -1$	M1		For correct values of d or d^2
		$r_s = 1 - \dfrac{6 \times 8}{5 \times 24} = 0.6$	M1		For use of the Spearman formula
			A1	5	For correct answer 0.6 or fractional equiv
	(ii)	 (e.g.)	B2	2	For 5 points, showing any non-linear 'increasing' relationship
				$\boxed{7}$	
3	(i)	$3! \times 3! = 36$	M1		For at least one factor of 3!
			A1	2	For correct answer
	(ii)	Ali, Bev and Carla must be in 1st, 3rd, 5th, posns	B1		For identifying this restriction
		Hence number of ways is $3! \times 2! = 12$	M1		For at least one of the factors present
			A1	3	For correct answer
	(iii)	Total number of possible arrangements is 5!	B1		For correct statement or use of 5!
		No. of ways with 2 together is $5! - 36 - 12 = 72$	M1		For subtraction of **(i)** and **(ii)** from total
		Hence probability is $\frac{72}{120} = \frac{3}{5}$	A1	3	For correct answer
				$\boxed{8}$	
4	(i)	Geometric distribution	B1		For 'geometric' or 'Geo(...)' stated
		$p = \frac{1}{5}$	B1		For correct parameter value
		Each packet is equally likely to contain any of the 5 animals, independently of other packets	B1	3	For either 'equally likely' or 'independent'
	(ii)	$\left(\frac{4}{5}\right)^2 \times \left(\frac{1}{5}\right) = \frac{16}{125}$ or 0.128	M1		For any numerical '$q^n p$' calculation
			A1	2	For correct answer
	(iii)	$\left(\frac{4}{5}\right)^4$ or $1 - \left\{\frac{1}{5} + \left(\frac{4}{5}\right)\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)\right\}$	M1		Allow M mark even if there is an error of 1 in the number of terms
			A1		For correct expression for the answer
		$\frac{256}{625}$ or 0.4096 or 0.410	A1	3	For correct answer
				$\boxed{8}$	

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5 (i) *EITHER*: $P(X=0)=\binom{7}{3}\Big/\binom{12}{3}=\frac{35}{220}=\frac{7}{44}$	M1	For ratio of relevant $\binom{n}{r}$ terms
	A1	For showing the given answer correctly
OR: $P(X=0)=\frac{7}{12}\times\frac{6}{11}\times\frac{5}{10}=\frac{7}{44}$	M1	For multiplication of relevant 'girl' probs
	A1 **2**	For showing the given answer correctly
(ii) *EITHER*: $P(X=2)=P(2\text{ boys and 1 girl})$	M1	For use of three $\binom{n}{r}$ terms relevant to the 2B, 1G case
$=\binom{7}{1}\times\binom{5}{2}\Big/\binom{12}{3}$		
$=\frac{7\times10}{220}=\frac{7}{22}$	B1	For both $\binom{5}{2}$ and $\binom{12}{3}$ correct
	A1	For showing the given answer correctly
OR: $P(X=2)=P(2\text{ boys and 1 girl})$	M1	For three probabilities multiplied relevant to the 2B, 1G case
$=\frac{5}{12}\times\frac{4}{11}\times\frac{4}{10}\times3=\frac{7}{22}$	B1	For inclusion of factor 3
	A1 **3**	For showing the given answer correctly
(iii) $E(X)=0\times\frac{7}{44}+1\times\frac{21}{44}+2\times\frac{7}{22}+3\times\frac{1}{22}=\frac{5}{4}$	M1	For correct calculation process
	A1	For correct answer
$E(X^2)=0\times\frac{7}{44}+1\times\frac{21}{44}+4\times\frac{7}{22}+9\times\frac{1}{22}=\frac{95}{44}$	B1	For correct numerical expression for $\Sigma x^2 p$
$\text{Var}(X)=\frac{95}{44}-\left(\frac{5}{4}\right)^2=\frac{105}{176}$ or 0.597 (to 3dp)	M1	For correct overall method for variance
	A1$\surd$ **5**	For correct answer
	$\boxed{\mathbf{10}}$	
6 (i) Medians correspond to 1000 candidates	M1	For reading off at 1000; may be implied
$m_1 = 38$, $m_2 = 63$	A1	For correct value for either median
	A1 **3**	For both correct
(ii) Paper 2 was easier	B1	For a correct statement
Marks were higher on paper 2	B1 **2**	For a correct justification
(iii) 66 marks on paper 1 corresponds to 1700 cands,	M1	For reading off at 66; may be implied
1700 cands on paper 2 corresponds to 82 marks	A1	For stating the correct mark
Proportion is $\frac{2000-1700}{2000}$, i.e. 15%	M1	For relevant subtraction from 2000
	A1 **4**	For correct answer 15% or equivalent
(iv) Possible valid comments include: Box plots give quick direct comparisons of medians and IQRs; Box plots don't include all the information that CF graphs do; CF graphs can be used to read off values both ways round	B1	For any one valid comment
etc	B1 **2**	For any other valid comment
	$\boxed{\mathbf{11}}$	

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7 (i) (a)	$1 - 0.7899 = 0.210(1)$	M1 A1 **2**	For complement of relevant tabular value For correct answer
(b)	$0.9209 - 0.7899 = 0.131$	M1 A1 **2**	For subtracting relevant tabular values For correct answer
(ii) (a)	$0.790^5 + 5\times0.790^4\times0.210 + 10\times0.790^3\times0.210^2$ $= 0.934$	M1 M1 A1√ A1 **4**	For recognition of B(5, 0.210) For identification of correct three cases For correct expression for the required prob For correct answer
(b)	Expectation is $5\times0.210 = 1.05$	M1 A1 **2**	For relevant use of *np* For correct answer
		$\boxed{10}$	
8 (i)	$r = \dfrac{1837.78 - \frac{43.3\times471.9}{12}}{\sqrt{\left(164.69 - \frac{43.3^2}{12}\right)\left(20915.75 - \frac{471.9^2}{12}\right)}}$ $= 0.956$ The value is close to $+1$, and the points in the diagram lie (fairly) close to a straight line with positive gradient	M1 A1 B1 B1 **4**	For correct formula or calculator use For correct value For relating the value to 1 For a reasonable comment about linearity
(ii)	Gradient of regression line is $\dfrac{1837.78 - \frac{43.3\times471.9}{12}}{164.69 - \frac{43.3^2}{12}} = 15.9789$ $y - \frac{471.9}{12} = 15.9789\left(x - \frac{43.3}{12}\right)$ $y = 16.0x - 18.3$	M1 A1 M1 A1 **4**	For correct formula or calculator use For correct value for the regression coeff For correct form of equn (may be implied) For correct (simplified) equation
(iii)	$y = 16.0\times4.2 - 18.3$ Current is $48.8\ \text{cm s}^{-1}$ Comments could include: Diagram indicates some uncertainty High value of pmcc suggests fairly reliable	M1 A1√ B1 **3**	For substitution into equation from **(ii)** For correct answer For any one reasonable comment
(iv)	As extrapolation is involved, the prediction would be (very) unreliable	M1 A1 **2**	For identifying extrapolation For correct conclusion
		$\boxed{13}$	

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OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4733

Probability & Statistics 2

Specimen Paper

Additional materials:
- Answer booklet
- Graph paper
- List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

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1 The standard deviation of a random variable F is 12.0. The mean of n independent observations of F is denoted by $\overline{F}$.

(i) Given that the standard deviation of $\overline{F}$ is 1.50, find the value of n. [3]

(ii) For this value of n, state, with justification, what can be said about the distribution of $\overline{F}$. [2]

2 A certain neighbourhood contains many small houses (with small gardens) and a few large houses (with large gardens). A sample survey of all houses is to be carried out in this neighbourhood. A student suggests that the sample could be selected by sticking a pin into a map of the neighbourhood the requisite number of times, while blindfolded.

(i) Give two reasons why this method does not produce a random sample. [2]

(ii) Describe a better method. [3]

3 Sixty people each make two throws with a fair six-sided die.

(i) State the probability of one particular person obtaining two sixes. [1]

(ii) Using a suitable approximation, calculate the probability that at least four of the sixty obtain two sixes. [5]

4 The random variable G has mean 20.0 and standard deviation σ. It is given that $P(G > 15.0) = 0.6$. Assume that G is normally distributed.

(i) **(a)** Find the value of σ. [4]

(b) Given that $P(G > g) = 0.4$, find the value of $P(G > 2g)$. [3]

(ii) It is known that no values of G are ever negative. State with a reason what this tells you about the assumption that G is normally distributed. [2]

5 The mean solubility rating of widgets inserted into beer cans is thought to be 84.0, in appropriate units. A random sample of 50 widgets is taken. The solubility ratings, x, are summarised by

$$n = 50, \qquad \Sigma x = 4070, \qquad \Sigma x^2 = 336\,100.$$

Test, at the 5% significance level, whether the mean solubility rating is less than 84.0. [10]

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6 On average a motorway police force records one car that has run out of petrol every two days.

(i) **(a)** Using a Poisson distribution, calculate the probability that, in one randomly chosen day, the police force records exactly two cars that have run out of petrol. [3]

(b) Using a Poisson distribution and a suitable approximation to the binomial distribution, calculate the probability that, in one year of 365 days, there are fewer than 205 days on which the police force records no cars that have run out of petrol. [6]

(ii) State an assumption needed for the Poisson distribution to be appropriate in part **(i)**, and explain why this assumption is unlikely to be valid. [2]

7 The time, in minutes, for which a customer is prepared to wait on a telephone complaints line is modelled by the random variable X. The probability density function of X is given by

$$\mathrm{f}(x) = \begin{cases} kx(9-x^2) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{4}{81}$. [2]

(ii) Find $\mathrm{E}(X)$. [3]

(iii) **(a)** Show that the value y which satisfies $\mathrm{P}(X < y) = \frac{3}{5}$ satisfies

$$5y^4 - 90y^2 + 243 = 0.$$ [4]

(b) Using the substitution $w = y^2$, or otherwise, solve the equation in part **(a)** to find the value of y. [3]

8 The proportion of left-handed adults in a country is known to be 15%. It is suggested that for mathematicians the proportion is greater than 15%. A random sample of 12 members of a university mathematics department is taken, and it is found to include five who are left-handed.

(i) Stating your hypotheses, test whether the suggestion is justified, using a significance level as close to 5% as possible. [8]

(ii) In fact the significance test cannot be carried out at a significance level of exactly 5%. State the probability of making a Type I error in the test. [2]

(iii) Find the probability of making a Type II error in the test for the case when the proportion of mathematicians who are left-handed is actually 20%. [2]

(iv) Determine, as accurately as the tables of cumulative binomial probabilities allow, the actual proportion of mathematicians who are left-handed for which the probability of making a Type II error in the test is 0.01. [2]

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MATHEMATICS **4733**

Probability & Statistics 2

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

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1	**(i)** $\dfrac{12.0}{\sqrt{n}} = 1.50 \Rightarrow \sqrt{n} = \dfrac{12.0}{1.50} = 8 \Rightarrow n = 64$	B1 M1 A1 **3**	For any correct equation involving n For correct solution method for n or $\sqrt{n}$ For correct answer 64
	(ii) n is large, the distribution of $\overline{F}$ can be taken to be normal, according to the Central Limit Theorem	M1 A1 **2**	For relating the size of n to normality For reference to the CLT
		$\boxed{5}$	
2	**(i)** Reasons for bias may include: Larger properties more likely to be picked Some regions of the map more/less likely	B1 B1 **2**	For stating one valid relevant reason For stating a second valid relevant reason
	(ii) Make a list of all the houses in the neighbourhood Number the houses from 1 upwards Select the sample using random numbers	B1 B1 B1 **3**	For stating or implying a sampling frame For numbering the sampling units For referring to use of random numbers
		$\boxed{5}$	
3	**(i)** $\frac{1}{36}$	B1 **1**	For correct probability
	(ii) Number obtaining two sixes ~ $\text{B}(60, \frac{1}{36})$ Approximate distribution is $\text{Po}(\frac{5}{3})$ $\text{P}(\geqslant 4) = 1 - e^{-\frac{5}{3}}\left\{1 + \frac{5}{3} + \frac{(5/3)^2}{2!} + \frac{(5/3)^3}{3!}\right\}$ $= 0.0883$	M1 A1√ M1 M1 A1 **5**	For stating or implying binomial distribution For the correct Poisson approximation For calculation of correct terms For correct use of Poisson formula For correct answer 0.088(3)
		$\boxed{6}$	
4	**(i)** **(a)** $\dfrac{15.0 - 20.0}{\sigma} = -0.253$ Hence $\sigma = \dfrac{5}{0.253} \approx 19.8$	M1 B1 M1 A1 **4**	For standardising and equating to $\Phi^{-1}(p)$ For correct value 0.253 (or 0.254) seen For solving equation for σ For correct value 19.8
	(b) $g = 25.0$, using symmetry Hence $\text{P}(G > 2g) = 1 - \Phi\left(\dfrac{50.0 - 20.0}{19.8}\right)$ $= 1 - 0.935 = 0.065$	B1 M1 A1 **3**	For stating (or finding) the value of g For correct process for upper tail prob For correct answer
	(ii) If normal, $\text{P}(G < 0)$ is substantial Hence the assumption seems unjustified	M1 A1 **2**	For considering relevant normal probability For stating the appropriate conclusion
		$\boxed{9}$	

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5	$\bar{x} = \frac{4070}{50} = 81.4$	B1	For correct value of sample mean
	$s^2 = \frac{336100}{49} - \frac{4070^2}{49 \times 50} = 98$	M1	For calculation of unbiased or biased estimate
		A1	For correct value of unbiased estimate
	$H_0 : \mu = 84.0$; $H_1 : \mu < 84.0$	B1	For correct statement of null hypothesis
		B1	For correct statement of alt hypothesis
	EITHER: $z = \frac{\bar{x} - 84.0}{\sqrt{(s^2/50)}} = -1.857$	M1	For standardising, including use of $\sqrt{50}$
		A1	For correct value 1.857
	This is significant, since $-1.857 < -1.645$	M1	For comparing z value to -1.645 or equiv
	OR: $\frac{c - 84.0}{\sqrt{(s^2/50)}} = -1.645 \Rightarrow c = 81.697$	M1	For critical value calculation, inc use of $\sqrt{50}$
		A1	For correct value 81.697
	$\bar{x}$ is in the critical region since $81.4 < 81.697$	M1	For comparing sample mean to critical region
	Hence H_0 is rejected	A1√	For stating or implying rejection of H_0
	There is sufficient evidence to conclude that the mean solubility rating is less than 84.0	A1√ **10**	For stating the outcome in context
		$\boxed{10}$	
6	**(i) (a)** For one day, the distribution is Po(0.5)	B1	For use of correct Poisson mean
	Hence P(exactly 2) $= 0.9856 - 0.9098$	M1	For relevant use of tables (or formula)
	$= 0.0758$	A1 **3**	For correct answer 0.0758
	(b) No. of days with no cars ~ B(365, 0.6065)	M1	For relevant Poisson probability of P(0)
		A1	For identifying correct binomial distribution
	Normal approximation is N(221.3725, 87.11)	A1√	For correct use of *np* and *npq*
	$P(<205) = P\left(Z < \frac{204.5 - 221.3725}{\sqrt{87.11}}\right)$	M1	For standardising (with or without c.c. here)
		A1	For completely correct expression
	$= \Phi(-1.808) = 0.0353$	A1 **6**	For correct answer 0.0353
	(ii) Events (cars running out of petrol) must occur at a constant average rate. This seems unlikely, given that there will be different volumes of traffic on different days of the week (e.g. weekdays and weekends)	B1	For correct statement of the condition
		B1 **2**	For a correct explanation
		$\boxed{11}$	

[Turn over

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7	**(i)** $1 = k\int_0^3 (9x - x^3)\,dx = k\left[\frac{9}{2}x^2 - \frac{1}{4}x^4\right]_0^3 = \frac{81}{4}k$	M1	For equating to 1 and integrating
	Hence $k = \frac{4}{81}$	A1 **2**	For showing given answer correctly
	(ii) $E(X) = \frac{4}{81}\int_0^3 x^2(9 - x^2)\,dx = \frac{4}{81}\left[3x^3 - \frac{1}{5}x^5\right]_0^3 = 1.6$	M1	For attempt at $\int_0^3 x\,f(x)\,dx$
		A1	For correct indefinite integral, in any form
		A1 **3**	For correct answer 1.6
	(iii) **(a)** $\frac{3}{5} = \frac{4}{81}\int_0^y x(9 - x^2)\,dx = \frac{4}{81}\left[\frac{9}{2}x^2 - \frac{1}{4}x^4\right]_0^y$	M1	For attempt at $\int_0^y f(x)\,dx = \frac{3}{5}$
		B1	For correct indefinite integral, in any form
	Hence $\frac{3}{5} = \frac{4}{81}\left\{\frac{9}{2}y^2 - \frac{1}{4}y^4\right\}$	M1	Use limits to produce relevant equation in y
	i.e. $5y^4 - 90y^2 + 243 = 0$	A1 **4**	For showing given answer correctly
	(b) $w = \dfrac{90 \pm \sqrt{(90^2 - 4\times5\times243)}}{10} = 3.31$ or 14.7	M1	For use of quadratic formula to find w
		A1	For either value found correctly
	Hence $y = \sqrt{3.31} = 1.82$	A1 **3**	For correct (unique) answer 1.82
		12	
8	**(i)** $H_0 : p = 0.15$; $H_1 : p > 0.15$	B1	For correct statement of null hypothesis
		B1	For correct statement of alt hypothesis
	Under H_0, number left-handed $L \sim B(12, 0.15)$	M1	For correct distribution stated or implied
	$P(L \geqslant 5) = 1 - 0.9761 = 0.0239$	M1	For calculation of relevant tail probability, or finding the critical region
		A1	For correct value 0.0239 or region $l \geqslant 5$
	This is significant, since $0.0239 < 0.05$	M1	For comparing tail probability with 0.05 or observed value with critical region
	Hence H_0 is rejected	A1√	For stating or implying rejection of H_0
	Accept the suggestion that the proportion of mathematicians who are left-handed is more than 15%	A1√ **8**	For stating the outcome in context
	(ii) $P_I = P(L \text{ in critical region}) = 0.0239$	M1	For evaluating $P(\text{reject } H_0)$
		A1 **2**	For correct answer 0.0239 or equivalent
	(iii) $P_{II} = P(L \leqslant 4 \mid p = 0.2) = 0.9274$	M1	For evaluating $P(\text{accept } H_0)$ with $p = 0.2$
		A1 **2**	For correct probability
	(iv) $P_{II} = 0.0188$ for $p = \frac{2}{3}$ and 0.0095 for $p = 0.7$	M1	For relevant use of tables
	So the proportion is between 67% and 70%	A1 **2**	For an appropriate conclusion
		14	

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Advanced Subsidiary General Certificate of Education
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MATHEMATICS 4734

Probability & Statistics 3

Specimen Paper

Additional materials:
- Answer booklet
- Graph paper
- List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

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1 A car repair firm receives call-outs both as a result of breakdowns and also as a result of accidents. On weekdays (Monday to Friday), call-outs resulting from breakdowns occur at random, at an average rate of 6 per 5-day week; call-outs resulting from accidents occur at random, at an average rate of 2 per 5-day week. The two types of call-out occur independently of each other. Find the probability that the total number of call-outs received by the firm on one randomly chosen weekday is more than 3. [5]

2 Boxes of matches contain 50 matches. Full boxes have mean mass 20.0 grams and standard deviation 0.4 grams. Empty boxes have mean mass 12.5 grams and standard deviation 0.2 grams. Stating any assumptions that you need to make, calculate the mean and standard deviation of the mass of a match. [7]

3 A random sample of 80 precision-engineered cylindrical components is checked as part of a quality control process. The diameters of the cylinders should be 25.00 cm. Accurate measurements of the diameters, x cm, for the sample are summarised by

$$\Sigma(x-25)=0.44, \qquad \Sigma(x-25)^2=0.2287\,.$$

(i) Calculate a 99% confidence interval for the population mean diameter of the components. [6]

(ii) For the calculation in part **(i)** to be valid, is it necessary to assume that component diameters are normally distributed? Justify your answer. [2]

4 The lengths of time, in seconds, between vehicles passing a fixed observation point on a road were recorded at a time when traffic was flowing freely. The frequency distribution in Table 1 is a summary of the data from 100 observations.

Time interval (x seconds)	$0<x\leqslant 5$	$5<x\leqslant 10$	$10<x\leqslant 20$	$20<x\leqslant 40$	$40<x$
Observed frequency	49	22	20	7	2

Table 1

It is thought that the distribution of times might be modelled by the continuous random variable X with probability density function given by

$$f(x)=\begin{cases}0.1e^{-0.1x} & x>0,\\ 0 & \text{otherwise.}\end{cases}$$

Using this model, the expected frequencies (correct to 2 decimal places) for the given time intervals are shown in Table 2.

Time interval (x seconds)	$0<x\leqslant 5$	$5<x\leqslant 10$	$10<x\leqslant 20$	$20<x\leqslant 40$	$40<x$
Expected frequency	39.35	23.87	23.25	11.70	1.83

Table 2

(i) Show how the expected frequency of 23.87, corresponding to the interval $5<x\leqslant 10$, is obtained. [5]

(ii) Test, at the 10% significance level, the goodness of fit of the model to the data. [5]

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5 The continuous random variable X has a triangular distribution with probability density function given by

$$f(x) = \begin{cases} 1 + x & -1 \leqslant x \leqslant 0, \\ 1 - x & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that, for $0 \leqslant a \leqslant 1$,

$$P(|X| \leqslant a) = 2a - a^2.$$ [3]

The random variable Y is given by $Y = X^2$.

(ii) Express $P(Y \leqslant y)$ in terms of y, for $0 \leqslant y \leqslant 1$, and hence show that the probability density function of Y is given by

$$g(y) = \frac{1}{\sqrt{y}} - 1, \quad \text{for } 0 < y \leqslant 1.$$ [4]

(iii) Use the probability density function of Y to find $E(Y)$, and show how the value of $E(Y)$ may also be obtained directly using the probability density function of X. [4]

(iv) Find $E(\sqrt{Y})$. [2]

6 Certain types of food are now sold in metric units. A random sample of 1000 shoppers was asked whether they were in favour of the change to metric units or not. The results, classified according to age, were as shown in the table.

	Age of shopper: Under 35	Age of shopper: 35 and over	Total
In favour of change	187	161	348
Not in favour of change	283	369	652
Total	470	530	1000

(i) Use a χ^2 test to show that there is very strong evidence that shoppers' views about changing to metric units are not independent of their ages. [7]

(ii) The data may also be regarded as consisting of two random samples of shoppers; one sample consists of 470 shoppers aged under 35, of whom 187 were in favour of change, and the second sample consists of 530 shoppers aged 35 or over, of whom 161 were in favour of change. Determine whether a test for equality of population proportions supports the conclusion in part **(i)**. [7]

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7 A factory manager wished to compare two methods of assembling a new component, to determine which method could be carried out more quickly, on average, by the workforce. A random sample of 12 workers was taken, and each worker tried out each of the methods of assembly. The times taken, in seconds, are shown in the table.

Worker	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
Time in seconds for Method 1	48	38	47	59	62	41	50	52	58	54	49	60
Time in seconds for Method 2	47	40	38	55	57	42	42	40	62	47	47	51

(i) **(a)** Carry out an appropriate t-test, using a 2% significance level, to test whether there is any difference in the times for the two methods of assembly. [8]

(b) State an assumption needed in carrying out this test. [1]

(c) Calculate a 95% confidence interval for the population mean time difference for the two methods of assembly. [3]

(ii) Instead of using the same 12 workers to try both methods, the factory manager could have used two independent random samples of workers, allocating Method 1 to the members of one sample and Method 2 to the members of the other sample.

(a) State one disadvantage of a procedure based on two independent random samples. [1]

(b) State any assumptions that would need to be made to carry out a t-test based on two independent random samples. [2]

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OXFORD CAMBRIDGE AND RSA EXAMINATIONS

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MATHEMATICS **4734**

Probability & Statistics 3

MARK SCHEME

Specimen Paper

MAXIMUM MARK	**72**

This mark scheme consists of 4 printed pages.

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1 Model for call-outs is Poisson	B1	For any implication of Poisson
Mean is $\frac{1}{5}(6+2)$	M1	For summing two relevant parameters
$= 1.6$	A1	For correct mean of 1.6
Probability is $1 - 0.9212$	M1	For relevant use of tables
$= 0.0788$	A1 **5**	For correct answer
	$\boxed{5}$	
2 Assume $F = E + M_1 + M_2 + \ldots + M_{50}$, where		(The relation itself may be implied)
the masses of the 50 matches in a box are independent	B1	For one relevant valid assumption
the mass of the empty box is independent of the masses of the matches	B1	For another relevant valid assumption
$20.0 = 12.5 + 50\mu$	M1	For attempting $\mathrm{E}(F)$ in terms of μ
Hence mean mass of a match is 0.15 grams	A1	For correct value 0.15
$0.4^2 = 0.2^2 + 50\sigma^2$	M1	For attempting $\mathrm{Var}(F)$ as a sum
	A1	For correct equation
Hence standard deviation is 0.049 grams	A1 **7**	For correct value 0.049
	$\boxed{7}$	
3 (i) $\bar{x} = 25.0055$	B1	For correct sample mean, or equivalent; the 25 may be taken into account later
$s^2 = \frac{1}{79}\left(0.2287 - \frac{0.44^2}{80}\right)$	M1	For correct unsimplified expression
$= 0.00286...$	A1	For correct unbiased estimate
Interval is $25.0055 \pm 2.576\sqrt{\frac{0.00286}{80}}$	M1	For calculation of the form $\bar{x} \pm z\sqrt{(s^2/n)}$
	B1	For relevant use of $z = 2.576$
Hence $24.99(0) < \mu < 25.02(1)$	A1 **6**	For correct interval, stated to an appropriate degree of accuracy
(ii) The sample size of 80 is sufficient large for the Central Limit Theorem to apply, so it is not necessary to assume a normal distribution	M1 A1 **2**	For mention of sample size and CLT For the correct conclusion and reason
	$\boxed{8}$	
4 (i) $f_e = 100 \times \int_5^{10} 0.1\mathrm{e}^{-0.1x}\,\mathrm{d}x$	M1	For attempting to integrate $\mathrm{f}(x)$
$= 100\left[-\mathrm{e}^{-0.1x}\right]_5^{10}$	A1	For correct indefinite integral
	M1	For multiplying by total frequency
$= 100(\mathrm{e}^{-0.5} - \mathrm{e}^{-1}) = 23.87$	M1	For use of correct limits
	A1 **5**	For obtaining given answer correctly
(ii) Combining: f_o: 49, 22, 20, 9; f_e: 39.35, 23.87, 23.25, 13.53	M1	For combining the last two classes
Test statistic is $\frac{9.65^2}{39.35} + \frac{1.87^2}{23.87} + \frac{3.25^2}{23.25} + \frac{4.53^2}{13.53}$	M1	For correct calculation process
$= 4.484$	A1	For correct value 4.48
This is less than 6.251	M1	For comparison with the correct critical value
Hence there is a satisfactory fit	A1√ **5**	For correct conclusion, in terms of the fit
	$\boxed{10}$	

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5	**(i)** $\mathrm{P}(\lvert X\rvert < a) = \mathrm{P}(-a < X < a)$	M1	For consideration of two areas, or equiv
	$= \int_{-a}^{0}(1+x)\,dx + \int_0^a (1-x)\,dx$	A1	For integrals or equivalent trapezia
	$= \left[x+\tfrac{1}{2}x^2\right]_{-a}^{0} + \left[x-\tfrac{1}{2}x^2\right]_0^a = 2a - a^2$	A1 **3**	For showing the given answer correctly
	(ii) $\mathrm{P}(Y \leqslant y) = \mathrm{P}(X^2 \leqslant y) = \mathrm{P}\left(\lvert X\rvert \leqslant \sqrt{y}\right) = 2\sqrt{y} - y$	M1	For expression of $\mathrm{P}\left(X^2 \leqslant y\right)$ in terms of y
		A1	For correct expression $2\sqrt{y} - y$
	Hence the pgf of Y is $\frac{d}{dy}\left(2\sqrt{y} - y\right) = \frac{1}{\sqrt{y}} - 1$	M1	For differentiation of previous expression
		A1 **4**	For showing the given answer correctly
	(iii) $\mathrm{E}(Y) = \int_0^1 y^{\frac{1}{2}} - y\,dy = \left[\tfrac{2}{3}y^{\frac{3}{2}} - \tfrac{1}{2}y^{\frac{1}{2}}\right]_0^1 = \tfrac{1}{6}$	M1	For the correct integral in terms of y
		A1	For correct answer $\frac{1}{6}$
	$\mathrm{E}(X^2) = \int_{-1}^{0}(x^2+x^3)\,dx + \int_0^1 (x^2-x^3)\,dx$	M1	For the correct integrals in terms of x
	$= \left[\tfrac{1}{3}x^3 + \tfrac{1}{4}x^4\right]_{-1}^{0} + \left[\tfrac{1}{3}x^3 - \tfrac{1}{4}x^4\right]_0^1 = \tfrac{1}{12} + \tfrac{1}{12} = \tfrac{1}{6}$	A1 **4**	For the correct answer correctly obtained
	(iv) $\mathrm{E}(\sqrt{Y}) = \int_0^1 y^{\frac{1}{2}}\ \mathrm{g}(y)\,dy = \int_0^1 (1 - y^{\frac{1}{2}})\,dy$	M1	For forming the correct integral
	$= \left[y - \tfrac{2}{3}y^{\frac{3}{2}}\right]_0^1 = \tfrac{1}{3}$	A1 **2**	For the correct answer $\frac{1}{3}$
		$\boxed{13}$	
6	**(i)** H_0: shoppers' views and age are independent,		
	H_1: shoppers' views and age are not independent	B1	For stating both hypotheses
	Exp frequencies under H_0 are $\begin{matrix} 163.56 & 184.44 \\ 306.44 & 345.56 \end{matrix}$	M1	For correct method for expected frequencies
		A1	For all four correct
	Test statistic is $\frac{22.94^2}{163.56} + \frac{22.94^2}{184.44} + \frac{22.94^2}{306.44} + \frac{22.94^2}{345.56}$	M1	For correct calculation process, inc Yates
	$= 9.31\ldots$	A1	For correct value of the test statistic
	This is greater than the critical 0.5% value of 7.879	M1	For a relevant (1 df) comparison
	Hence there is very strong evidence to reject H_0 and conclude that views about changing to metric units are not independent of age	A1√ **7**	For correctly justifying the given answer (the final two marks remain available if Yates' correction is omitted)
	(ii) $\mathrm{H}_0: p_1 = p_2$, $\mathrm{H}_1: p_1 \neq p_2$	B1	For both hypotheses stated
	Under H_0 the sample value of the common proportion is $\frac{187+161}{1000} = 0.348$	B1	For correct value of estimated p
	Test statistic is $\dfrac{\frac{187}{470} - \frac{161}{530}}{\sqrt{0.348 \times 0.652 \times \left(\frac{1}{470} + \frac{1}{530}\right)}}$	M1	For num $p_1 - p_2$ and denom using attempted s.d. based on a common estimate of p
		A1	For completely correct expression
	$= 3.118$	A1	For correct value of the test statistic
	This is greater than the 0.2% (two-tail) critical value of 3.090	M1	For a relevant comparison using the normal distribution
	Hence this test supports the conclusion of part **(i)**	A1√ **7**	For any relevant comparison or comment
		$\boxed{14}$	

[Turn over

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7	**(i) (a)**	$H_0 : \mu_d = 0,\ \ H_1 : \mu_d \neq 0$	B1		For both hypotheses stated
		$\bar{d} = 4.1667$	B1		For correct mean difference (subtraction can be either way round)
		$s^2 = \dfrac{486}{11} - \dfrac{50^2}{11 \times 12} = 25.2424$	M1		For calculation of unbiased variance estimate
			A1		For correct value 25.24...
		Test statistic is $\dfrac{4.1667 - 0}{\sqrt{(25.2424/12)}}$	M1		For correct standardising process
		$= 2.873$	A1		For correct value of test statistic
		This is greater than the critical value 2.718	M1		For a relevant comparison using t tables
		Hence there is enough evidence to reject H_0 and conclude that there is a difference between the times for the two methods	A1√	**8**	For correctly stated conclusion in context
	(b)	Population of differences is normal	B1	**1**	For correct statement
	(c)	Interval is $4.1667 \pm 2.201\sqrt{\dfrac{25.2424}{12}}$	M1		For calculation of the form $\bar{d} \pm t\sqrt{(s^2/n)}$
			B1		For relevant use of $t = 2.201$
		Hence $0.97 < \mu_d < 7.36$	A1	**3**	For correct interval
	(ii) (a)	Variation in the speed of individual workers is not eliminated, and may be large compared with the difference between the methods that is being tested	B1	**1**	For any relevant valid statement
	(b)	Both samples are from normal populations	B1		For a correct statement about normality
		The population variances are equal	B1	**2**	For a correct statement about the variances
				$\boxed{15}$	

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MATHEMATICS **4735**

Probability & Statistics 4

Specimen Paper

Additional materials:
- Answer booklet
- Graph paper
- List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

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1 A continuous random variable X has moment generating function given by

$$M_X(t) = \frac{9}{(3-t)^2}.$$

Find the mean and variance of X. [5]

2 The events A and B are independent, and $P(A) = P(B) = p$, where $0 < p < 1$.

(i) Express $P(A \cup B)$ in terms of p. [3]

(ii) Given that $P\big((A \cap B) \mid (A \cup B)\big) = \frac{1}{2}$, find the value of $P\big((A \cap B') \cup (A' \cap B)\big)$. [5]

3 A University's Department of Computing is interested in whether students who have passed A level Mathematics perform better in Computing examinations that those who have not.

A random sample of 19 students was taken from those students who took a particular first year Computing examination. This sample included 12 students who have passed A level Mathematics and 7 students who have not. The marks gained in the Computing examination were as follows:

Students who have passed A level Mathematics: 27, 34, 39, 41, 45, 47, 55, 59, 66, 75, 78, 86.

Students who have not passed A level Mathematics: 17, 21, 28, 35, 37, 54, 64.

Use a suitable non-parametric test to determine if there is evidence, at the 5% significance level, that students who have passed A level Mathematics gain a higher average mark than students who have not passed A level Mathematics. (A normal approximation may be used.) [10]

4 The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} kx & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant and the value of the parameter a is unknown.

(i) Show that $k = \dfrac{2}{a^2}$. [2]

The random variable U is defined by $U = \frac{3}{2}X$.

(ii) Show that U is an unbiased estimator of a. [3]

(iii) Find, in terms of a, the variance of U. [4]

The random variable λX^n, where n is a positive integer and λ is a constant, is an unbiased estimator of a^n.

(iv) Express λ in terms of n. [2]

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5 **(i)** Explain briefly the circumstances under which a non-parametric test of significance should be used in preference to a parametric test. [1]

The acidity of soil can be measured by its pH value. As a part of a Geography project a student measured the pH values of 14 randomly chosen samples of soil in a certain area, with the following results.

5.67 5.73 6.64 6.76 6.10 5.41 5.80 6.52 5.16 5.10 6.71 5.89 5.68 5.37

(ii) Use a Wilcoxon signed-rank test to test whether the average pH value for soil in this area is 6.24. Use a 10% level of significance. [5]

Some time later, the pH values of soil samples taken at exactly the same locations as before were again measured. It was found that, for 3 of the 14 locations, the new pH value was higher than the previous value, while for the other 11 locations the new value was lower.

(iii) Test, at the 5% significance level, whether there is evidence that the average pH value of soil in this area is lower than previously. [5]

6 The joint probability distribution of the discrete random variables X and Y is shown in the following table.

		x	
		-1	0
y	2	$\frac{1}{6}$	$\frac{2}{9}$
	3	$\frac{5}{18}$	$\frac{1}{3}$

(i) Show that $\mathrm{E}(X) = -\frac{4}{9}$ and find $\mathrm{Var}(X)$. [4]

(ii) Write down the distributions of X conditional on $Y = 2$ and X conditional on $Y = 3$. Find the means of these conditional distributions, and hence verify that

$$\mathrm{E}(X) = \mathrm{E}(X \mid Y = 2) \times \mathrm{P}(Y = 2) + \mathrm{E}(X \mid Y = 3) \times \mathrm{P}(Y = 3).$$ [3]

It is given that $\mathrm{E}(Y) = \frac{47}{18}$ and $\mathrm{Var}(Y) = \frac{77}{324}$.

(iii) Find $\mathrm{Cov}(X, Y)$ and state, with a reason, whether X and Y are independent. [4]

(iv) Find $\mathrm{Var}(X + Y)$. [2]

[Turn over

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7 The random variable X has a geometric distribution with parameter p.

(i) Show that the probability generating function $\mathrm{G}_X(t)$ of X is given by

$$\mathrm{G}_X(t) = \frac{pt}{1 - t(1-p)}.$$ [3]

(ii) Hence show that $\mathrm{E}(X) = \frac{1}{p}$ and that $\mathrm{Var}(X) = \frac{1-p}{p^2}$. [5]

A child has 4 fair, six-sided dice, one white, one yellow, one blue and one red.

(iii) The child rolls the white die repeatedly until the die shows a six. The number of rolls up to and including the roll on which the white die first shows a six is denoted by W. Write down an expression for $\mathrm{G}_W(t)$. [1]

(iv) The child then repeats this process with the yellow die, then with the blue die and then with the red die. By finding an appropriate probability generating function, find the probability that the total number of rolls of the four dice, up to and including the roll on which the red die first shows a six, is exactly 24. [4]

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Advanced Subsidiary General Certificate of Education
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MATHEMATICS **4735**

Probability & Statistics 4

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

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1 *EITHER*: $M'_X(t) = \frac{18}{(3-t)^3}$	B1	For correct differentiation of the mgf
Hence $E(X) = M'_X(0) = \frac{2}{3}$	B1√	For correct value for the mean
$M''_X(t) = \frac{54}{(3-t)^4}$	B1	For correct second derivative
Hence $\text{Var}(X) = M''_X(0) - \{E(X)\}^2 = \frac{2}{3} - \frac{4}{9} = \frac{2}{9}$	M1	For correct method for the variance
	A1√	For correct answer
OR: $M_X(t) = 1 + \frac{2}{3}t + \frac{1}{3}t^2 + \ldots$	M1	For attempting binomial expansion of mgf
	A1	For first three terms correct (unsimplified)
Hence $E(X) = \frac{2}{3}$	A1√	For correct value for the mean
$\text{Var}(X) = (2!) \times \frac{1}{3} - \{E(X)\}^2 = \frac{2}{3} - \frac{4}{9} = \frac{2}{9}$	M1	For correct method for the variance
	A1√ **5**	For correct answer
	$\boxed{5}$	
2 **(i)** $P(A \cup B) = p + p - p \times p = 2p - p^2$	M1	For use of $P(A) + P(B) - P(A \cap B)$
	B1	For $P(A \cap B) = P(A)P(B)$ since independent
	A1 **3**	For correct expression $2p - p^2$
(ii) $\frac{p^2}{2p - p^2} = \frac{1}{2} \Rightarrow 2p = 2 - p \Rightarrow p = \frac{2}{3}$	B1√	For equation $\frac{P(A \cap B)}{P(A \cup B)} = \frac{1}{2}$
	M1	For solving relevant equation for p
	A1	For correct value
Hence $P((A \cap B') \cup (A' \cap B)) = 2 \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{9}$	M1	For calculation of $2p(1-p)$ or equivalent
	A1 **5**	For correct answer $\frac{4}{9}$
	$\boxed{8}$	
3 H_0 : population medians equal , H_1 : higher median for those who passed Mathematics	B1	For both hypotheses stated correctly
Ranking: Pass: 3, 5, 8, 9, 10, 11, 13, 14, 16, 17, 18, 19 Not pass: 1, 2, 4, 6, 7, 12, 15	M1	For attempt at ranking correctly
Sum of ranks of those not passing is 47	A1	For correct sum of ranks
$R_m \sim N(\frac{1}{2} \times 7 \times 20, \frac{1}{12} \times 7 \times 12 \times 20) = N(70, 140)$	M1	For using the appropriate normal approx
	A1	For both parameters correct
EITHER: Test statistic is $\frac{47.5 - 70}{\sqrt{140}} = -1.902$	M1	For standardising
	A2	For correct value of test statistic (allow A1 if correct apart from missing or wrong c.c.)
This is less than -1.645	M1	For comparison with correct critical value
OR: Critical region is $\frac{X + 0.5 - 70}{\sqrt{140}} < -1.645$	M1	For setting up the appropriate inequality
i.e. $X \leqslant 50$	A2	For correct critical region (allow A1 if correct apart from missing or wrong c.c.)
Sample value 47 lies in the critical region	M1	For comparing 47 with critical region
Hence there is evidence that those passing Mathematics have a higher average score	A1√ **10**	For conclusion stated in context
	$\boxed{10}$	

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4	**(i)** $\int_0^a kx\,\mathrm{d}x = 1 \Rightarrow \tfrac{1}{2}ka^2 = 1 \Rightarrow k = \dfrac{2}{a^2}$	M1 A1 **2**	For use of $\int_0^a \mathrm{f}(x)\,\mathrm{d}x = 1$ For showing the given answer correctly
	(ii) $\mathrm{E}(U) = \tfrac{3}{2}\int_0^a kx^2\,\mathrm{d}x = \tfrac{3}{2}\times\tfrac{1}{3}ka^3 = a$ Hence U is an unbiased estimator of a	B1 M1 A1 **3**	For stating or implying $\mathrm{E}(U) = \tfrac{3}{2}\mathrm{E}(X)$ For use of $\int_0^a x\mathrm{f}(x)\,\mathrm{d}x$ For showing the given result correctly
	(iii) $\mathrm{E}(U^2) = \int_0^a \left(\tfrac{3}{2}x\right)^2 kx\,\mathrm{d}x = \tfrac{9}{16}ka^4 = \tfrac{9}{8}a^2$ Hence $\mathrm{Var}(U) = \tfrac{9}{8}a^2 - a^2 = \tfrac{1}{8}a^2$	M1 A1 M1 A1√ **4**	For correct process for $\mathrm{E}(U^2)$ For correct value $\tfrac{9}{8}a^2$ For correct process for $\mathrm{Var}(U)$ For correct answer (Alternatively via $\mathrm{Var}(U) = \tfrac{9}{4}\mathrm{Var}(X)$.)
	(iv) $\dfrac{2\lambda}{a^2}\int_0^a x^{n+1}\,\mathrm{d}x = a^n \Rightarrow \dfrac{2\lambda}{a^2}\times\dfrac{a^{n+2}}{n+2} = a^n$ Hence $\lambda = \tfrac{1}{2}(n+2)$	M1 A1 **2**	For using $\lambda\mathrm{E}(X^n) = a^n$ For correct answer
		$\boxed{11}$	
5	**(i)** A non-parametric test is needed when there is no information (or reasonable assumption) available about an underlying distribution	B1 **1**	For a correct statement
	(ii) H_0: population median pH is 6.24, H_1: population median pH is not 6.24 Deviations from NH value 6.24 are: –0.57 –0.51 0.40 0.52 –0.14 –0.83 –0.44 0.28 –1.08 –1.14 0.47 –0.35 –0.56 –0.87 Signed ranks are: –10 –7 4 8 –1 –11 –5 2 –13 –14 6 –3 –9 –12 Test statistic is $2+4+6+8=20$ This is less than the critical value of 25, so we conclude that there is evidence to suggest that the average pH value is not 6.24	B1 M1 M1 A1 M1 A1 **6**	For both hypotheses stated correctly For calculating signed differences from 6.24 For calculating signed ranks For the correct value of the test statistic For comparing with the correct critical value For correct conclusion based on correct work
	(iii) H_0: same average pH as before; H_1: lower value $\mathrm{P}(\leqslant 3 \text{ out of } 14 \mid \mathrm{H}_0) = 0.0287$ This is less than 0.05, so we reject H_0 and conclude that the average pH is now lower	B1 M1 A1 M1 A1 **5**	For both hypotheses stated correctly For relevant use of $\mathrm{B}(14, \tfrac{1}{2})$ For correct value 0.0287 For comparing with 0.05 For correct conclusion based on correct work
		$\boxed{12}$	

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6 **(i)**	Marginal probabilities for X are $\frac{4}{9}, \frac{5}{9}$	B1		For appropriate addition
	Hence $\mathrm{E}(X) = -1\times\frac{4}{9} + 0\times\frac{5}{9} = -\frac{4}{9}$	B1		For showing the given answer correctly
	$\mathrm{Var}(X) = (-1)^2 \times \frac{4}{9} - \left(-\frac{4}{9}\right)^2 = \frac{20}{81}$	M1		For correct process for variance
		A1	4	For correct value
(ii)	$\begin{array}{\|c\|c\|c\|}\hline x & -1 & 0 \\ \hline \mathrm{P}_2(X=x) & \frac{3}{7} & \frac{4}{7} \\ \hline\end{array}$ $\begin{array}{\|c\|c\|c\|}\hline x & -1 & 0 \\ \hline \mathrm{P}_3(X=x) & \frac{5}{11} & \frac{6}{11} \\ \hline\end{array}$	B1		For both conditional distributions correct
	Hence $\mathrm{E}(X \mid Y=2) = -\frac{3}{7}$, $\mathrm{E}(X \mid Y=3) = -\frac{5}{11}$	B1		For both conditional expectations correct
	$\mathrm{RHS} = -\frac{3}{7}\times\frac{7}{18} - \frac{5}{11}\times\frac{11}{18} = -\frac{4}{9} = \mathrm{E}(X)$	B1	3	For correct verification
(iii)	$\mathrm{E}(XY) = -2\times\frac{1}{6} - 3\times\frac{5}{18} = -\frac{7}{6}$	M1		For evaluation of $\mathrm{E}(XY)$
	$\mathrm{Cov}(X, Y) = -\frac{7}{6} - (-\frac{4}{9})\times\frac{47}{18} = -\frac{1}{162}$	M1		For correct method for $\mathrm{Cov}(X, Y)$
		A1		For correct value (fraction or decimal)
	X and Y are not independent, as $\mathrm{Cov}(X, Y) \neq 0$	B1	4	For correct conclusion, with correct reason
(iv)	$\mathrm{Var}(X+Y) = \frac{20}{81} + \frac{77}{324} - \frac{2}{162} = \frac{17}{36}$	M1		For use of $\mathrm{Var}(X) + \mathrm{Var}(Y) + 2\mathrm{Cov}(X, Y)$
		A1√	2	For correct value
			$\boxed{13}$	
7 **(i)**	$\mathrm{G}_X(t) = \sum_{r=1}^{\infty} q^{r-1} p t^r$, where $q = 1-p$	B1		For correct statement of the required sum
	$= pt\sum_{r=1}^{\infty}(qt)^{r-1} = \dfrac{pt}{1-qt} = \dfrac{pt}{1-(1-p)t}$	M1		For summing the relevant GP
		A1	3	For showing the given answer correctly
(ii)	$\mathrm{G}'_X(t) = \dfrac{p}{(1-qt)^2}$	B1		For correct derivative, in any form
	Hence $\mathrm{E}(X) = \mathrm{G}'_X(1) = \dfrac{p}{p^2} = \dfrac{1}{p}$	B1		For showing the given answer correctly
	$\mathrm{G}''_X(t) = \dfrac{2pq}{(1-qt)^3}$	B1√		For correct second derivative, in any form
	Hence $\mathrm{Var}(X) = \mathrm{G}''_X(1) + \dfrac{1}{p} - \dfrac{1}{p^2}$	M1		For use of $\mathrm{G}''(1) + \mathrm{G}'(1) - \{\mathrm{G}'(1)\}^2$
	$= \dfrac{2pq}{p^3} + \dfrac{1}{p} - \dfrac{1}{p^2} = \dfrac{q}{p^2} = \dfrac{1-p}{p^2}$	A1	5	For showing the given answer correctly
(iii)	$\mathrm{G}_W(t) = \dfrac{\frac{1}{6}t}{1-\frac{5}{6}t}$	B1	1	For correct expression, in any form
(iv)	Required pgf is $\left(\dfrac{\frac{1}{6}t}{1-\frac{5}{6}t}\right)^4$	B1		For stating fourth power of $\mathrm{G}_W(t)$
	Required probability is the coefficient of t^{24}	B1		For stating or implying the required coeff
	This is $\left(\frac{1}{6}\right)^4 \times \dfrac{(-4)(-5)(-6)\ldots(-23)}{20!} \times \left(\frac{5}{6}\right)^{20}$	M1		For use of appropriate binomial coefficient
	≈ 0.0356	A1	4	For correct value
			$\boxed{13}$	

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OXFORD CAMBRIDGE AND RSA EXAMINATIONS

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MATHEMATICS **4736**

Decision Mathematics 1

Specimen Paper

Additional materials:
- Answer booklet
- Graph paper
- List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages and an insert.

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1 The graph K_5 has five nodes, A, B, C, D and E, and there is an arc joining every node to every other node.

(i) Draw the graph K_5 and state how you know that it is Eulerian. [2]

(ii) By listing the arcs involved, give an example of a **path** in K_5. (Your path must include more than one arc.) [1]

(iii) By listing the arcs involved, give an example of a **cycle** in K_5. [1]

2 This question is about a simply connected network with at least three arcs joining 4 nodes. The weights on the arcs are all different and any direct paths always have a smaller weight than the total weight of any indirect paths between two vertices.

(i) Kruskal's algorithm is used to construct a minimum connector. Explain why the arcs with the smallest and second smallest weights will always be included in this minimum connector. [3]

(ii) Draw a diagram to show that the arc with the third smallest weight need not always be included in a minimum connector. [4]

3 **(i)** Use the shuttle sort algorithm to sort the list

$$6 \quad 3 \quad 8 \quad 3 \quad 2$$

into increasing order. Write down the list that results from each pass through the algorithm. [5]

(ii) Shuttle sort is a quadratic order algorithm. Explain briefly what this statement means. [3]

4 [Answer this question on the insert provided.]

An algorithm involves the following steps.

Step 1: Input two positive integers, A and B.
Let $C = 0$

Step 2: If B is odd, replace C by $C + A$.

Step 3: If $B = 1$, go to step 6.

Step 4: Replace A by $2A$.
If B is even, replace B by $B \div 2$, otherwise replace B by $(B-1) \div 2$.

Step 5: Go back to step 2.

Step 6: Output the value of C.

(i) Demonstrate the use of the algorithm for the inputs $A = 6$ and $B = 13$. [5]

(ii) When $B = 8$, what is the output in terms of A? What is the relationship between the output and the original inputs? [4]

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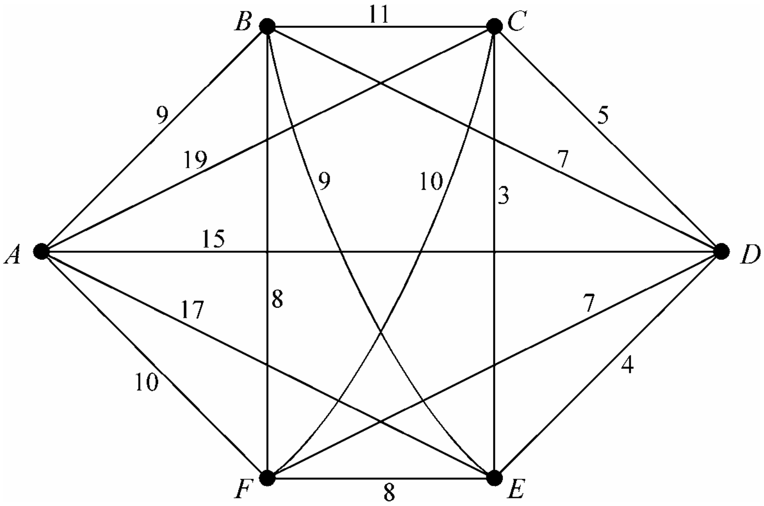
5 [Answer this question on the insert provided.]



In this network the vertices represent towns, the arcs represent roads and the weights on the arcs show the shortest distances in kilometres.

(i) The diagram on the insert shows the result of deleting vertex F and all the arcs joined to F. Show that a lower bound for the length of the travelling salesperson problem on the original network is 38 km. [4]

The corresponding lower bounds by deleting each of the other vertices are:

A : 40 km, B : 39 km, C : 35 km, D : 37 km, E : 35 km .

The route A–B–C–D–E–F–A has length 47 km.

(ii) **Using only this information**, what are the best upper and lower bounds for the length of the solution to the travelling salesperson problem on the network? [2]

(iii) By considering the orders in which vertices C, D and E can be visited, find the best upper bound given by a route of the form A–B– … –F–A. [3]

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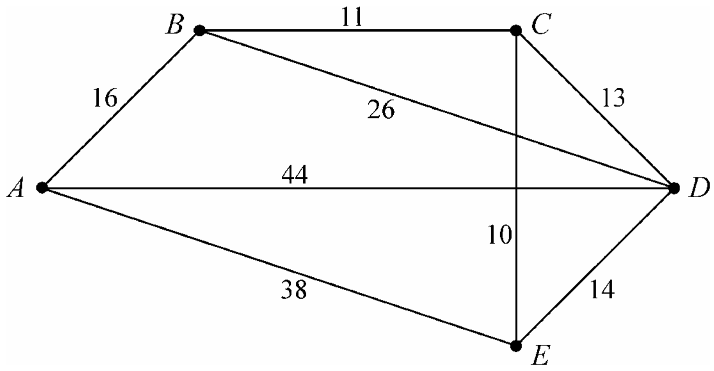
6 [Answer part **(i)** of this question on the insert provided.]

The diagram shows a simplified version of an orienteering course. The vertices represent checkpoints and the weights on the arcs show the travel times between checkpoints, in minutes.



(i) Use Dijkstra's algorithm, **starting from checkpoint** A, to find the least travel time from A to D. You must show your working, including temporary labels, permanent labels and the order in which permanent labels were assigned. Give the route that takes the least time from A to D. [6]

(ii) By using an appropriate algorithm, find the least time needed to travel **every arc** in the diagram starting and ending at A. You should show your method clearly. [6]

(iii) Starting from A, apply the nearest neighbour algorithm to the diagram to find a cycle that visits every checkpoint. Use your solution to find a path that visits every checkpoint, starting from A and finishing at D. [3]

7 Consider the linear programming problem:

$$\begin{array}{ll} \text{maximise} & P = 4y - x, \\ \text{subject to} & x + 4y \leqslant 22, \\ & x + y \leqslant 10, \\ & -x + 2y \leqslant 8, \\ \text{and} & x \geqslant 0, \; y \geqslant 0. \end{array}$$

(i) Represent the constraints graphically, shading out the regions where the inequalities are **not** satisfied. Calculate the value of x and the value of y at each of the vertices of the feasible region. Hence find the maximum value of P, clearly indicating where it occurs. [8]

(ii) By introducing slack variables, represent the problem as an initial Simplex tableau and use the Simplex algorithm to solve the problem. [10]

(iii) Indicate on your diagram for part **(i)** the points that correspond to each stage of the Simplex algorithm carried out in part **(ii)**. [2]

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MATHEMATICS **4736**

Decision Mathematics 1

INSERT for Questions 4, 5 and 6

Specimen Paper

INSTRUCTIONS TO CANDIDATES

- This insert should be used to answer Questions **4**, **5** and **6 (i)**.
- Write your Name, Centre Number and Candidate Number in the spaces provided at the top of this page.
- Write your answers to Questions **4**, **5** and **6 (i)** in the spaces provided in this insert, and attach it to your answer booklet.

This insert consists of 4 printed pages.

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4 **(i)**

STEP	*A*	*B*	*C*
1			
2			

(ii)

STEP	*A*	*B*	*C*
1			
2			

....

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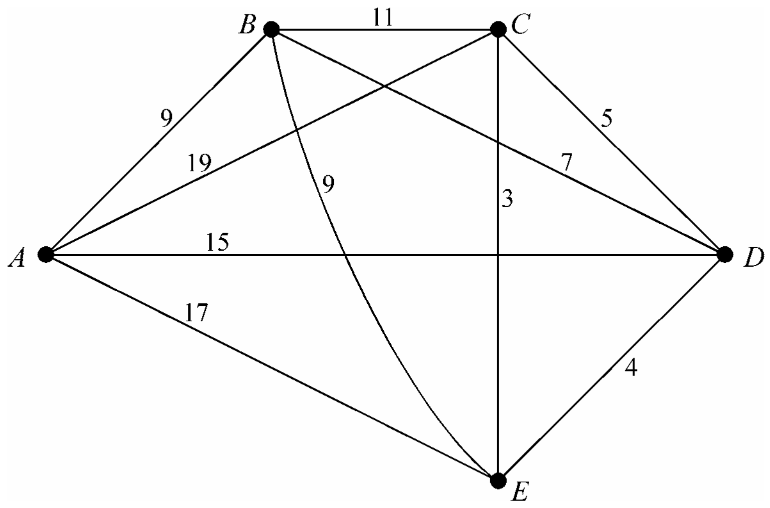
5 **(i)**



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(ii) Upper bound = ... km

Lower bound = ... km

(iii) ....

....

....

....

Best upper bound = ... km

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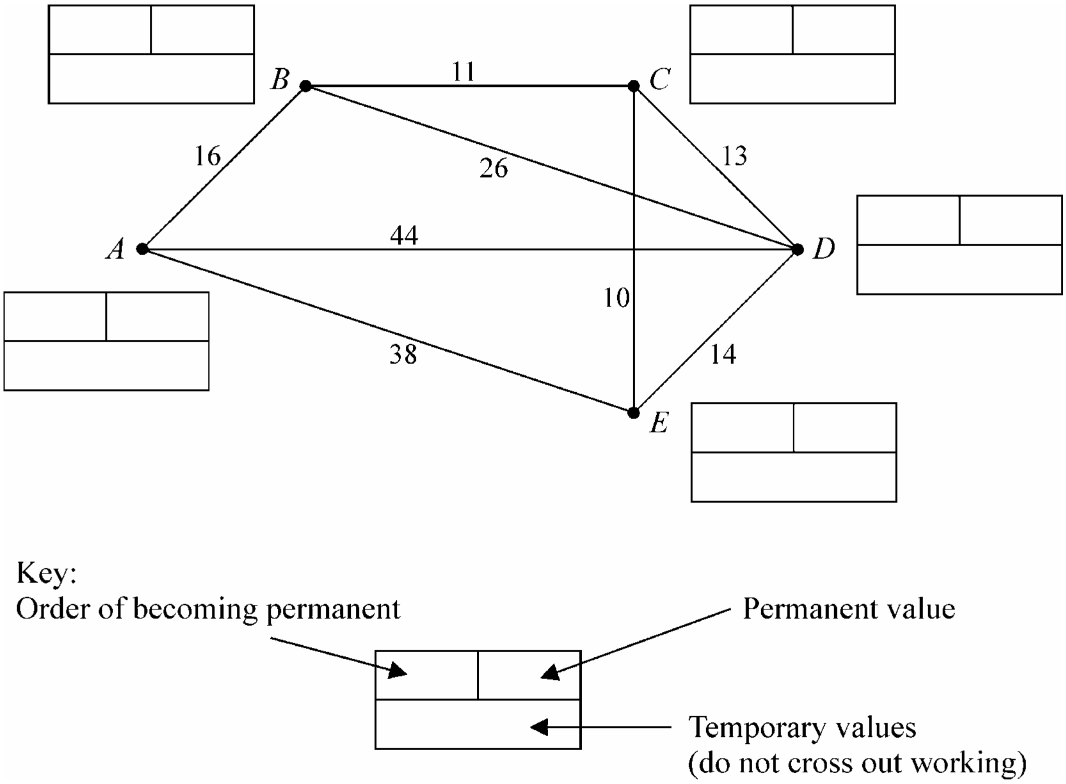
6 **(i)**



....

....

....

....

....

Least travel time = ... minutes

Route: A – .... – D

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MATHEMATICS 4736

Decision Mathematics 1

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

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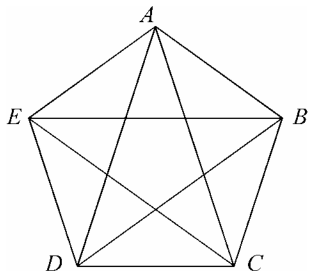
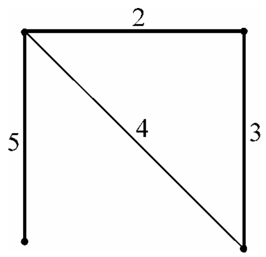
1	**(i)**		B1		For correct graph
		K_5 is Eulerian since every node is even	B1	**2**	For a correct statement
	(ii)	A path is (e.g.) *A–B–C*	B1	**1**	For any correct path
	(iii)	A cycle is (e.g.) *A–B–C–A*	B1	**1**	For any correct cycle
				$\boxed{4}$	
2	**(i)**	Using Kruskal's algorithm, the arc of least weight is chosen first and so is certainly included	B1		For identifying the first choice
		The arc of second least weight is chosen next	B1		For identifying the second choice
		since just two arcs cannot form a cycle	B1	**3**	For correct justification
	(ii)		B1		For any connected graph with 4 nodes and at least 3 arcs
			M1		For including a cycle
			A1		For a network having the required property
			A1	**4**	For making the minimum connector clear
				$\boxed{7}$	
3	**(i)**	1st pass: $\underline{6\ \ 3}\ \ 8\ \ 3\ \ 2$ giving $3\ \ 6\ \ 8\ \ 3\ \ 2$	B1		For correct result of first pass
		2nd pass: $3\ \ \underline{6\ \ 8}\ \ 3\ \ 2$ giving $3\ \ 6\ \ 8\ \ 3\ \ 2$	B1		For correct result of second pass
		3rd pass: $3\ \ 6\ \ \underline{8\ \ 3}\ \ 2$			
		$3\ \ \underline{6\ \ 3}\ \ 8\ \ 2$			
		$\underline{3\ \ 3}\ \ 6\ \ 8\ \ 2$ giving $3\ \ 3\ \ 6\ \ 8\ \ 2$	M1		For correct shuttle process in third pass
		4th pass: $3\ \ 3\ \ 6\ \ \underline{8\ \ 2}$			
		$3\ \ 3\ \ \underline{6\ \ 2}\ \ 8$			
		$3\ \ \underline{3\ \ 2}\ \ 6\ \ 8$	M1		For correct shuttle process in final pass
		$\underline{3\ \ 2}\ \ 3\ \ 6\ \ 8$ giving $2\ \ 3\ \ 3\ \ 6\ \ 8$	A1	**5**	For shuttle sort completed correctly
	(ii)	The number of operations to be carried out, and thus the time to complete the algorithm, is	M1		For idea of dependency on 'size' of problem
		(approximately) proportional to the square of the	A1		For number of operations, or time required
		number of items to be sorted	A1	**3**	For square of list size
				$\boxed{8}$	

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4 (i)

STEP	A	B	C
1	6	13	0
2	6	13	6
4	12	6	6
4	24	3	6
2	24	3	30
4	48	1	30
2	48	1	78
3	48	1	78
6	Output 78		

B1 — For assigning value to C in first Step 2
M1 — For updating A and B in first Step 4
M1 — For continuing algorithm and updating C
A1 — For correct new value 30 for C
A1 **5** — For correct output

(ii)

STEP	A	B	C
1	A	8	0
4	$2A$	4	0
4	$4A$	2	0
4	$8A$	1	0
2	$8A$	1	$8A$
3	$8A$	1	$8A$
6	Output $8A$		

M1 — For values of A doubling
M1 — For values of B halving
A1 — For output $8A$

The output is the product of the inputs — B1 **4** — For identifying multiplication

$\boxed{9}$

5 (i) A minimum connector on reduced network has arcs CE, ED, BD, AB, giving length 23 km
Two shortest arcs from F have weights 7, 8
Hence lower bound is $23 + 7 + 8 = 38$ km

M1 — For attempt at a relevant minimum connector
A1 — For correct weight 23
M1 — For identifying the two shortest arcs at F
A1 **4** — For showing given answer correctly

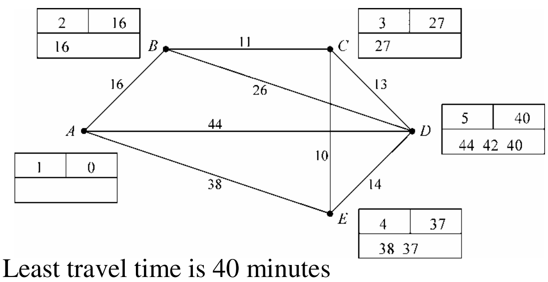
(ii) The best upper bound is 47 km — B1 — For the correct answer
The best lower bound is 40 km — B1 **2** — For the correct answer

(iii) Other orders are CED, DCE, DEC, ECD, EDC
Shortest is $ABDCEFA$, of length 42 km

M1 — For calculation of at least one other length
A1 — For any correct bound less than 47 km
A1 **3** — For the correct value 42

$\boxed{9}$

6 (i)



M1 — For correct use of temporary labels
M1 — For updating E and D
A1 — For all permanent labels correct
B1 — For correct order of assignment stated

Least travel time is 40 minutes — B1 — For correct value 40
Route is A–B–C–D — B1 **6** — For correct route

(ii) The Route Inspection algorithm is used — B1 — For stating or implying the correct algorithm
A, B, C and E are odd nodes — B1 — For identifying the odd nodes

$$AB = 16 \quad AC = 27 \quad AE = 37$$
$$CE = \underline{10} \quad BE = \underline{21} \quad BC = \underline{11}$$
$$26 \quad 48 \quad 48$$

M1 — For pairing odd nodes correctly

Double up on AB and CE — M1 — For selecting appropriate pair for doubling
Sum of arcs is 172 — M1 — For adding weights on all the arcs
Hence shortest time is $172 + 26 = 198$ minutes — A1 **6** — For correct value 198

(iii) Nearest neighbour algorithm gives A–B–C–E–D–A — M1 — For starting the algorithm correctly, up to C
A1 — For the correct cycle A–B–C–E–D–A
Hence required path is A–B–C–E–D — B1 **3** — For a correct path

$\boxed{15}$

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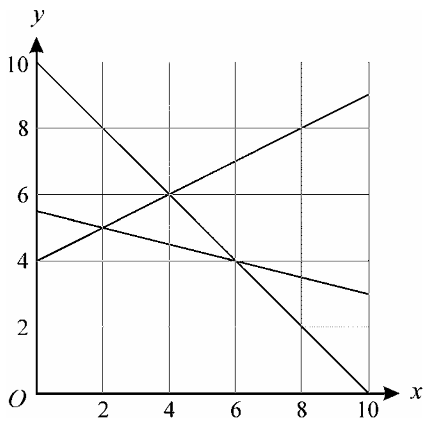
7 **(i)**



Answer	Marks	Total	Guidance
	M1		For lines $x+4y=22$ and $x+y=10$
	M1		For line $-x+2y=8$
	A1		For correct diagram including shading
	B1√		For vertices $(0, 0), (0, 4), (10, 0)$
	B1√		For vertex $(2, 5)$
	B1√		For vertex $(6, 4)$
Hence maximum $P = 18$, occurring at $(2, 5)$	B1		For the correct value 18
	B1	**8**	For identifying the correct vertex

(ii)

P	x	y	s	t	u	
1	1	–4	0	0	0	0
0	1	4	1	0	0	22
0	–1	2	0	0	1	8

B1 For the correct pay-off row
M1 For the use of three slack variables
A1 For all constraints correct

Pivot on 2 in row 3 — M1 For choice of pivot

1	–1	0	0	0	2	16
0	3	0	1	0	–2	6
0	$1\frac{1}{2}$	0	0	1	$-\frac{1}{2}$	6
0	$-\frac{1}{2}$	1	0	0	$\frac{1}{2}$	4

M1 For pivoting correctly
A1√ For correct tableau

Now pivot on 3 in row 1 — M1 For choice of pivot

1	0	0	$\frac{1}{3}$	0	$1\frac{1}{3}$	18
0	1	0	$\frac{1}{3}$	0	$-\frac{2}{3}$	2
0	0	0	$-\frac{1}{2}$	1	$\frac{1}{2}$	3
0	0	1	$\frac{1}{6}$	0	$\frac{1}{6}$	5

M1 For pivoting correctly
A1 For correct tableau

Hence $P = 18$ when $x = 2$, $y = 5$ — B1√ **10** For reading off correctly from final tableau

(iii) Vertices $(0, 0) \to (0, 4) \to (2, 5)$ indicated — M1 For indication of starting at the origin
A1 **2** For the correct correspondence indicated

$\boxed{20}$

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OXFORD CAMBRIDGE AND RSA EXAMINATIONS

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MATHEMATICS **4737**

Decision Mathematics 2

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 6 printed pages, 2 blank pages and an insert.

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1 [Answer this question on the insert provided.]

Six neighbours have decided to paint their houses in bright colours. They will each use a different colour.

- Arthur wants to use lavender, orange or tangerine.
- Bridget wants to use lavender, mauve or pink.
- Carlos wants to use pink or scarlet.
- Davinder wants to use mauve or pink.
- Eric wants to use lavender or orange.
- Ffion wants to use mauve.

Arthur chooses lavender, Bridget chooses mauve, Carlos chooses pink and Eric chooses orange. This leaves Davinder and Ffion with colours that they do not want.

(i) Draw a bipartite graph on the insert, showing which neighbours (A, B, C, D, E, F) want which colours (L, M, O, P, S, T). On a separate diagram on the insert, show the incomplete matching described above. [3]

(ii) By constructing alternating paths obtain the complete matching between the neighbours and the colours. Give your paths and show your matching on the insert. [4]

(iii) Fill in the table on the insert to show how the Hungarian algorithm could have been used to find the complete matching. (You do **not** need to carry out the Hungarian algorithm.) [2]

2 A company has organised four regional training sessions to take place at the same time in four different cities. The company has to choose four of its five trainers, one to lead each session. The cost (£1000's) of using each trainer in each city is given in the table.

		City			
		London	Glasgow	Manchester	Swansea
Trainer	Adam	4	3	2	4
	Betty	3	5	4	2
	Clive	3	6	3	3
	Dave	2	6	4	3
	Eleanor	2	5	3	4

(i) Convert this into a square matrix and then apply the Hungarian algorithm, **reducing rows first**, to allocate the trainers to the cities at minimum cost. [7]

(ii) Betty discovers that she is not available on the date set for the training. Find the new minimum cost allocation of trainers to cities. [2]

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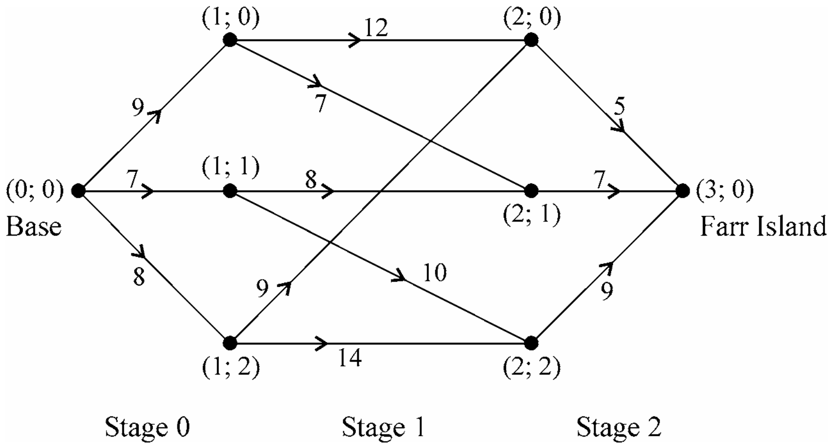
3 [Answer this question on the insert provided.]

A flying doctor travels between islands using small planes. Each flight has a weight limit that restricts how much he can carry. A plague has broken out on Farr Island and the doctor needs to take several crates of medical supplies to the island. The crates must be carried on the same planes as the doctor.

The diagram shows a network with (stage; state) variables at the vertices representing the islands, arcs representing flight routes that can be used, and weights on the arcs representing the number of crates that the doctor can carry on each flight.



(i) It is required to find the route from (0; 0) to (3; 0) for which the minimum number of crates that can be carried on any stage is a maximum (the maximin route). The insert gives a dynamic programming tabulation showing stages, states and actions, together with columns for working out the route minimum at each stage and for indicating the current maximin.

Complete the table on the insert sheet and hence find the maximin route and the maximum number of crates that can be carried. [7]

(ii) It is later found that the number of crates that can be carried on the route from (2; 0) to (3; 0) has been recorded incorrectly and should be 15 instead of 5. What is the maximin route now, and how many crates can be carried? [3]

 [Turn over

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4 Henry is planning a surprise party for Lucinda. He has left the arrangements until the last moment, so he will hold the party at their home. The table below lists the activities involved, the expected durations, the immediate predecessors and the number of people needed for each activity. Henry has some friends who will help him, so more than one activity can be done at a time.

Activity	Duration (hours)	Preceded by	Number of people
A: Telephone other friends	2	–	3
B: Buy food	1	A	2
C: Prepare food	4	B	5
D: Make decorations	3	A	3
E: Put up decorations	1	D	3
F: Guests arrive	1	C, E	1

(i) Draw an activity network to represent these activities and the precedences. Carry out forward and reverse passes to determine the minimum completion time and the critical activities. If Lucinda is expected home at 7.00 p.m., what is the latest time that Henry or his friends can begin telephoning the other friends? [7]

(ii) Draw a resource histogram showing time on the horizontal axis and number of people needed on the vertical axis, assuming that each activity starts at its earliest possible start time. What is the maximum number of people needed at any one time? [3]

(iii) Now suppose that Henry's friends can start buying the food and making the decorations as soon as the telephoning begins. Construct a timetable, with a column for 'time' and a column for each person, showing who should do which activity when, in order than the party can be organised in the minimum time using a total of only six people (Henry and five friends). When should the telephoning begin with this schedule? [3]

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5 [Answer this question on the insert provided.]

Fig. 1 shows a directed flow network. The weight on each arc shows the capacity in litres per second.

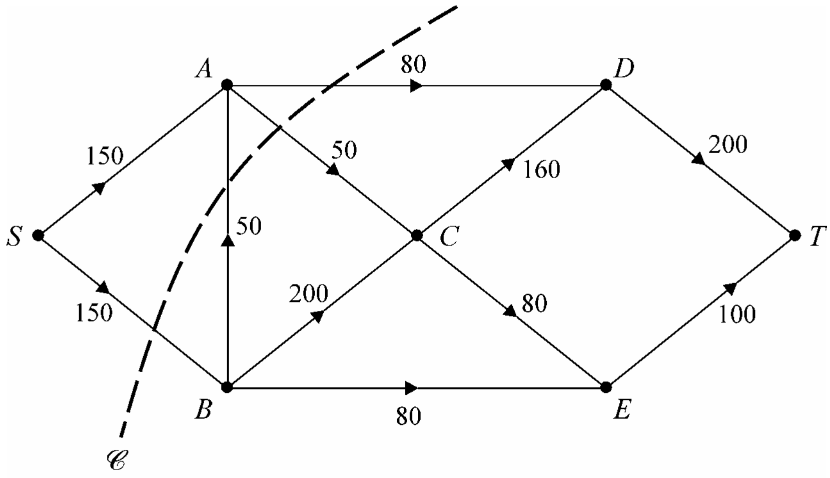

Fig. 1

(i) Find the capacity of the cut 𝒞 shown. [2]

(ii) Deduce that there is no possible flow from S to T in which both arcs leading into T are saturated. Explain your reasoning clearly. [2]

Fig. 2 shows a possible flow of 160 litres per second through the network.

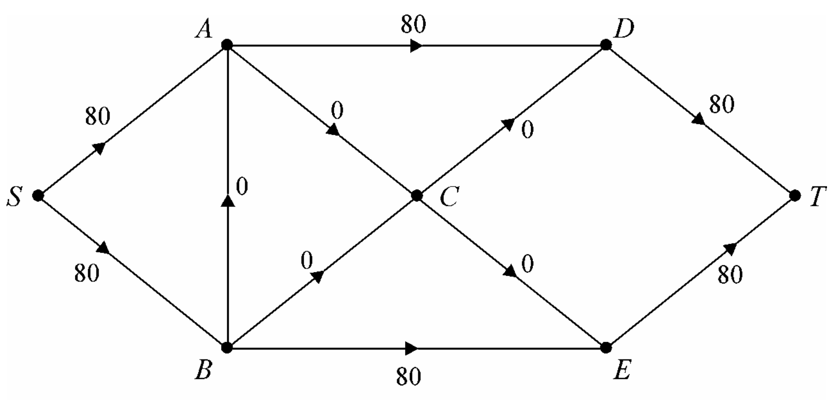

Fig. 2

(iii) On the diagram in the insert, show the excess capacities and potential backflows for this flow. [3]

(iv) Use the labelling procedure to augment the flow as much as possible. Show your working clearly, but do not obscure your answer to part **(iii)**. [4]

(v) Show the final flow that results from part **(iv)**. Explain clearly how you know that this flow is maximal. [3]

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6 Rose is playing a game against a computer. Rose aims a laser beam along a row, A, B or C, and, at the same time, the computer aims a laser beam down a column, X, Y or Z. The number of points won by Rose is determined by where the two laser beams cross. These values are given in the table. The computer loses whatever Rose wins.

		Computer		
		X	Y	Z
Rose	A	1	3	4
	B	4	3	2
	C	3	2	1

(i) Find Rose's play-safe strategy and show that the computer's play-safe strategy is Y. How do you know that the game does not have a stable solution? [3]

(ii) Explain why Rose should never choose row C and hence reduce the game to a 2×3 pay-off matrix. [2]

(iii) Rose intends to play the game a large number of times. She decides to use a standard six-sided die to choose between row A and row B, so that row A is chosen with probability a and row B is chosen with probability $1-a$. Show that the expected pay-off for Rose when the computer chooses column X is $4-3a$, and find the corresponding expressions for when the computer chooses column Y and when it chooses column Z. Sketch a graph showing the expected pay-offs against a, and hence decide on Rose's optimal choice for a. Describe how Rose could use the die to decide whether to play A or B. [6]

The computer is to choose X, Y and Z with probabilities x, y and z respectively, where $x+y+z=1$. Graham is an AS student studying the D1 module. He wants to find the optimal choices for x, y and z and starts off by producing a pay-off matrix for the computer.

(iv) Graham produces the following pay-off matrix.

3	1	0
0	1	2

Write down the pay-off matrix for the computer and explain what Graham did to its entries to get the values in his pay-off matrix. [2]

(v) Graham then sets up the linear programming problem:

maximise $P = p - 4$,

subject to
$$p - 3x - y \leqslant 0,$$
$$p - y - 2z \leqslant 0,$$
$$x + y + z \leqslant 1,$$

and $p \geqslant 0,\ x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0.$

The Simplex algorithm is applied to the problem and gives $x = 0.4$ and $y = 0$. Find the values of z, p and P and interpret the solution in the context of the game. [4]

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MATHEMATICS **4737**

Decision Mathematics 2

INSERT for Questions 1, 3 and 5

Specimen Paper

INSTRUCTIONS TO CANDIDATES

- This insert should be used to answer Questions **1**, **3** and **5**.
- Write your Name, Centre Number and Candidate Number in the spaces provided at the top of this page.
- Write your answers to Questions **1**, **3** and **5** in the spaces provided in this insert, and attach it to your answer booklet.

This insert consists of 4 printed pages.

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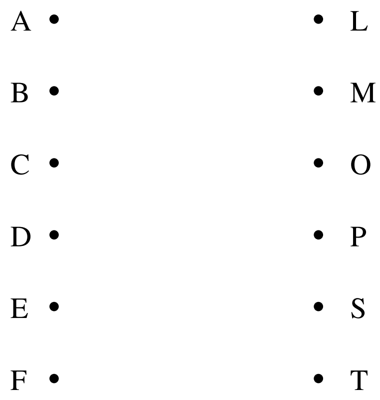
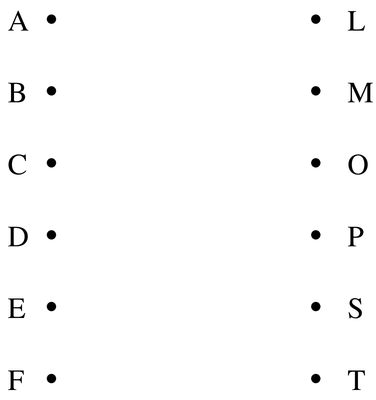
1 **(i)**



Bipartite graph

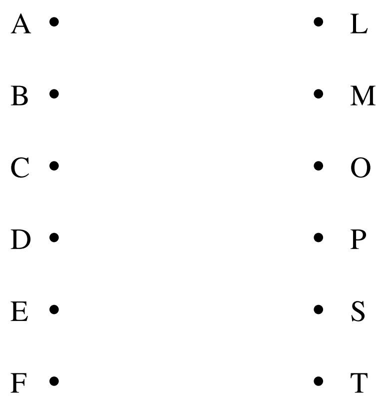


Matching described in question

(ii) ....

....

....



(iii)

	A	B	C	D	E	F
L						
M						
O						
P						
S						
T						

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3 **(i)**

Stage	State	Action	Route minimum	Current maximin
2	0	0		
	1	0		
	2	0		
1	0	0		
		1		
	1	1		
		2		
	2	0		
		2		
0		0		
		1		
		2		

Route: ....

Maximum number of crates that can be carried: ....

(ii) ....

....

....

....

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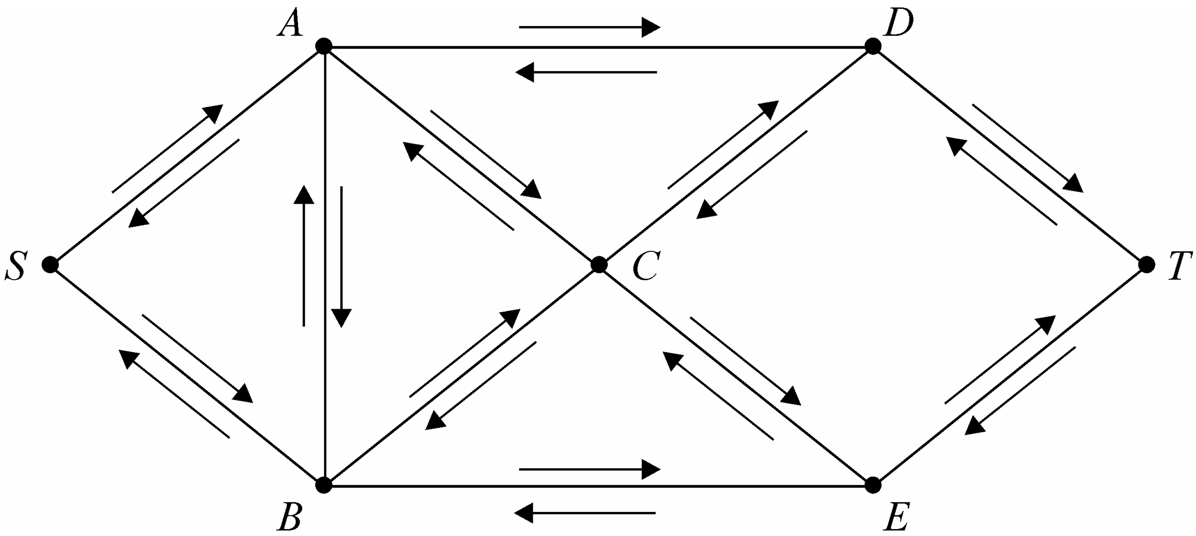
5 (i) Capacity of $\mathscr{C}$: ....

(ii) ....

....

(iii)

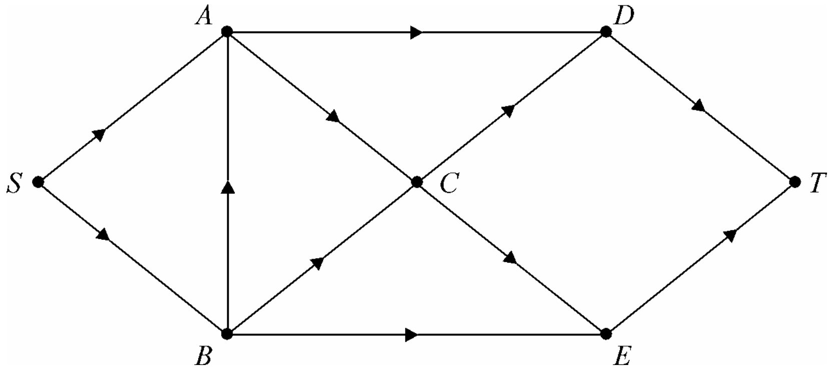


(iv) ....

....

....

(v) Final flow:



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MATHEMATICS **4737**

Decision Mathematics 2

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

This mark scheme consists of 5 printed pages and 3 blank pages.

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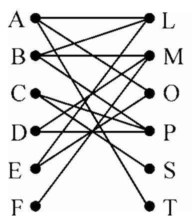
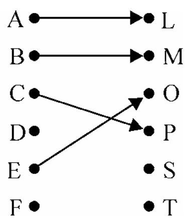
1 (i)



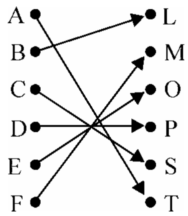


M1 — For attempt at the bipartite graph
A1 — For correct graph
B1 — **3** — For the correct incomplete matching

(ii) Alternating paths are:

	S–C=P–D	and	T–A=L–B=M–F
or	S–C=P–B=M–D	and	T–A=L–B=P–D=M–F
or	S–C=P–B=M–F	and	T–A=L–B=P–D
or	T–A=L–B=M–D	and	S–C=P–D=M–F
or	T–A=L–B=M–F	and	S–C=P–D

M1 — For attempt at an alternating path
A1 — For one correct path
A1 — For the second path correcct



B1 — **4** — For correct matching

(iii)

	A	B	C	D	E	F
L	0	0	1	1	0	1
M	1	0	1	0	1	0
O	0	1	1	1	0	1
P	1	0	0	0	1	1
S	1	1	0	1	1	1
T	0	1	1	1	1	1

M1 — For appropriate zeros and ones (e.g.) to correspond with minimum cost matching
A1 — **2** — For a correct table

$\boxed{9}$

2 (i) Adding a dummy column gives $\begin{matrix} 4 & 3 & 2 & 4 & 6 \\ 3 & 5 & 4 & 2 & 6 \\ 3 & 6 & 3 & 3 & 6 \\ 2 & 6 & 4 & 3 & 6 \\ 2 & 5 & 3 & 4 & 6 \end{matrix}$

B1 — For a dummy column (equal entries, $\geqslant 6$)

Reducing: $\begin{matrix} 2 & 1 & 0 & 2 & 4 \\ 1 & 3 & 2 & 0 & 4 \\ 0 & 3 & 0 & 0 & 3 \\ 0 & 4 & 2 & 1 & 4 \\ 0 & 3 & 1 & 2 & 4 \end{matrix}$ then $\begin{matrix} 2 & 0 & 0 & 2 & 1 \\ 1 & 2 & 2 & 0 & 1 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 3 & 2 & 1 & 1 \\ 0 & 2 & 1 & 2 & 1 \end{matrix}$

M1 — For reducing rows
M1 — For reducing columns

Four lines are needed to cover zeros

M1 — For covering zeros in the reduced matrix

Augmenting: $\begin{matrix} 3 & 0 & 0 & 3 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 0 & 2 & 0 \end{matrix}$ or $\begin{matrix} 3 & 0 & 0 & 2 & 1 \\ 2 & 2 & 2 & 0 & 1 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{matrix}$

M1 — For correct augmentation process

Hence Adam–Glasgow and Betty–Swansea

A1 — For these two allocations correct

and either Clive–Manchester, Dave–London
or Clive–Manchester, Eleanor–London
or Dave–London, Eleanor–Manchester

A1 — **7** — For any one of the correct possibilities

(ii) Without Betty, reduced matrix is $\begin{matrix} 2 & 0 & 0 & 2 \\ 0 & 2 & 0 & 0 \\ 0 & 3 & 2 & 1 \\ 0 & 2 & 1 & 2 \end{matrix}$

M1 — For new reduced matrix

Hence either Adam–Glasgow, Clive–Swansea, Dave–London, Eleanor–Manchester
or Adam–Glasgow, Clive–Manchester, Dave–Swansea, Eleanor–London

A1 — **2** — For either of the correct new possibilities

$\boxed{9}$

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3 (i)

2	0	0	5	5
	1	0	7	7
	2	0	9	9
1	0	0	min(12, 5) = 5	7
		1	min(7, 7) = 7	
	1	1	min(8, 7) = 7	9
		2	min(10, 9) = 9	
	2	0	min(9, 5) = 5	9
		2	min(14, 9) = 9	
0	0	0	min(9, 7) = 7	8
		1	min(7, 9) = 7	
		2	min(8, 9) = 8	

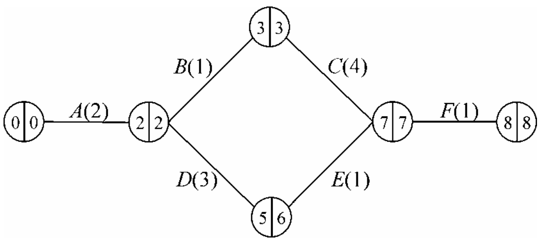
Route is (0; 0)–(1; 2)–(2; 2)–(3; 0)
Maximum number of crates is 8

M1		For dealing with route min column
A1		For at least 6 minima correct
M1		For dealing with maximin column
A1		For Stage 1 section of table all correct
A1		For completely correct table
B1		For correct route
B1	7	For correct number

(ii) New maximin values are 15, 7, 9, 12, 9, 9, 9
Hence new route is (0; 0)–(1; 0)–(2; 0)–(3; 0)
New maximum number of crates is 9

M1		For appropriate re-calculation
A1		For correct new route
A1	3	For correct number
	$\boxed{10}$	

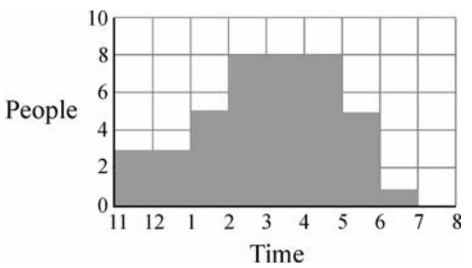
4 (i)



Minimum completion time is 8 hours
Critical activities are *A*, *B*, *C*, *F*
Start telephoning at 11.00 am

B1		For correct arcs and activities (activity on arc network or equivalent with activity at node)
M1		For correct process for forward pass
M1		For correct process for reverse pass
A1		For all early and late times correct
A1		For correct minimum time stated
B1		For correct critical activities
B1√	7	For stating the appropriate time of day

(ii)



Maximum number of people needed is 8

M1		For resource histogram with axes labelled
A1		For correct heights 3, 3, 5, 8, 8, 8, 5, 1
B1	3	For correct number stated

(iii)

Time	H	1	2	3	4	5
10–11	*A*	*A*	*A*	*D*	*D*	*D*
11–12	*A*	*A*	*A*	*D*	*D*	*D*
12– 1		*B*	*B*	*D*	*D*	*D*
1– 2		*C*	*C*	*C*	*C*	*C*
2– 3		*C*	*C*	*C*	*C*	*C*
3– 4		*C*	*C*	*C*	*C*	*C*
4– 5		*C*	*C*	*C*	*C*	*C*
5– 6	*E*	*E*	*E*			
6– 7	*F*					

Start telephoning at 10.00 am

M1		For substantially correct attempt
A1		For a correct schedule
B1	3	For correct time stated
	$\boxed{13}$	

[Turn over

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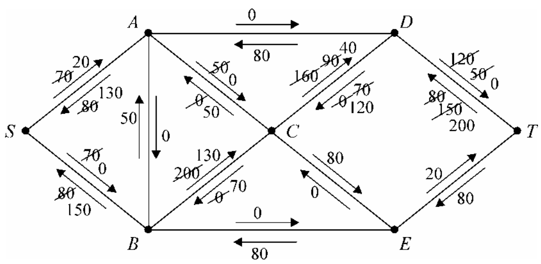
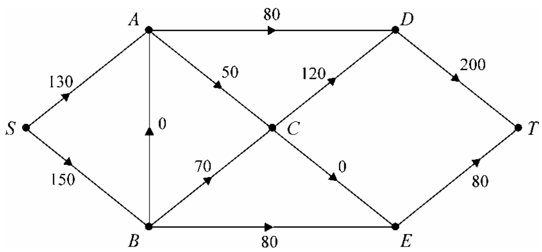
5	(i)	Capacity is $150+0+50+80=280$ litres/sec	M1 A1 **2**	For correct use of zero from *AB* For correct value 280
	(ii)	Maximum flow is $\leqslant 280$ So flow of $200+100=300$ is not possible	M1 A1 **2**	For relevant use of max flow/min cut For completely correct proof
	(iii)		M1 A1 A1 **3**	For correct method for excess and backflow For all initial excess capacities correct For all initial backflows correct
	(iv)	Augment by 70 along *SBCDT* (e.g.) New excesses and backflows are as shown above Now augment by 50 along *SACDT* (e.g.) Final excesses and backflows are as shown above	B1 M1 M1 A1 **4**	For identifying a correct augmentation For modifying excesses and backflows For continuing the process as far as possible For a completely correct solution
	(v)		B1√	For showing the augmented flow correctly
		The value of the augmented flow is 280 litres/sec, and so is the maximum possible	M1 A1 **3**	For comparing with results from **(i)** or **(ii)** For a completely correct explanation (Or equivalent explanation based on the disconnectedness of *S* and *T* in **(iii)**)
			$\boxed{14}$	

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6 **(i)**

	X	Y	Z	row min	
A	1	3	4	1	
B	4	3	2	2	←
C	3	2	1	1	
– col max	–4	–3	–4		
		↑			

Play-safe for Rose is B — B1 — For correct statement

Play-safe for computer is Y — B1 — For correct statement

Not stable as $-3+2\neq 0$ — B1 — **3** — For use of max row min and min col max

(ii) Row C is dominated by row B — B1 — For correct statement or explanation

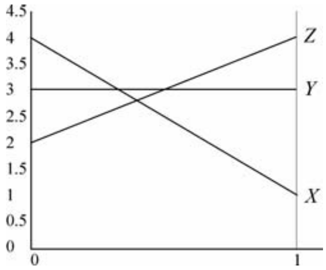
	X	Y	Z
A	1	3	4
B	4	3	2

B1 — **2** — For new matrix

(iii) Expected pay-off with X is $1\times a+4(1-a)=4-3a$ — B1 — For showing given answer correctly

and with Y is $3a+3(1-a)=3$ — B1 — For correct value 3

and with Z is $4a+2(1-a)=2+2a$ — B1 — For correct expression $2+2a$



B1$\surd$ — For correct diagram

Required value of a is 0.4 — B1$\surd$ — For correct value

Score of 1 or 2: play A; score of 3, 4 or 5: play B;
Throw die again if it shows a six — B1$\surd$ — **6** — For a correct decision rule

(iv)

	X	Y	Z
A	–1	–3	–4
B	–4	–3	–2

B1 — For correct pay-off matrix for the computer

Add 4 to each value — B1 — **2** — For a correct explanation

(v) $z=0.6$ — B1 — For the correct value of z

$p=1.2\Rightarrow P=-2.8$ — B1 — For the correct values of both p and P

The computer should choose X with probability 0.4 and Z with probability 0.6 — B1 — For a correct description of the strategy

On average the computer will lose no more than 2.8 points per game — B1 — **4** — For a correct interpretation of P

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